

# System Analysis and Modeling of a Robotic Cell

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## Abstract

This paper studies a robotic cell for automatic inspection of precise cylindrical parts. The cell is considered as a complex object. The analysis and modeling of functions of the cell are performed from a system point of view. Conceptual functional-purposeful information, including general-system and subsystem functions and goals are defined. A system graph-model and a formal mathematical model of the cell are proposed.

**Keywords:** robotics, manufacturing systems, system models.

## 1 Introduction

An ever increasing usage of robotic systems to solve problems of production automation is noticed in the contemporary industry. The necessity of precise parts in machinery and scientific instruments makes it necessary to solve the problems of their quality inspection within the manufacturing stage. It is expedient the inspection processes to be realized by specialized robotic complexes which could provide the highest degree of automation, to exclude subjective factors and to maximize the authenticity of the inspection results.

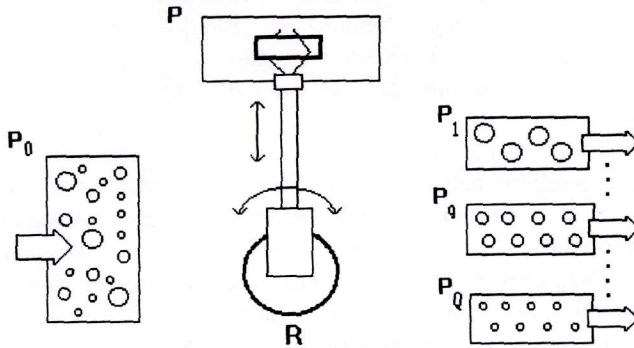
Successful design of complex systems such as flexible manufacturing systems and unmanned manufacture is possible only with the help of a computer-aided modeling and simulation of their operations. In the last years many researchers have directed their studies to this field. Different methods and tools of modeling and analysis of the manufacture are examined in (Naylor, 1987). Software tools for simulation and comprehensive design of such systems can be found in (Drake, 1995).

In this paper a robotic cell for inspection and classification of cylindrical parts is examined. The aim of the study is a formal mathematical model of the cell functioning to be developed. As a result of a system modeling general-system and subsystem functions and goals are formally described. The proposed functional model takes into account interactions between subsystems and their connections with the environment. It can be used for the evaluation of the system design working capacity and of the adopted performance criteria before the physical creation of the cell itself.

## 2 A robotic cell

A robotic cell for automatic inspection of the inner cylindrical surface of precise machine parts and a classification in accordance to their size is depicted in Fig.1. The

cell consists a measuring unit P, a transport robot R, an incoming conveyor P<sub>0</sub>, and outgoing conveyors P<sub>q</sub>, q = 1,2,...Q.



**Fig.1:** robotic cell for inspection of cylindrical parts

The cell operates as follows:

- a) A flow of mixed cylindrical parts arrives at the input P<sub>0</sub> of the cell;
- b) The robot moves each individual part to the measuring unit P;
- c) The inner surface of every part is measured in discrete points;
- d) The obtained measurement information is processed and the classification group q of the part is determined;
- e) The robot moves the part to an outgoing conveyor P<sub>q</sub>, corresponding to the classification group, q = 1,2,...Q.

### 3 System Model Information Concept

The concept of the functional-purposeful information for the cell contains the general-system and subsystem functions and goals of control. The information is defined as:

General system function : post-operational quality control;

General system purpose: classification of cylindrical parts.

Subsystem functions: transportation of the cylindrical parts, acquisition and processing of the measurement information;

Subsystem purpose: classification of the parts in size groups.

The cell is considered as composed of functional subsystems. The relevant information needed for the three subsystems "Transportation", "Measurement" and "Classification" is determined as:

#### 3.1 For Subsystem "Transportation"

a/ subsystem function - moving of details into defined points of operation space ,

b/ subsystem purpose - ensuring motion of physical flow.



### 3.2 For Subsystem "Measurement"

- a/ subsystem function - realization of measuring path ,
- b/ subsystem purpose - obtaining measurement information.

### 3.3 For Subsystem "Classification" :

- a/ subsystem function - processing of measurement information ,
- b/ subsystem purpose - determination of the classification group of every part.

## 4 A Model of the First Level of Integration

### 4.1 System Variables

Conditional variables of a subsystem  $S^m$  to reflect its performance and the interaction with the environment are shown in Fig. 2. :

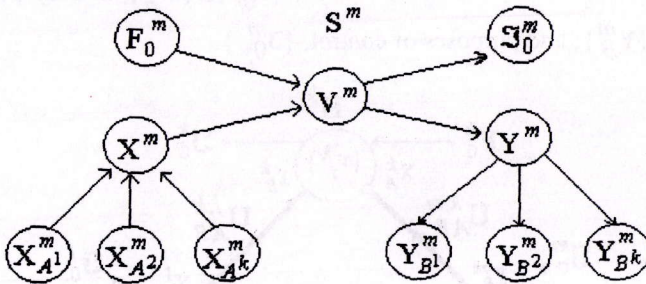


Fig.2: scheme of connections of model variables of a subsystem  $S^m$  and its environment

Vectors of independent variables at the input of the subsystem  $S^m$  :

$$F_0^m = \{F_{0r}^m; r = 1, 2, \dots, n_F^m\} \quad (1)$$

$$X^m = \{X_A^m; A = 1, 2, \dots, n_A^m\} \quad (2)$$

$$X_A^m = \{x_{A\alpha}^m; \alpha = 1, 2, \dots, n_{A\alpha}^m\} \quad (3)$$

Vectors of dependent variables at the output of the subsystem  $S^m$ :

$$S_0^m = \{S_{0\gamma}^m; \gamma = 1, 2, \dots, n_S^m\} \quad (4)$$

$$Y^m = \{Y_B^m; B = 1, 2, \dots, n_B^m\} \quad (5)$$

$$Y_B^m = \{y_{B\beta}^m; \beta = 1, 2, \dots, n_{B\beta}^m\} \quad (6)$$

Vector of internal variables of the subsystem  $S^m$ :

$$\mathbf{V}^m = \{v_i^m; i = 1, 2, \dots, n_v^m\} \quad (7)$$

where  $n_\varphi^m$ ,  $\varphi = F, A, A\alpha, \mathfrak{I}, B, B\beta, v$  are the cardinalities of the corresponding sets,  $m = 1, 2, \dots, N_m$ ,  $N_m$  is the number of the subsystems in a complex system.

#### 4.2 System Graph

The system graph  $G_{L=1}^\Sigma$  of the first level ( $L=1$ ) of integration is shown in Fig.3. (Vavilov, 1983). The first level of a functional-purposeful integration realizes an incorporation of subsystem variables  $\{v_i^m\}$  in subsystems  $S^m$ . These subsystems take into account the influence of the environment  $\{F_0^m\}$ ,  $\{X_A^m\}$  as they perform their own functions,  $\{Y_B^m\}$ , and purposes of control,  $\{\mathfrak{I}_0^m\}$ .

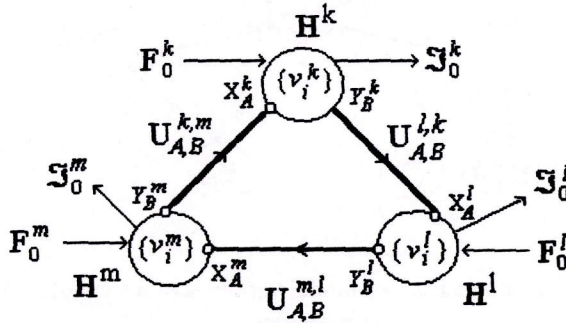


Fig.3: system graph of first level of integration built on three subsystems

The system graph  $G_{L=1}^\Sigma$  is defined as:

$$G_{L=1}^\Sigma = \langle \{ H^m; m = \overline{1, N_m} \}; \{ U_{AB}^{mk}; m, k = \overline{1, N_{mk}} \}; m \neq k \rangle \quad (8)$$

where  $\{H^m\}$  is a set of system vertices,  $\{U_{AB}^{mk}\}$  is a set of system edges,  $N_m$ ,  $N_{mk}$  are the cardinalities of the corresponding sets.

The vertices  $rH^m$  represent the functional subsystems  $S^m$  for which  $\{Y^m\}$  and  $\{X^m\}$  are sets of output and input vector variables,  $\{V^m\}$  is the vector set of the internal variables:

$$H^m = \langle \{Y^m\}; \{V^m\}; \{X^m\} \rangle \quad (9)$$

A system edge of the system graph (1) is defined as:

$$U_{AB}^{mk} = \langle \{x_{A\alpha}^m; \alpha \in [1, n_{A\alpha}^m]\}; \{y_{B\beta}^k; \beta \in [1, n_{B\beta}^k]\}; W_{AB}^{mk} \rangle \quad (10)$$

The system edge connects the system vertices  $H^m$  and  $H^k$  and represents a new graph of a connection with output vertices  $\{X_A^m\}$  and input vertices  $\{Y_B^k\}$  and an operator relation:

$$X_A^m = W_{A,B}^{m,k} Y_B^k \quad (11)$$

where  $W_{A,B}^{m,k}$  is a transfer operator.

### 4.3 System Formalism

The input-output connections for the system  $S^m$  is given by:

$$A_{VV}^m V^m = B_{VX}^m X^m + B_{VF}^m F_0^m \quad (12)$$

$$C_{YV}^m V^m = Y^m \quad (13)$$

$$C_{ZV}^m V^m = Z_0^m \quad (14)$$

where  $A_{VV}^m$  is  $(n_V^m \times n_V^m)$  square matrix over the internal variables  $V^m$  for the subsystem  $S^m$ ;

$B_{VX}^m$ ,  $B_{VF}^m$  are  $(n_V^m \times n_A^m)$ ,  $(n_V^m \times n_F^m)$  rectangular transfer matrixes of the connection of the environment with the input of the subsystem  $S^m$ ;

$C_{YV}^m$ ,  $C_{ZV}^m$  are  $(n_B^m \times n_V^m)$  and  $(n_Z^m \times n_V^m)$  rectangular transfer matrixes of the connection of the output of the subsystem  $S^m$  with the environment.

The relations (12), (13), (14) with respect to the output system variables are:

$$V^m = \Phi_{VX}^m X^m + \Phi_{VF}^m F_0^m \quad (15)$$

$$Y^m = \Phi_{YX}^m X^m + \Phi_{YF}^m F_0^m \quad (16)$$

$$Z_0^m = \Phi_{ZX}^m X^m + \Phi_{ZF}^m F_0^m \quad (17)$$

The matrix form of is:



$$\begin{bmatrix} \mathbf{V}^m \\ \mathbf{Y}^m \\ \mathfrak{Z}_0^m \end{bmatrix} = \begin{bmatrix} \Phi_{VX}^m & \Phi_{VF}^m \\ \Phi_{YX}^m & \Phi_{YF}^m \\ \Phi_{\mathfrak{Z}X}^m & \Phi_{\mathfrak{Z}F}^m \end{bmatrix} \begin{bmatrix} \mathbf{X}^m \\ \mathbf{F}_0^m \end{bmatrix} \quad (18)$$

where  $\Phi_{ij}^m$ ,  $i = V, Y, \mathfrak{Z}$ ;  $j = X, F$  are transfer matrix operators:

$$\begin{aligned} \Phi_{VX}^m &= \mathbf{D}^m (\mathbf{A}_{VV}^m)^* \mathbf{B}_{VX}^m, & \Phi_{VF}^m &= \mathbf{D}^m (\mathbf{A}_{VV}^m)^* \mathbf{B}_{VF}^m \\ \Phi_{YX}^m &= \mathbf{C}_{YV}^m \mathbf{D}^m (\mathbf{A}_{VV}^m)^* \mathbf{B}_{VX}^m, & \Phi_{YF}^m &= \mathbf{C}_{YV}^m \mathbf{D}^m (\mathbf{A}_{VV}^m)^* \mathbf{B}_{VF}^m \\ \Phi_{\mathfrak{Z}X}^m &= \mathbf{C}_{\mathfrak{Z}V}^m \mathbf{D}^m (\mathbf{A}_{VV}^m)^* \mathbf{B}_{VX}^m, & \Phi_{\mathfrak{Z}F}^m &= \mathbf{C}_{\mathfrak{Z}V}^m \mathbf{D}^m (\mathbf{A}_{VV}^m)^* \mathbf{B}_{VF}^* \end{aligned}$$

with  $\det \mathbf{A}_{VV}^m = (D^m)^{-1}$ ,  $D^m$  is a weight of the subsystem  $S^m$ ,  $m = 1, 2, \dots, N_m$ .

## 5 System Analysis of the Robotic Cell

The robotic cell (Fig. 1) is considered as a complex object. The system analysis is carried out on the basis of the system concept, as described above. The aim of the analysis is to transform the concept model information, given in section 3 to a relevant system model architecture (Tzvetkova, 1984). It leads to construction of a system model of the first level of functional-purposeful integration, as described in section 4.

### 5.1 Subsystem "Transportation", ( $S^1$ )

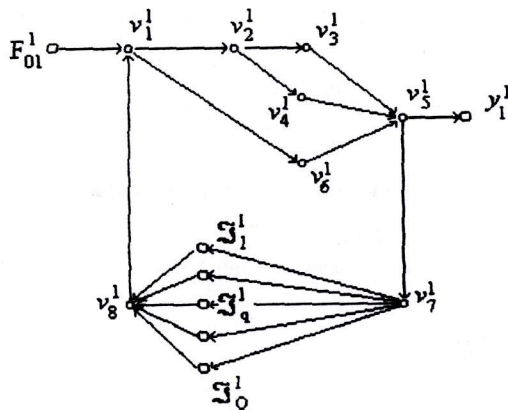


Fig. 4: system graph-model of subsystem "Transportation"

#### 5.1.1 Input Variables

Vector of the environment  $F_0^1$ :

$$F_0^1 = \{F_{01}^1\}$$

where  $F_{01}^1$  is considered as an environment of the subsystem  $S^1$  and indicate a part at the input of the cell.

Vector variables from other subsystems  $X^1$ :

$$X^1 = \{X_A^1; A = 1\} = \{X_1^1\} = [x_{11}^1 \ x_{12}^1]$$

where  $x_{11}^1$  is a task to transport the part to the measuring unit P,

$x_{12}^1$  is a task to transport the part to the corresponding outgoing conveyor  $P_q$ ,  $q = 1, 2, \dots, Q$ .

### 5.1.2 Internal Variables

Vector of the internal variables (7) is:

$$V^1 = \{v_1^1, v_2^1, v_3^1, v_4^1, v_5^1, v_6^1, v_7^1\}$$

where  $v_1^1$  - robot is in its initial position,  $v_2^1$  - robot moves to the input conveyor,  $v_3^1$  - robot grasps the part,  $v_4^1$  - robot moves the part to the measurement unit,  $v_5^1$  - classification task,  $v_6^1$  - robot moves the part to the corresponding outgoing conveyor,  $v_7^1$  - robot moves to its initial position.

### 5.1.3 Output Variables

Output vector variables  $Y^1$  for other subsystems:

$$Y^1 = \{Y_B^1; B = 1\} = \{Y_1^1\} = \{y_{1\gamma}^1; \gamma = 1\} = y_{11}^1$$

where  $y_{11}^1$  is an information variable that the part is at the measuring unit.

The output of the complex is:

$$\mathfrak{Z}_0^1 = \{\mathfrak{Z}_{0\gamma}^1; \gamma = 1, 2, \dots, q, \dots, Q\} = \{\mathfrak{Z}_1^1, \dots, \mathfrak{Z}_q^1, \dots, \mathfrak{Z}_Q^1\}$$

where  $\mathfrak{Z}_q^1$  - space coordinates of the  $q$ -th outgoing conveyor.

The operator input-output relations are:

$$V^1 = \Phi_{VX}^1 X^1 + \Phi_{VF}^1 F_0^1$$

$$Y^1 = \Phi_{YF}^1 F_0^1$$

$$\mathfrak{Z}_0^1 = \Phi_{\mathfrak{Z}F}^1 F_0^1$$

(19)

$$\begin{bmatrix} \mathbf{V}^1 \\ \mathbf{Y}^1 \\ \mathfrak{J}_0^1 \end{bmatrix} = \begin{bmatrix} \Phi_{VX}^1 & \Phi_{VF}^1 \\ 0 & \Phi_{YF}^1 \\ 0 & \Phi_{\mathfrak{J}F}^1 \end{bmatrix} \begin{bmatrix} \mathbf{X}^1 \\ \mathbf{F}_0^1 \end{bmatrix} \quad (20)$$

## 5.2 Subsystem "Measurement", ( $S^2$ )

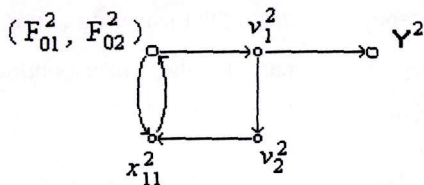


Fig.5: system graph-model of subsystem "Measurement"

### 5.2.1 Input Variables

Variables from the environment  $\mathbf{F}_0^2$ :

$$\mathbf{F}_0^2 = \{\mathbf{F}_{0\gamma}^2; \gamma = 1,2\} = \{F_{01}^2, F_{02}^2\}$$

where  $F_{01}^2, F_{02}^2$  are variables considered as external environment for the subsystem  $S^2$ ,

$F_{01}^2$  is a number of measurements at a cross direction of a cylindrical part,

$F_{02}^2$  is number of measurements at a longitudinal direction of a cylindrical part.

Input variables from other subsystems  $\mathbf{X}^2$ :

$$\mathbf{X}^2 = \{\mathbf{X}_A^2; A = 1\} = \{x_{A\alpha}^2; \alpha = 1\} = x_{11}^2$$

where  $x_{11}^2$  is a variable to give a permission to start a process of measurements.

### 5.2.2 Internal Variables

$$\mathbf{V}^2 = \{v_1^2, v_2^2\}$$

where  $v_1^2$  - performs a single measurement at a point of the profile of the part,  $v_2^2$  - movement to the next point to measure.

### 5.2.3 Output Variables

Vector of output variables to other subsystems  $\mathbf{Y}^2$ :



$$\mathbf{Y}^2 = \{\mathbf{Y}_B^2; B=1\} = \{Y_{B\beta}^2; \beta=1, \dots, NL\} = [y_{11}^2, \dots, y_{1\beta}^2, \dots, y_{1,NL}^2]$$

where  $y_{1\beta}^2$  are point coordinates of the profile of the part,  $NL$  is the number of measurement points of the whole cylindrical surface.

Operator input-output relation:

$$\begin{aligned} \mathbf{V}^2 &= \Phi_{VX}^2 \mathbf{X}^2 + \Phi_{VF}^2 \mathbf{F}_0^2 \\ \mathbf{Y}^2 &= \Phi_{YX}^2 \mathbf{X}^2 + \Phi_{YF}^2 \mathbf{F}_0^2 \end{aligned} \tag{21}$$

$$\begin{bmatrix} \mathbf{V}^2 \\ \mathbf{Y}^2 \\ 0 \end{bmatrix} = \begin{bmatrix} \Phi_{VX}^2 & \Phi_{VF}^2 \\ \Phi_{YX}^2 & \Phi_{YF}^2 \\ 0 & \Phi_{ZF}^2 \end{bmatrix} \begin{bmatrix} \mathbf{X}^2 \\ \mathbf{F}_0^2 \end{bmatrix} \tag{22}$$

### 5.3 Subsystem "Classification", ( $S^3$ )

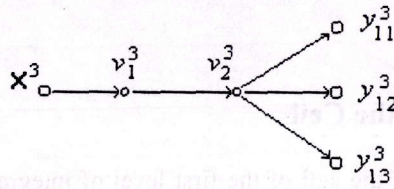


Fig.6: system graph-model of subsystem "Classification"

#### 5.3.1 Input Variables, $\mathbf{X}^3$ :

$$\mathbf{X}^3 = \{\mathbf{X}_A^3; A=2\} = \{x_{A\alpha}^3; \alpha=1, \dots, NL\} = [x_{21}^3 \dots x_{2\alpha}^3 \dots x_{2,NL}^3]$$

where  $[x_{11}^3 \dots x_{1\alpha}^3 \dots x_{1,NL}^3]$  is an array of the point value measurements of the cylindrical surface received from the subsystem  $S^2$ .

#### 5.3.2 Internal Variables

$$\mathbf{V}^3 = \{v_1^3, v_2^3\}$$

where  $v_1^3$  - processes an array of measurement information,  $v_2^3$  - determines the group of size of a part.

5.3.3 Output Variables,  $Y^3$ :

$$Y^3 = \{Y_B^3; B = 1\} = \{y_{B\beta}^3; \beta = 1, \dots, NL\} = [y_{11}^3 \dots y_{1\beta}^3 \dots y_{1Q}^3]$$

where  $y_{1\beta}^3$ ,  $\beta = 1, \dots, Q$  is the size group of the tested part.

Input-output operator relation:

$$V^3 = \Phi_{VX}^3 X^3 \tag{23}$$

$$Y^3 = \Phi_{YX}^3 X^3 \tag{24}$$

$$\begin{bmatrix} V^3 \\ Y^3 \\ 0 \end{bmatrix} = \begin{bmatrix} \Phi_{VX}^3 & 0 \\ \Phi_{YX}^3 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X^3 \\ 0 \end{bmatrix} \tag{24}$$

## 6 System Modeling of the Cell

The system graph-model of the cell of the first level of integration is shown in Fig.7 (Tzvetkova, 1995):

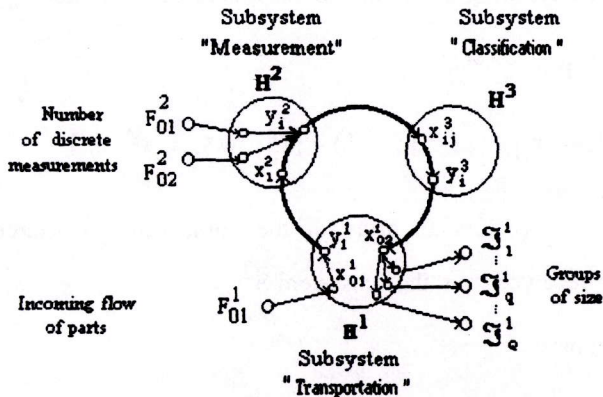


Fig.7: system graph-model of the cell of first level of integration



Vertices of the system graph-model  $H^1, H^2, H^3$  represent the subsystem functions of cell and edges are associated with exchange of information among them. Variables of the graph-model have the following meaning:

$$H^1: \begin{cases} F_{01}^1 - \text{presence of a part at the input } P_0; \\ x_{11}^1 - \text{task for transportation to a conveyor } P_q; \\ y_{11}^1 - \text{the part is on the measuring unit } P; \\ z_1^1 \dots z_q^1 \dots z_Q^1 - \text{coordinates of conveyors } P_q. \end{cases}$$

$$H^2: \begin{cases} F_{01}^2 - \text{number of measurements, } N, \text{ in a cross} \\ \text{direction of a cylindrical surface;} \\ F_{02}^2 - \text{number of measurements, } L, \text{ in a} \\ \text{longitudinal direction;} \\ x_1^2 - \text{measurement is allowable;} \\ Y_1^2 = \{y_{11}^2, \dots, y_{N \times L}^2\} - \text{measurements of } N \times L \\ \text{points of a cylindrical surface.} \end{cases}$$

$$H^3: \begin{cases} X_1^3 = \{x_{ij}^3\} - \text{an array of measurements;} \\ y_1^3 - \text{number } q, q = 1, 2, \dots, Q, \text{ of the size} \\ \text{group of a tested part.} \end{cases}$$

The structure of the system model is defined by (6) and its variable are determined in section 5.

$$\begin{bmatrix} y_{11}^1 \\ \underline{y}_1^2 \\ \underline{y}_1^3 \\ \underline{z}_1^1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \Gamma_{YX}^1 & 0 & 0 & \Gamma_{YF}^1 & 0 & 0 \\ 0 & \Gamma_{YX}^2 & 0 & 0 & \Gamma_{YF}^2 & 0 \\ 0 & 0 & \Gamma_{YX}^3 & 0 & 0 & \Gamma_{YF}^3 \\ \Gamma_{ZX}^1 & 0 & 0 & \Gamma_{ZF}^1 & 0 & 0 \\ 0 & \Gamma_{ZX}^2 & 0 & 0 & \Gamma_{ZF}^2 & 0 \\ 0 & 0 & \Gamma_{ZX}^3 & 0 & 0 & \Gamma_{ZF}^3 \end{bmatrix} \begin{bmatrix} \underline{x}_1^1 \\ x_{11}^2 \\ \underline{x}_1^3 \\ F_1^1 \\ \underline{F}^2 \\ 0 \end{bmatrix}$$

where  $\Gamma_{ij}^m$  means a transitive closure between the variables of the subsystem  $S^m$ ,  $m = 1, 2, 3$ .

## **7 Conclusion**

A system model first level of functional-purposeful integration is developed. The model takes into account interactions between subsystems and their connections with the environment. An operator model of a robotic cell for inspection and classification of cylindrical parts is proposed.

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