An Approach to Solution of the Dark Energy Problem

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Abstract

The problem is to explain the real physical source of the dark energy and what the value of the rate of the expansion is. In that sense, the analytical expression of $\Lambda = (GM/r^2c^2)^2$ has been derived as the function of the universe mass M and radius r. Meanwhile, the Einstein's field equations require that Λ should be a constant. Therefore, the structure of the universe should include a new mass-radius balancing constant $K_{mr} = M/r^2$, with the consequence that universe mass is producing proportional to r^2. Related scalar field, determined by Λ, is the real physical source of the dark energy. The velocity and acceleration equations show the value of the rate of the accelerated expansion. Thus, this model solves the dark energy problem, the initial singularity problem and gives a unique solution: a closed universe. It should be verified by the observation data.

Keywords: Dark Energy, Cosmological Constant, Mass-Radius Balancing Constant, Einstein's Field Equations, Dynamics of the Universe Motion.

1 Introduction

There are a lot of dynamics models of the universe motion [1-23] that have been derived in order to explain the present accelerated expansion of the universe and also to find out the answers to the following questions (Steinhard and Turok, 2002 [8]): what occurred at the initial singularity?, how old is the universe?, how big is the universe? , and what is its ultimate fate? The main problem is in the differences between expected critical density and observed present density of the universe matter. Namely, the density of the luminous matter (Ω_{lum}) of the universe is now less than one percent of the total density (Ω_{tot}) of all forms of matter and energy put together. The remaining 99% or more consists of dark mater and energy, which can not be seen directly, but are inferred to exist from their gravitational influence on luminous matter, as well as the geometry of the universe. The recent observations of distant supernovae and CMB fluctuations increasingly imply that dark energy is real and its present energy density ($\Omega_{\Lambda,0}$) exceeds that of all other forms of matter ($\Omega_{m,0}$) and radiation ($\Omega_{r,0}$) put together [493 ref. in 4].

A dark energy has many alternative names (the zero-point field, vacuum energy, quintessence, the cosmological constant Λ, phantom scalar field and so on). Recently in [23] it is shown to successfully rule out a wide range of non – Λ dark energy models. Thus, if we consider that the cosmological constant Λ is the dark energy then we have to have an analytic expression for Λ in terms of time or other cosmological parameters.

This analytical expression of Λ has been derived and presented in our previous papers (CASYS'03, CASYS'05) in the form $\Lambda = (GM/r^2c^2)^2$, where G, M, r and c are Newton's gravitational constant, a universe mass, a universe radius and the speed of the light in vacuum, respectively. This relation tells us that Λ is a function of the universe radius. Meanwhile, in the Einstein's field equations Λ should be a constant if the conservation lows are valid for the universe. This means that the structure of the universe includes a new mass-radius balancing constant $K_{mr} = M / r^2$, with the consequence that universe mass is producing following the law: $M = \Lambda^{1/2} c^2 r^2 / G$. The physical meaning of K_{mr} is the mass-space conservation. Thus, one can say: a mass needs a space and a space needs a mass. In that context Λ is an important property of the universe that is strongly coupled with the universe mass and radius (i.e. with the universe energy and geometry) and has a fundamental role in the control of the universe dynamics. In fact, Λ is the composition of the three fundamental universe constants: the Newton's gravitational constant G, the Einstein's speed of light constant c and the mass-space conservation constant K_{mr}. In that sense, one can describe the cosmological constant with the form $\Lambda = (G K_{mr} / c^2)^2$.

Starting with a non-diagonal line element of the universe as a function of the field parameters $\alpha = (1\text{-}K_m r)$ and $\alpha' = (1+K_m r)$, where K_m is a constant, it is found out that this line element satisfies the Einstein's field equations in a vacuum including Λ, with the solution $\Lambda = 3(K_m)^2$. This non-diagonal line element has been diagonalized [15] (for the convenience) and the related Lagrangean has been derived that was the source of the generalized energy equation. The velocity equation of the universe motion and the related scalar potential have been derived from the generalized energy equation also as the function of parameters K_m, r and k. Here k is energy conservation constant. Finally, the acceleration equation of the universe motion is obtained from the mentioned scalar potential. Thus, the dark energy, presented by the cosmological constant Λ, has its real physical source in a scalar field determined by the mentioned scalar potential $V_p = f(K_m , r)$, where $K_m = (\Lambda/3)^{1/2}$. Since we know the analytic expression of the cosmological constant Λ and the related scalar potential V_p as source of the dark energy, we can conclude that the universe dark energy is not so dark. Further more, the cosmological constant Λ solves the singularity problem of the universe at the initial radius. Namely, from the present observation we know that the new mass-radius balancing constant $K_{mr} = M / r^2 > 0$. Since, this constant should have the same value at the initial and present time, one can conclude that the initial universe mass M_o and radius r_o should satisfy the conditions: $M_o > 0$ and $r_o > 0$. This is very important discovery because it proves that at the initial radius of the universe we have no a singularity problem. Of course, at the initial radius a mass density and a temperature may be very, very large but finite quantities. The cosmological constant Λ gives also the answer to the question: how big is the universe? It can be seen from the velocity equation from where the minimal and the maximal radius of the universe are determined: $r_o = r_{min} = (1\text{- } k) / K_m$, $r_{max} = (1+k) / K_m$. The Special Relativity tells us that the maximal velocity of the mass particles satisfies $v < c$. Since $v_{max} = kc$, we conclude that the energy conservation constant $k < 1$. It gives $r_o=r_{min}>0$ and $r_{max} < 2/K_m$.

Following the acceleration equation one can see that the acceleration of the universe at r_{min} is positive, at the radius $r_c = 1/K_m$ is equal to zero and at the radius r_{max} is negative. These facts give the answer to the question: what is the ultimate fate of the Universe? The answer is a very simple. The universe will expand with the acceleration till the radius r_c and will continue by expanding but with deceleration till the radius r_{max}. At this point the velocity is equal to zero, acceleration is negative and the expansion is change into the contraction, following the trajectory symmetrical to the expansion phase, but in opposite direction. The contraction phase is finished at the radius r_{min}, and the new cycle is started at this point. All the time the mass of the universe is changing from the initial to the maximal value and vice versa, following the law: $M = 3^{1/2} K_m c^2 r^2 / G$. The mass density of the universe is also changing following the law of the inverse proportional to the universe radius r. That is much more less then the expected changing that follows the law of the inverse proportional to r^3. The answer to the last question: how old is the universe?, has been obtained by employing the proper time relation. This time is related to the present cycle of the universe motion. How many cycles was before and how many will be in the future no one knows at the time. The presented model of the universe motion supports the well known Big Bang scenario of the universe evolution. But, what is very important, it solves the initial singularity problem, dark energy problem and tell us what the ultimate fate of the universe really is giving a unique solution: a closed universe. Related scalar field, determined by Λ, is the real physical source of the dark energy. The velocity and acceleration equations show the value of the rate of the accelerated expansion at the present time. Finally, the presented model should be verified by the cosmological observation data. This will be done in the next paper.

This paper is organized as follows. Determination of Λ by solution of the Einstein's field equations in a vacuum, including Λ, is presented in the section 2. Derivation of velocity and acceleration equations of the universe motion is considered in the section 3. The procedures of the estimation of the universe parameters at the present time are presented in the section 4. Finally, the conclusions are emphasized in the section 5.

2 Determination of Lambda by Solution of the Einstein's Field Equations in a Vacuum

Following the GLT-model [14, 15, 16], the general line element, ds, of the full form of the GLT-model has been derived in spherical polar coordinates and presented as a function of the two field parameters α and α′ in the nondiagonal form [16, 17]:

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \delta(\alpha - \alpha')\, c\, dt\, dr + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2. \quad (1)$$

Here, c is the speed of the light in a vacuum, t is a time, r is a radius vector, θ is an angle between radius vector r and z-axis, and ϕ is an angle between projection of a radius vector r on (x-y) plane and x-axis. The observation parameter δ is equal to one (δ = 1) if an observation signal is emitted from the origin of the system K, and minus one (δ = - 1) if an observation signal is emitted from the origin of the system K' (see [14, 15]). The unknown field parameters α and α' can be identified in a gravitational field by employing the full form of the Einstein's field equations in the General Theory of Relativity in the tensor relation [1]:

$$R_{\eta\mu} - \frac{1}{2} g_{\eta\mu} R + \Lambda g_{\eta\mu} = \frac{8\pi G}{c^4} T_{\eta\mu} . \tag{2}$$

In this relation $R_{\eta\mu}$ is the Ricci tensor, R is the Ricci scalar, $g_{\eta\mu}$ is the metric tensor, Λ is the Einstein's cosmological constant, G is the Newton's gravitational constant and $T_{\eta\mu}$ is the Einstein's energy-momentum tensor. In the reference [19] it is shown the solution of the field parameters α and α' for a vacuum (i.e. $T_{\mu\beta} = 0$) and including Λ:

$$\begin{aligned} \delta &= 1, \quad \alpha = 1 - \frac{GM}{rc^2}, \quad \alpha' = 1 + \frac{GM}{rc^2} , \\ \delta &= -1, \quad \alpha = 1 + \frac{GM}{rc^2}, \quad \alpha' = 1 - \frac{GM}{rc^2} . \end{aligned} \tag{3}$$

In the equations (3) M is a total gravitational mass and r is a gravitational radius. Thus, the field parameters α and α' are functions of the gravitational radius r and therefore a metric in (1) is changing in the field-changing sense. The related solution of Λ tells us that Λ is also a function of the gravitational radius r:

$$\Lambda = \left(\frac{GM}{r^2 c^2} \right)^2 . \tag{4}$$

Meanwhile, the covariant derivative of the Einstein's field equations (2) can be written in the following form with the help of the Bianchi identities:

$$\nabla^{\mu} (R_{\eta\mu} - \frac{1}{2} R g_{\eta\mu}) = 0, \quad \rightarrow \quad \partial_{\eta} \Lambda = \frac{8\pi G}{c^4} \nabla^{\mu} T_{\eta\mu} . \tag{5}$$

Within Einstein's theory, it follows that Λ = constant as long as matter and energy (contained in $T_{\eta\mu}$) are conserved. Assuming that matter and energy of the universe are conserved and following (5) one can conclude that Λ from (4) should be constant. Consequently, this leads to the conclusion that the universe mass is changing in accordance to the following law:

$$\Lambda = \left(\frac{GM}{r^2 c^2} \right)^2 = K_{\Lambda}^2, \quad \rightarrow \quad M = \Lambda^{1/2} \frac{c^2 r^2}{G} = K_{\Lambda} \frac{c^2 r^2}{G} . \tag{6}$$

Here K_{Λ} is a constant. On that way Λ from (6) satisfies both the Einstein's field equations (2) and the related covariant derivative (5). Now, the main question is: what is the form of the field parameters α and α' that gives Λ = constant? The answer to this question is presented in the proposition 1.

The proposition 1: If the field parameters α and α' of the line element (1) have the following form:

$$\alpha = 1 - K_m r, \qquad \alpha' = 1 + K_m r, \tag{7}$$

where K_m is a constant, then the line element (1), when it is applied to the Einstein's field equations (2) in the vacuum case (i.e. $T\eta\mu = 0$) and including Λ, gives the solution of Λ = constant.

Proof of the proposition 1. Let start with the line element in the nondiagonal form (1) with the following substitutions (for the convenience) of its parameters:

$$\nu = \alpha\alpha', \qquad \lambda = \delta(\alpha' - \alpha)/2. \tag{8}$$

In that case the nondiagonal line element (1) is transformed into the new relation:

$$ds^2 = -\nu\, c^2 dt^2 + 2\lambda\ \, c dt\, dr + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \,. \tag{9}$$

By employing coordinate system:

$$dx^0 = c\,dt, \qquad dx^1 = dr, \qquad dx^2 = d\theta, \qquad dx^3 = d\phi, \tag{10}$$

the related covariant metric tensor, g_{ij}, of the line element (9) is given by the expression:

$$\left[g_{ij}\right] = \begin{bmatrix} -\nu & \lambda & 0 & 0 \\ \lambda & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{bmatrix}. \tag{11}$$

From the Einstein's field equations in General Relativity (2) we know that the following condition should be satisfied:

$$\sqrt{-\det\left[g_{ij}\right]} = \sqrt{r^4\left(\nu + \lambda^2\right)\sin^2\theta} = 1\,. \tag{12}$$

Including the normalization of the radius, $r = 1$, and $\theta = 90^\circ$ into the equation (12) we obtain the relations between the parameters ν and λ :

$$\begin{gathered} \nu + \lambda^2 = 1, \quad \nu = 1 - \lambda^2, \quad \nu' = -2\lambda\,\lambda', \quad \nu'' = -2\left(\lambda'^2 + \lambda\lambda''\right), \\ \nu' = \partial\nu/\partial r, \qquad \nu'' = \partial^2\nu/\partial r^2, \qquad \lambda' = \partial\lambda/\partial r, \qquad \lambda'' = \partial^2\lambda/\partial r^2. \end{gathered} \tag{13}$$

The related Ricci tensor $R_{\eta\mu}$ and Ricci scalar R for the line element (9) are derived and presented in the reference [19]:

$$R_{00} = (\lambda^2 - 1)\left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r}\right), \quad R_{01} = R_{10} = \lambda\left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r}\right),$$

$$R_{11} = \left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r}\right), \quad R_{22} = 2\lambda\lambda' r + \lambda^2, \quad R_{33} = (2\lambda\lambda' r + \lambda^2)\sin^2\theta,$$

$$R = 2\left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r}\right) + 2\left(\frac{2\lambda\lambda'}{r} + \frac{\lambda^2}{r^2}\right), \tag{14}$$

The first of the Einstein's field equations (2) for a vacuum (where the energy-momentum tensor vanishes, i.e. $T_{\eta\mu} = 0$) and including Λ, is given by the following scalar relation:

$$R_{00} - \frac{1}{2} g_{00} R + \Lambda g_{00} = 0. \tag{15}$$

Taking into account g_{00}, R_{00} and R for the line element (9), given by (11) and (14), respectively, and including condition (12), the equation (15) is transformed into the form:

$$\left(\frac{2\lambda\lambda'}{r} + \frac{\lambda^2}{r^2}\right) - \Lambda = 0 \ . \tag{16}$$

From the relations (7) and (8) we obtain the following equations for the parameter λ:

$$\lambda = \delta K_m r, \qquad \lambda' = \delta K_m, \qquad \lambda'' = 0. \tag{17}$$

By using the application of (17) to the equation (16) and including $\delta^2 = 1$, one obtains the solution of the cosmological constant Λ:

$$\Lambda = 3K_m^2, \quad \rightarrow \quad K_m = \sqrt{\Lambda/3} = K_\Lambda/\sqrt{3}\,. \tag{18}$$

The second and the third of the Einstein's vacuum field equations (2), including Λ, have the forms:

$$R_{01} - \frac{1}{2} g_{01} R + \Lambda g_{01} = 0, \qquad R_{11} - \frac{1}{2} g_{11} R + \Lambda g_{11} = 0. \tag{19}$$

Taking into account g_{01}, g_{11}, R_{01}, R_{11} and R for the line element (9), given by (11) and (14), and including condition (12), the both equations in (19) give the differential equation equal to the equation (16). Therefore, the equations (19) give also the solutions of Λ equal to (18). Finally, employing the fourth and the fifth of the Einstein's vacuum field equations (2), including Λ, we obtain the relations:

$$R_{22} - \frac{1}{2} g_{22} R + \Lambda g_{22} = 0, \qquad R_{33} - \frac{1}{2} g_{33} R + \Lambda g_{33} = 0\,. \tag{20}$$

Taking into account g_{22}, g_{33}, R_{22}, R_{33} and R for the line element (9), given by (11) and (14), and including condition (12), the both relations in (20) are transformed into the following differential equation:

$$\left(\lambda'^2+\lambda\lambda''+\frac{2\lambda\lambda'}{r}\right)-\Lambda=0\ . \tag{21}$$

Applying the relations (17) to the equation (21) one obtains the solution of the cosmological constant Λ also equal to (18). Now, the all of the Einstein's vacuum field equations give the same solution of Λ, presented by (18). Since K_m is a constant we conclude from (18) that Λ is also a constant. On that way the proof of the proposition 1 is finished. Consequently, the relations (17) and (18) transform the Ricci scalar equation (14) into the new form:

$$R=4\Lambda=12K_m^2. \tag{22}$$

This relation is in accordance with the solution of R in General Theory of Relativity [1].

3 Derivation of Velocity and Acceleration Equations of the Universe Motion

It has been shown in the reference [15] that the general non-diagonal form of the line element (1) can be diagonalized into the form $\underline{ds}$ and described (for δ=1) by the relation:

$$\underline{ds}^2=-\alpha^2c^2dt^2+\alpha^{-2}dr^2+r^2d\theta^2+r^2\sin^2\theta\,d\phi^2\ . \tag{23}$$

Including the field parameter α from (7) this relation is transformed into the following diagonal line element:

$$\underline{ds}^2=-(1-K_mr)^2c^2dt^2+(1-K_mr)^{-2}dr^2+r^2d\theta^2+r^2\sin^2\theta\,d\phi^2\ . \tag{24}$$

This equation is related to the space-time metric of the homogeneous and isotropic universe in the General Theory of Relativity. In order to find out the velocity and acceleration equations of the universe motion one can start with the Lagrangean L of the line element (24):

$$L=\left[-\frac{ds^2}{c^2d\tau^2}\right]^{1/2}=\frac{1}{c}\left[-g_{ij}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\right]^{1/2},i,j=0,1,2,3. \tag{25}$$

Here $d\tau$ is the differential of the proper time τ, and dx^i is the i-th component of the displacement four-vector dX in (10). Applying the relation (24) to (25) one obtains the Lagrangean in the following form:

$$L=\begin{pmatrix}(1-K_m r)^2\left(\frac{dt}{d\tau}\right)^2-\frac{1}{c^2}(1-K_m r)^{-2}\left(\frac{dr}{d\tau}\right)^2-\\ -\frac{r^2}{c^2}\left(\frac{d\theta}{d\tau}\right)^2-\frac{r^2}{c^2}\sin^2\theta\left(\frac{d\phi}{d\tau}\right)^2\end{pmatrix}^{1/2}. \qquad (26)$$

The related Euler – Lagrange equations are given by the relations:

$$\frac{\partial L}{\partial z^i}=\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot z^i}\right), \qquad i=0,1,2,3,$$

$$z^o=t, \quad z^1=r, \quad z^2=\theta, \quad z^3=\phi, \quad \dot z^i=dz^i/d\tau. \qquad (27)$$

Applying index $i=0$ to the relation (27) one obtains an energy conservation equation:

$$\frac{\partial L}{\partial t}=0 \Rightarrow \frac{\partial L}{\partial \dot t}=\text{const.}=k \Rightarrow (1-K_m r)^2\frac{dt}{d\tau}=k, \ \frac{dk}{d\tau}=\dot k=0, \qquad (28)$$

where k is energy conservation constant. Applying index i = 3 to the relation (27) one obtains an angular momentum conservation equation:

$$\frac{\partial L}{\partial \phi}=0 \Rightarrow \frac{\partial L}{\partial \dot\phi}=\text{const.}=h \Rightarrow -\frac{r^2\dot\phi\sin^2\theta}{c^2}=h, \ \frac{dh}{d\tau}=\dot h=0. \qquad (29)$$

Here h is angular momentum conservation constant. In the case $\theta = \pi / 2$ (as in Newtonian theory) the angular momentum conservation equation (29) is transformed into the well-known relation:

$$\frac{\partial L}{\partial \phi}=0 \Rightarrow \frac{\partial L}{\partial \dot\phi}=\text{const.}=h \Rightarrow -\frac{r^2\dot\phi}{c^2}=h, \ \frac{dh}{d\tau}=\dot h=0. \qquad (30)$$

Now, the following equation can be derived from the relations (28):

$$\frac{dt}{d\tau}=k(1-K_m r)^{-2}. \qquad (31)$$

Substituting (31) into the equation (26) and employing $\varepsilon = L$ (where $\varepsilon = 1$ for time-like geodesics and $\varepsilon = 0$ for null geodesics) one obtains the following relation:

$$\frac{1}{2}\left[\dot r^2+r^2\left(\dot\theta^2+\dot\phi^2\sin^2\theta\right)(1-K_m r)^2\right]-K_m r c^2\varepsilon^2\left(1-\frac{K_m r}{2}\right)=\frac{c^2}{2}\left(k^2-\varepsilon^2\right),$$

$$\dot r=\frac{dr}{d\tau}, \qquad \dot\theta=\frac{d\theta}{d\tau}, \qquad \dot\phi=\frac{d\phi}{d\tau}. \qquad (32)$$

This relation represents the generalized energy equation of the universe motion. It describes the law of the energy conservation during the motion of the homogeneous and isotropic universe. The coordinates (τ, r, θ, ϕ) are commoving coordinates in the universe space-time during the expansion or contracting phase of the universe motion. The generalized energy equation (32) can be transformed into the new relation:

$$\frac{v^2}{2} - K_m r c^2 \varepsilon^2 \left(1 - \frac{K_m r}{2}\right) = \frac{c^2}{2}\left(k^2 - \varepsilon^2\right),$$
$$v^2 = \left[\dot{r}^2 + r^2\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right)\left(1 - K_m r\right)^2\right]. \quad (33)$$

Here v is a velocity. Consequently, the velocity equation can be derived from the first equation in (33):

$$v = \pm\left[2K_m r c^2 \varepsilon^2 \left(1 - \frac{K_m r}{2}\right) + \left(k^2 - \varepsilon^2\right)c^2\right]^{1/2}. \quad (34)$$

The radial velocity ($\dot{r}$) equation can also be derived from (33), or directly from (32):

$$\dot{r} = \pm\left[2K_m r c^2 \varepsilon^2 \left(1 - \frac{K_m r}{2}\right) + \left(k^2 - \varepsilon^2\right)c^2 - r^2\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right)\left(1 - K_m r\right)^2\right]^{1/2}. \quad (35)$$

From the observation we know that the universe motion is in the radial direction only. Therefore, one can substitute $\dot{\theta} = 0$ and $\dot{\phi} = 0$ into the relation (35). As the result we have the new radial velocity equation of the universe motion in the following form:

$$\dot{r} = v = \pm\left[2K_m r c^2 \varepsilon^2 \left(1 - \frac{K_m r}{2}\right) + \left(k^2 - \varepsilon^2\right)c^2\right]^{1/2}. \quad (36)$$

From this point the radial velocity $\dot{r}$ will be denoted with v and will be called just velocity, for the simplicity. The hypothetical possibility that the universe is expanding with orbiting can be considered in the next paper by starting with relation (35).

Now, one can ask the question: which geodesic follows the universe motion? In the case of a null geodesic (ε = 0), the velocity equation (36) is transformed into the relation:

$$v = \pm kc\ . \quad (37)$$

This means that a velocity is constant because the parameters k and c are constants. Meanwhile, we know from the observations that our universe is expanding, even at an accelerating rate. Therefore, it is naturally to assume that the universe motion follows time-like geodesic (ε = 1). In that case, the velocity equation (36) is transformed into the universe velocity relation:

$$\dot{r} = v = \pm\left[2K_m r c^2 \left(1 - \frac{K_m r}{2}\right) + \left(k^2 - 1\right)c^2\right]^{1/2}. \quad (38)$$

The sign (+) is valid for an expanding phase, while the sign (-) is related to the contracting one. The velocity equation (38) has two zeros at the positions r_1 and r_2 :

$$r_1 = r_{min} = \frac{1-k}{K_m}, \qquad r_2 = r_{max} = \frac{1+k}{K_m}. \tag{39}$$

Since the universe radius can not be negative, the first relation in (39) tells us that the energy conservation constant should be less or equal to one ($k \le 1$). This means that the universe velocity (38) has two real finite zeros and that we are leaving in the closed universe. If the energy conservation constant is equal to one ($k = 1$), then the initial universe radius r_1 is equal to zero ($r_1 = 0$). Otherwise, if the energy conservation constant is less than one ($k < 1$), then the initial radius r_1 is greater than zero ($r_1 > 0$). Anyway, the universe motion started at initial radius r_1 and will expanding to the largest radius r_2. It follows the contracting phase in opposite direction from r_2 to the initial radius r_1. The equation for calculation of the energy conservation constant k has been derived by employing the relation (33) and assuming that the universe motion follows time-like geodesic ($\varepsilon = 1$):

$$k = \left((1-K_m r)^2 + \frac{v^2}{c^2} \right)^{1/2}, \quad v = v_0 = 0 \;\rightarrow\; k = 1 - K_m r_0 . \tag{40}$$

Thus, for calculation of k we have to know the constant K_m and the present values of r and v, or K_m and the initial universe radius r_0.

The generalized energy equation of the universe motion (32) can also be transformed into the new form:

$$\frac{1}{2}\left[\dot{r}^2 + r^2\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right)\left(1-K_m r\right)^2\right] + \frac{c^2\varepsilon^2}{2}\left(1-K_m r\right)^2 = \frac{c^2k^2}{2}. \tag{41}$$

For the time-like geodesic ($\varepsilon = 1$) and radial motion only ($\dot{\theta} = 0$, $\dot{\phi} = 0$), it describes expansion or contraction motion of the homogeneous and isotropic universe and represents the law of the energy conservation during these processes. From the relation (41) one can recognize the related very important scalar potential V_p of a potential field that controls the expansion and contraction motions of the universe:

$$V_p = \frac{c^2\varepsilon^2\alpha^2}{2} = \frac{c^2\varepsilon^2}{2}\left(1-K_m r\right)^2, \;\rightarrow\; \alpha = 1 - K_m r . \tag{42}$$

Thus, assuming that the universe motion follows time-like geodesic ($\varepsilon = 1$), and taking into account that $\dot{\theta} = 0$ and $\dot{\phi} = 0$, the related radial acceleration equation of the universe expansion or contraction motion, a_c, can be derived by employing the following relation :

$$a_c = \ddot{r} = -\frac{\partial V_p}{\partial r}, \qquad \rightarrow \qquad a_c = \ddot{r} = K_m c^2\left(1-K_m r\right). \tag{43}$$

From the relation (43) one can see that the zero acceleration ($a_c = 0$) is occurred at the radius $r_c = 1 / K_m$. For $r < r_c$ the universe acceleration is positive (accelerated expansion) and for $r > r_c$ the universe acceleration is negative (de-accelerated expansion). From the observation we know that at the present time we have accelerated expansion of the universe. This means that the present universe radius is less than r_c. Including $r = r_c = 1/ K_m$ into the relation (38) one obtains the maximal velocity relation of the universe motion:

$$r = r_c = 1/K_m , \qquad \rightarrow \qquad v = v_{max} = kc . \tag{44}$$

The Special Theory of Relativity tells us that maximal velocity of the mass particles should be less than the speed of the light. Applying this theory to the relation (44) one can conclude that the energy conservation constant k should be less than one ($k < 1$). As the consequence we have the following relations for initial universe radius r_1 and the largest universe radius r_2:

$$\begin{aligned} r_1 = r_{min} &= \frac{1-k}{K_m} > 0, \quad \rightarrow \quad a_c(r_1) = kK_m c^2 < K_m c^2 , \\ r_2 = r_{max} &= \frac{1+k}{K_m} < \frac{2}{K_m} , \quad \rightarrow \quad a_c(r_2) = -kK_m c^2 > -K_m c^2 . \end{aligned} \tag{45}$$

At the maximal universe radius $r_2 = r_{max}$ the expansion velocity is equal to zero. Since at this point the acceleration is negative, $a_c(r_2)$, we have the condition for starting of the contracting phase. Between r_2 and r_c we have accelerated contraction and in the interval r_c and r_1 it follows de-accelerated contraction. At the minimal universe radius $r_1 = r_{min}$ the contraction velocity is equal to zero. Since at this point the acceleration is positive, $a_c(r_1)$, we have the condition for starting of the new expansion phase. The equation of the universe proper time τ has been derived from the relation (31) with the substitution $dt = dr/c$:

$$\tau = \frac{1}{kc}\left[r - r_1 - K_m\left(r^2 - r_1^2\right) + \frac{K_m^2}{3}\left(r^3 - r_1^3\right)\right]. \tag{46}$$

Further, including (4), (6) and (18), one can derive the following relations:

$$\begin{aligned} K_m &= \sqrt{\Lambda/3} = \frac{GM}{\sqrt{3}c^2 r^2} , \qquad M = \rho_m \frac{4r^3\pi}{3} = \rho \frac{4r^3\pi}{3c^2} , \\ K_m &= \frac{4\pi G}{3\sqrt{3}c^2}\rho_m r , \quad \rightarrow \quad \rho_m r = const. \end{aligned} \tag{47}$$

Here ρ_m is a mass density and ρ is the related energy density. The last equation in (47) tell us the important information that product of the mass density ρ_m and universe radius r is always constant. This means that during the universe expansion phase the universe mass should be created following the law (6). On the other side, the related annihilation process should follow the contracting universe phase.

Now, one can introduce the normalization of the universe radius R in the form:

$$R = K_m r, \qquad R_1 = K_m r_1 = (1-k), \qquad R_2 = K_m r_2 = (1+k),$$
$$R_c = K_m r_c = 1, \qquad (1-k) \le R \le (1+k). \tag{48}$$

The last relation in (48) shows the region in which R should be changed. Applying this radius to the equations (38), (42) and (43) and using $\varepsilon = 1$, one obtains the normalized equations for velocity v, scalar potential V_p and radial acceleration a_c of the universe motion, respectively:

$$v = \pm\left[2c^2R\left(1-\frac{R}{2}\right)+\left(k^2-1\right)c^2\right]^{1/2},$$
$$V_p = \frac{\alpha^2c^2}{2} = \frac{c^2}{2}(1-R)^2, \rightarrow \alpha = 1-R, \quad a_c = K_m c^2(1-R). \tag{49}$$

Including (48) and (49) one can calculate v, V_p, and a_c at the initial, middle and the largest normalized radius R_1, R_c and R_2, respectively:

$$v(R_1) = v(R_2) = 0, \quad v(R_c) = kc, \quad V_p(R_1) = V_p(R_2) = \frac{k^2c^2}{2},$$
$$V_p(R_c) = 0, \quad a_c(R_1) = kK_m c^2, \quad a_c(R_2) = -kK_m c^2, \quad a_c(R_c) = 0. \tag{50}$$

These values determine the velocity, scalar potential and acceleration at the three important points on the trajectory of the universe motion, belonging to the closed universe structure.

Following the Friedmann - Robertson - Walker approach [20] one can introduce the cosmological scale factor S on the way that the radius of the sphere r can be obtained from an initial radius r_0 by employing the equation $r = Sr_0$. For that case, the velocity of the universe expansion is given by the relation $v = \dot{r} = \dot{S}r_0 = \dot{S}r/S = Hr$, where $H = \dot{S}/S$ is the well known Hubble parameter. Applying the cosmological scale factor S to the relation (41) and including $\varepsilon = 1, \dot{\theta} = 0$ and $\dot{\phi} = 0$, one obtains the equation that describes expansion of the homogenous and isotropic universe in the following form:

$$\frac{\dot{S}^2}{2} + \frac{c^2}{2r_0^2}\left(1-\sqrt{\frac{\Lambda}{3}}r_0S\right)^2 = \frac{c^2k^2}{2r_0^2}. \tag{51}$$

This equation represents the law of the energy conservation during the universe expansion phase. If one uses (for the simplicity) the normalization $r_0 = 1$, then it follows that $r = S$. For that case the universe radius r describes at the same time the scale of the universe expansion. Now, applying the cosmological scale factor S to the relation (43) one obtains the state equation of the motion of the homogenous and isotropic universe in the following form:

$$\frac{\ddot{S}}{S}=\frac{4\pi G}{3c^2}\left(\frac{\rho}{\sqrt{3}}-\frac{c^4}{4\pi G}\Lambda\right)=\frac{4\pi G}{3}\left(\frac{\rho_m}{\sqrt{3}}-\frac{c^2}{4\pi G}\Lambda\right),\quad \rho_m=\frac{\rho}{c^2}. \qquad (52)$$

Here ρ is energy density, while ρ_m is the related mass density of the universe. The relation (52) shows the accelerated expansion phase for $\rho_m > \sqrt{3}\, c^2\, \Lambda \,/\, 4\pi G$. On the other side the de-accelerated expansion phase is occurred for $\rho_m < \sqrt{3}\, c^2\, \Lambda \,/\, 4\pi G$. The detailed analysis of the relations (51) and (52) will be presented in the next paper.

4 Estimation of the Universe Parameters at the Present Time

Some of the universe present parameters are known from the observation, like velocity v_p, mass or energy density ρ_{mp} or ρ_p, the age of the universe τ_p and the Hubble parameter H_{Bp}. Here Hubble parameter is not reliable because of the large region of its value. The other parameters should be estimated by using the related equations of the universe dynamics. Thus, applying the relations (40), (47) and (48) we obtain new equations for calculation of the energy conservation constant k and present radius r_p:

$$k=\sqrt{\left(1-R_p\right)^2+\frac{v_p^2}{c^2}},\qquad r_p=\sqrt{\frac{3\sqrt{3}\,c^2R_p}{4\pi G\rho_{mp}}},\qquad R_p=K_m r_p. \qquad (53)$$

Applying $v_p = 0.956c$ m/s and $\tau_p = 13.7e+9$ years and using the iteration procedure, with initial R_p and ρ_{mp}, one can calculate k and r_p from (53) and the other present universe parameters by the following relations:

$$K_m=\frac{4\pi G}{3\sqrt{3}\,c^2}\rho_{mp}\,r_p,\quad a_{cp}=K_m c^2\left(1-R_p\right),\quad r_1=\frac{1-k}{K_m},\quad r_2=\frac{1+k}{K_m},$$

$$r_c=\frac{1}{K_m},\quad V_{P_p}=\frac{\left(1-R_p\right)^2c^2}{2},\quad M_p=\frac{\sqrt{3}K_m c^2}{G}r_p^2,\quad \Lambda=3K_m^2, \qquad (54)$$

The numerical values of the estimated present universe parameters are shown in the table 1. In this table, in the second column, the iteration procedure is stopped when we obtain the initial value of v_p and τ_p. In the third column the iteration procedure is stopped when the energy conservation constant k is increasing to the value close to one, k = 0.9999999999999999. In the table 1, parameter h_o is as usual the normalized value of the Hubble constant. This calculation shows that the present values of the universe parameters should be measured very precisely, if we want to be sure about obtained calculation results. The related universe velocity and acceleration are presented in the Fig. 1. On this figure the velocity shows the cyclic scenario of the universe motion and the acceleration confirms the present accelerated expansion of the universe. This confirms that our universe is not static but exhibits at present the accelerated expansion in accordance with the cosmological observations. Finally, the process of the universe proper time changing is presented in Fig. 2. Considering this figure one can see the relativistic effect on the proper time: very small changing of the proper time in the region where the universe velocity is close to the speed of the light in a vacuum.

Table 1: The present parameters of the universe obtained by the iteration procedures

ρ_{mp}, kg/m^3	0.4946111000000000e-026	0.4959919105000000e-026
k	0.9996136918947285	0.9999999999999999
r_p, m	2.823447601087397e+026	2.816885899244881e+026
K_m, m^{-1}	0.2507405690634053e-026	0.2508562151617445e-026
R_p	0.7079528582173605	0.7066333352270580
r_1, m	1.540668535269333e+023	4.425734574323058e+010
r_c, m	3.988185891638174e+026	3.986347315952409e+026
r_2, m	7.974831114741077e+026	7.972694631904819e+026
v_p, m/s	2.866015898480001e+008	2.866015898480001e+008
a_{cp}, m/s^2	6.581410401725731e-011	6.614195611564151e-011
M_p, kg	4.663281845660758e+053	4.643772871117975e+053
τ_p, y	13.70009046047019e+009	13.70000000662603e+009
H_{Bp}, s^{-1}	1.015076708835046e-018	1.017441245755921e-018
h_o	0.3138379101257933	0.3145689694823955
Λ, m^{-2}	1.886124989227209e-053	1.887865220558263e-053

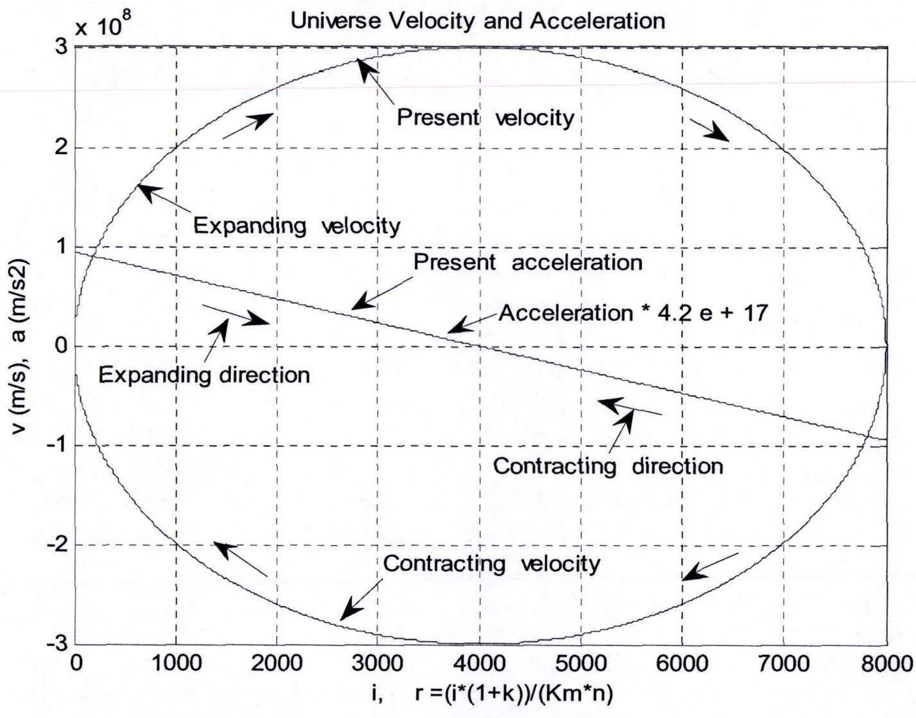

Figure 1: The velocity, eq. 38, and acceleration, eq. 43, of the universe, n = 8000.

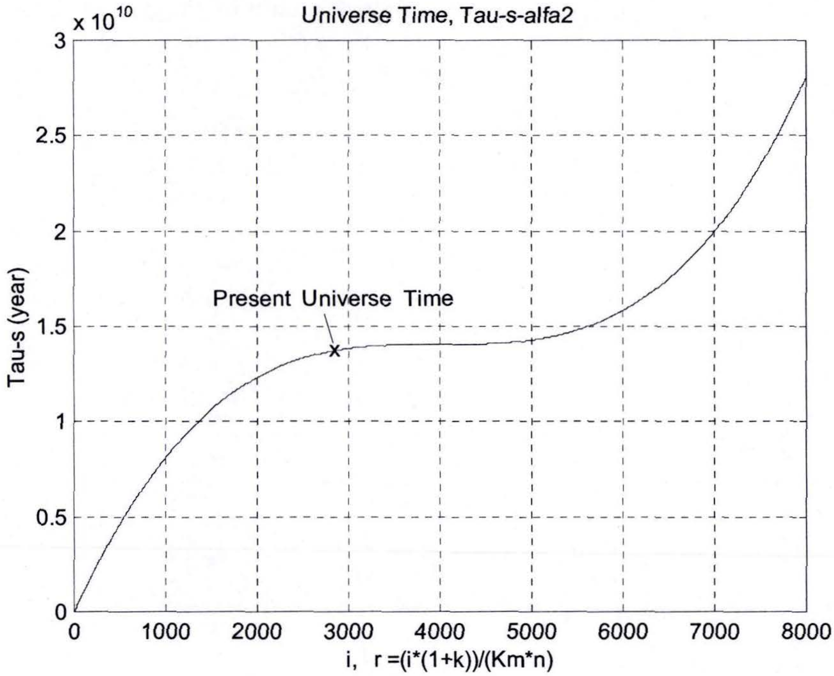

Figure 2: The change of the universe proper time, eq. 46, n = 8000.

5 Conclusion

The problems of the explanation of the real physical source of the dark energy and investigation of the value of the rate of the universe expansion are solved. In that sense, an analytic expression for Λ has been derived in terms of the universe mass M and radius r: $\Lambda = (GM/r^2c^2)^2$. Following the Einstein's field equations, Λ should be a constant if the conservation laws are valid for the universe. Therefore, the structure of the universe should include a new mass-radius balancing constant $K_{mr} = M / r^2$, with the consequence that universe mass is producing proportional to r^2: $M = \Lambda^{1/2} c^2 r^2 / G$. The physical meaning of K_{mr} is the mass-space conservation. Starting with a line element of the universe as a function of $K_m = (\Lambda / 3)^{1/2}$, the related velocity equation, scalar potential and acceleration equations of the universe motion have been derived as the functions of K_m, r and k. Here k is the energy conservation constant. Thus, the dark energy, presented by the cosmological constant Λ, has its physical source in a scalar field determined by the mentioned scalar potential. The velocity and acceleration equations show the value of the rate of the present accelerated expansion of the universe. This unique solution of the universe motion belongs to the closed universe structure without an initial singularity problem. The obtained results should be verified by comparing with the cosmological observation data.

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