Stochastic Models for Prediction of Fatigue-Crack Growth in Aircraft Structure Components

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Abstract
For important fatigue-sensitive structures of aircraft whose breakdowns cause serious accidents, it is required to keep their reliability extremely high. In this paper, we discuss inspection strategies for such important structures against fatigue failure. The focus is on the case when there are fatigue-cracks unexpectedly detected in a fleet of aircraft within a warranty period (prior to the first inspection). The paper examines this case and proposes stochastic models for prediction of fatigue-crack growth to determine appropriate inspections intervals. We also do not assume known parameters of the underlying distributions, and the estimation of that is incorporated into the analysis and decision-making. Numerical example is provided to illustrate the procedure.
Keywords: Aircraft, Fatigue crack, Inspection interval.

1 Introduction

Fatigue is one of the most important problems of aircraft arising from their nature as multiple-component structures, subjected to random dynamic loads. The analysis of fatigue crack growth is one of the most important tasks in the design and life prediction of aircraft fatigue-sensitive structures (for instance, wing, fuselage) and their components (for instance, aileron or balancing flap as part of the wing panel, stringer, etc.). An example of in-service cracking from B727 aircraft (year of manufacture 1981; flight hours not available; flight cycles 39,523) [1] is given on Figure 1.

Several probabilistic or stochastic models have been employed to fit the data from various fatigue crack growth experiments. Among them, the Markov chain model [2], the second-order approximation model [3], and the modified second-order polynomial model [4]. Each of the models may be the most appropriate one to depict a particular set of fatigue growth data but not necessarily the others. All models can be improved to depict very accurately the growth data but, of course, it has to be at the cost of increasing computational complexity. Yang's model [3] and the polynomial model [4] are considered more appropriate than the Markov chain model [2] by some researchers through the introduction of a differential equation which indicates that fatigue crack growth rate is a function of crack size and other parameters. The parameters, however, can only be determined through the observation and measurement of many crack growth samples. If fatigue crack growth samples are observed and measured, descriptive statistics can then be applied directly to the data to find the distributions of the desired random quantities. Thus, these models still lack prediction algorithms. Moreover, they are mathematically too complicated for fatigue researchers as well as design engineers.

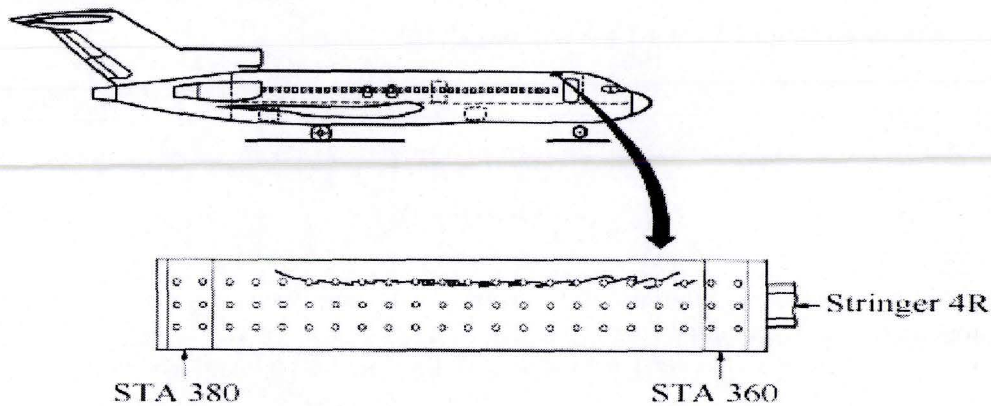

Figure 1: Example of in-service cracking from B727 aircraft.

A large gap still needs to be bridged between the fatigue experimentalists and researchers who use probabilistic methods to study the fatigue crack growth problems.

2 Problem Description

Let us assume that a fatigue-sensitive component has been found cracked on n aircraft within a warranty period. The cracking had not yet caused an accident, but the safety experts have told the manager that had this item failed, an accident was possible. It is clear that the part would have to be redesigned and replaced. The manager's dilemma is that redesigning the part, manufacturing the new design, and installing it in the fleet would take, say, at least two years. The manager must decide how to manage risk for the next two years. The alternatives include doing nothing and accepting the risk of continued cracking and the possibility of an accident. An inspection program is usually instigated, which should reduce the risk of failure, but due to uncertainties in aircraft loading histories, provides no direct measurement of the criticality of the detected cracks. Generally, such a program would lead to some aircraft being grounded, eliminating risk for those aircraft and reducing overall risk, but reducing operational capability. This would leave precious few aircraft to spare before the service's ability to accomplish its mission became impaired. In such a scenario, the decision process involves a complex probability problem concerning the likelihood of additional failures and acceptable risk. To compound the difficulty little guidance is provided in aircraft design specifications for this situation. The situation presented is not uncommon.

The purpose of this paper is to present a more accurate stochastic crack growth analysis method, while maintaining the simplicity of the proposed stochastic fatigue models, for the above problem. We discuss the optimal relationship between the inspection time and the prespecified minimum level of reliability. To illustrate the proposed technique, a numerical example is given.

3 Paris-Erdogan Law as a Starting Point

The basis of most of the fatigue models is the Paris-Erdogan law [5] relating the rate of growth of crack size a to N cycles:

$$\frac{\mathrm{d}a(N)}{\mathrm{d}N} = q[a(N)]^b \tag{1}$$

in which q and b are parameters depending on loading spectra, structural/material properties, etc. We fit da/dN vs $a(N)$ with a function that we can integrate between limits (initial crack size, a_0, and any given crack size, a) to get a life prediction.

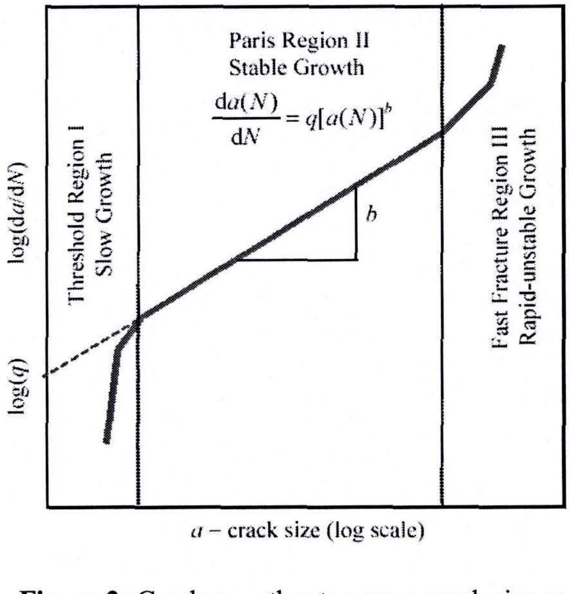

Figure 2: Crack growth rate versus crack size curve
(I = near-threshold region; II = linear region; III = instability region).

In the linear region (see Figure 2) we use the Paris-Erdogan Equation (1) as follows. Integrating

$$\int_{N_0}^{N} \mathrm{d}N = \int_{a_0}^{a(N)} \frac{\mathrm{d}a}{qa^b}, \tag{2}$$

we have

$$N - N_0 = \frac{1}{q(-b+1)} \left[a(N)^{-b+1} - a_0^{-b+1} \right]. \tag{3}$$

Thus, the crack growth equation representing the solution of the differential equation for the Paris-Erdogan law is given by

$$N - N_0 = \frac{1}{q(b-1)} \left(\frac{1}{a_0^{b-1}} - \frac{1}{a(N)^{b-1}} \right). \tag{4}$$

3.1 Sensitivity Analysis

Consider the solution of the differential equation for the Paris-Erdogan law written in the form of (4) as:

$$N(a) = \frac{1}{q(b-1)} \left(\frac{1}{a_0^{b-1}} - \frac{1}{a^{b-1}} \right), \tag{5}$$

where a_0 is the initial crack size at N_0=0. The derivatives of the number of load cycles with respect to the parameters q and b read:

$$\frac{\mathrm{d}N(a)}{\mathrm{d}q} = -\frac{N(a)}{q} \tag{6}$$

and

$$\frac{\mathrm{d}N(a)}{\mathrm{d}b} = \frac{1}{b-1} \left[\frac{1}{q} \left(\frac{\ln a}{a^{b-1}} - \frac{\ln a_0}{a_0^{b-1}} \right) - N(a) \right]. \tag{7}$$

From this one can see that the number of cycles to reach a certain crack size is very sensitive to changes of the parameter q.

4 Statistical Variability of Fatigue-Crack Growth

The traditional analytical method of engineering fracture mechanics (EFM) usually assumes that crack size, stress level, material property and crack growth rate, etc. are all deterministic values which will lead to conservative or very conservative outcomes. However, according to many experimental results and field data, even in well-controlled laboratory conditions, crack growth results usually show a considerable statistical variability (as shown in Figure 3).

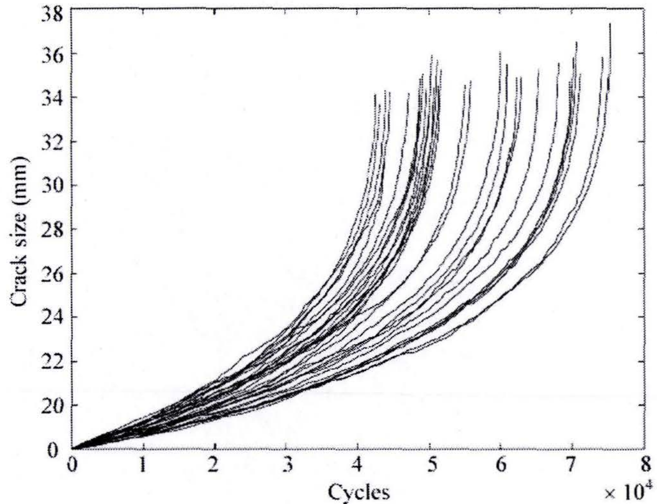

Figure 3: Constant amplitude loading fatigue test data curves.

Yet more considerable statistical variability is the case under variable amplitude loading (as shown in Figure 4).

The basis of most data analyses seems to be to take logarithms in (1) and estimate b and q by least squares in the equation

$$\ln(\mathrm{d}a(N)/\mathrm{d}N) = \ln q + b \ln a(N). \tag{8}$$

Unfortunately to use this equation estimates of $\mathrm{d}a(N)/\mathrm{d}N$ are required. Estimates of derivatives are notoriously unreliable. If several repetitions of an experiment under the same conditions are made it is not always clear how to combine the results. Moreover, as a regression model the properties of the estimates of the coefficients in (8) are not the same as those of estimates of the coefficients in (4). Thus it is sensible to ask why the estimation does not proceed directly from the data on crack size and cycles through equation (4).

It is interesting to note that if b were known q could be estimated from a straight line

$$\frac{1}{a_0^{b-1}} - \frac{1}{a^{b-1}} = (b-1)q(N - N_0) \tag{9}$$

and indeed such a plot for a few values of q is indicative of the nature of the Paris-Erdogan equation in a particular case.

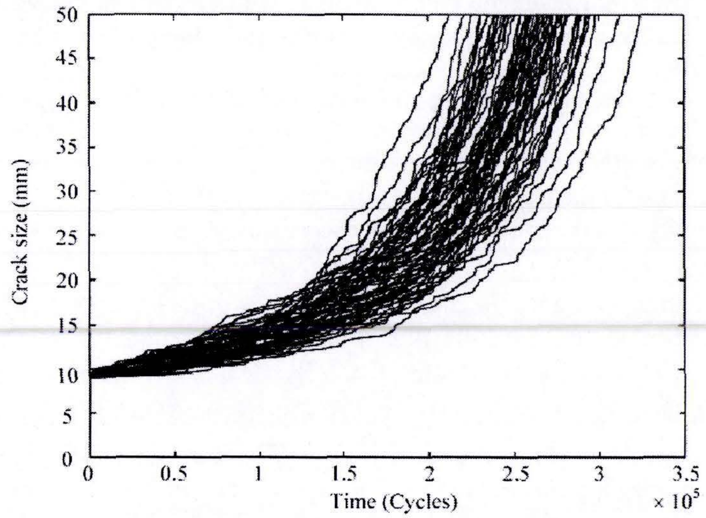

Figure 4: Variable amplitude loading fatigue test data curves.

During the service of the components being assessed, there may be uncertainties in the applied loading conditions, extrapolation of the material data to service conditions, component dimensions, and nature, size and location of detected (postulated) defects, etc. These uncertainties/ variations are critical inputs to the crack growth assessment and can be taken into account using probabilistic methodologies. There is now an extensive literature on the subject of the statistical nature of crack growth. Most of the literature is concerned with model building and the agreement between the general features of the model and the observed behaviour of the crack. However, little use has been made of the statistical nature of the models to analyze experimental results.

While most industrial failures involve fatigue, the assessment of the fatigue reliability of industrial components subjected to various dynamic loading situations is one of the most difficult engineering problems that remains. Material degradation processes due to fatigue depend upon material characteristics, component geometry, loading history and environmental conditions. As a result, stochastic models for crack growth have been suggested by many investigators in the last 15 years. These include evolutionary probabilistic models, cumulative jump models and differential equation (DE) models. DE models are the most widely used models for predicting stochastic crack growth accumulation in the reliability and durability analyses of fatigue critical components.

In practical applications of the stochastic crack growth analysis, either one of the following two distribution functions is needed: the distribution of the crack size at any service time or the distribution of the service time to reach any given crack size.

Unfortunately, when the crack growth rate is modeled as a random process, these two distribution functions are not amenable to analytical solutions. As a result, numerical simulation procedures have been used to obtain accurate results. The simulation approach is a very powerful tool; in particular, with modern high-speed computers. However, it is a very time consuming procedure and therefore simple approximate analytical solutions are very useful in engineering.

The purpose of this paper is to present a more useful stochastic fatigue crack growth models by using the solution of the Paris-Erdogan law equation, which result in a simple analytical solution for either the distribution of the service time to reach any given crack size or the distribution of the crack size at any service time.

The probability that crack size $a(N)$ will exceed any given crack size $a^\bullet$ in the service interval $(N_0,N^\bullet)$, $\Pr\{a(N^\bullet)> a^\bullet\}$, is frequently referred to as crack exceedance probability and can be found based on the stochastic fatigue crack growth model. In addition to this probability distribution of crack size, the probability distribution of cycles (or time) for a crack to grow from size a_0 to $a^\bullet$, $\Pr\{N(a^\bullet)\leq N^\bullet\}$, can also be found based on the above model. In fact, the probability that service time $N(a^\bullet)$ will be within the interval $(N_0, N^\bullet)$ for crack size to reach $a^\bullet$ is identical to $\Pr\{a(N^\bullet)> a^\bullet\}$. That is $\Pr\{N(a^\bullet) \leq N^\bullet\}= \Pr\{a(N^\bullet) > a^\bullet\}$. To summarize the concept of the above derivation, the readers can refer to Figure 5.

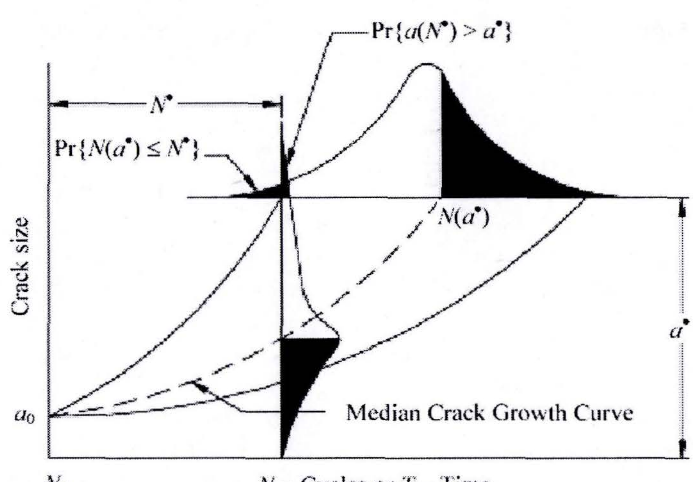

Figure 5: Schematic diagram of crack size distribution and random time distribution.

5 Stochastic Fatigue-Crack Growth Parameter Variability Models

These models allow one to describe the uncertainties in one or two parameters of the solution (4) of the differential equation (1) for the Paris-Erdogan law via parameters modelled as random variables in order to characterize the random properties, which seem to vary from specimen to specimen (see Figure 3). In other words, the stochastic fatigue-crack growth parameter variability models (with respect to the parameters b and q modelled as random variables) are given by

$$N - N_0 = \frac{1}{Q(B-1)}\left(\frac{1}{a_0^{B-1}} - \frac{1}{a^{B-1}}\right), \tag{10}$$

where $(N–N_0)$ is a joint random variable of B and Q. In fact, these models are suited to account for this type of variability. The ones however cannot explain the variability of the crack growth rate during the crack growth process. In particular, crack growth data (crack size versus service time and vice versa) may been analyzed using Eq. (10) by considering, for instance, two different approaches:

(i) B is identical for each specimen and Q varies from specimen to specimen, referred to as Case 1;

(ii) both B and Q vary from specimen to specimen, referred as Case 2.

For Case 1, with B=1, the crack growth data for each specimen are best fitted by equation

$$N - N_0 = \frac{\ln\left[\dfrac{a(N)}{a(N_0)}\right]}{Q} \tag{11}$$

to obtain a sample value of Q, where $a(N) \equiv a$, $a(N_0) \equiv a_0$. For Case 2 equation (10) is used to best fit the crack growth data for each specimen to obtain a set of sample values of B and Q. From the statistical standpoint, B is considered to be a deterministic value and Q to be a statistical (random) variable in Case 1, while both B and Q are considered to be statistical variables in Case 2. It is found that the lognormal or Weibull distribution provides a reasonable fit for B and Q in both cases.

5.1 Weibull Crack Growth Parameter Variability Model

Consider Case 1. The Weibull probability distribution function, $F(q;\sigma,\delta)$, *of* Q is expressed as

$$F(q;\sigma,\delta) = \begin{cases} 1-\exp[-(q/\sigma)^{\delta}], & q \geq 0, \\ 0, & \text{otherwise}, \end{cases} \tag{12}$$

in which $F(q;\sigma,\delta)$ is the probability that Q is smaller than or equal to an arbitrary value q; σ and δ are distribution parameters representing the scale parameter and the shape parameter, respectively.

5.2 Crack Exceedance Probability

For B=1, the probability that crack size $a(N)$ will exceed any given (say, maximum allowable) crack size $a^\bullet$ can be derived and expressed as

$$\Pr\{a(N) > a^\bullet\} = \Pr\left\{Q > \frac{\ln[a^\bullet / a(N_0)]}{N - N_0}\right\} = \exp\left[-\left(\frac{\ln[a^\bullet / a(N_0)]}{\sigma(N - N_0)}\right)^\delta\right] \tag{13}$$

For $B=b\neq 1$ the maximum allowable crack size exceedance probability for a single crack is given by

$$\Pr\{a(N) > a^\bullet\} = \Pr\left\{Q > \frac{[a(N_0)]^{-(b-1)} - [a^\bullet]^{-(b-1)}}{(b-1)(N - N_0)}\right\} = \exp\left[-\left(\frac{[a(N_0)]^{-(b-1)} - [a^\bullet]^{-(b-1)}}{\sigma(b-1)(N - N_0)}\right)^\delta\right]. \tag{14}$$

It will be noted that the crack exceedance probability can be used for assigning sequential in-service inspections [6].

6 Stochastic Fatigue-Crack Growth Lifetime Variability Models

These models allow one to characterize the random properties, which seem to vary during crack growth (see Figure 4), via crack growth equation with a stochastic noise V dependent, in general, on the crack size a:

$$\frac{1}{(b-1)q}\left(\frac{1}{a_0^{b-1}} - \frac{1}{a^{b-1}}\right) = N - N_0 + V \tag{15}$$

$$\ln\left(\frac{1}{(b-1)q}\left(\frac{1}{a_0^{b-1}} - \frac{1}{a^{b-1}}\right)\right) = \ln(N - N_0) + V \tag{16}$$

$$\frac{1}{a_0^{b-1}} - \frac{1}{a^{b-1}} = (b-1)q(N - N_0) + V, \tag{17}$$

$$\ln\left(\frac{1}{a_0^{b-1}} - \frac{1}{a^{b-1}}\right) = \ln[(b-1)q(N - N_0)] + V, \tag{18}$$

and so on, where $V \sim N(0,\sigma^2(b,q,N))$, $a_0 \equiv a(N_0)$, $a \equiv a(N)$. They are suited to account for this type of variability. The ones however cannot explain the variability of the crack growth rate from specimen to specimen.

6.1 Crack Exceedance Probability

If $V \sim N(0,\sigma^2)$ in (15), then the probability that crack size $a(N)$ will exceed any given (say, maximum allowable) crack size $a^\bullet$ can be derived and expressed as

$$\Pr\{a(N) > a^\bullet\} = \Phi\left(\left[N - N_0 - \frac{[a_0]^{-(b-1)} - [a^\bullet]^{-(b-1)}}{(b-1)q}\right] \Big/ \sigma\right), \tag{19}$$

where $\Phi(.)$ is the standard normal distribution function,

$$\Phi(\eta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\eta} \exp(-x^2/2)dx. \tag{20}$$

If $V \sim N(0,[(b\text{-}1)\sigma(N\text{-}N_0)^{1/2}]^2)$ in (17), then the probability that crack size $a(N)$ will exceed any given (say, maximum allowable) crack size $a^\bullet$ can be derived and expressed as

$$\Pr\{a(N) > a^\bullet\} = \Phi\left(\left[\frac{(b-1)q(N-N_0) - ([a_0]^{-(b-1)} - [a^\bullet]^{-(b-1)})}{(b-1)\sigma(N-N_0)^{1/2}}\right]\right). \tag{21}$$

In this case, the conditional probability density function of a is given by

$$f(a;a_0,N_0,N) = \frac{a^{-b}}{\sigma[2\pi(N-N_0)]^{1/2}} \exp\left(-\frac{1}{2}\left[\frac{(a_0^{-(b-1)} - a^{-(b-1)}) - (b-1)q(N-N_0)}{(b-1)\sigma(N-N_0)^{1/2}}\right]^2\right). \tag{22}$$

6.2 Data Analysis for a Single Crack

Consider the regression model corresponding to (17). Because the variance is non-constant (17) is a non-standard model; however, on dividing by $(N\text{–}N_0)^{1/2}$ the model becomes

$$\frac{a_0^{1-b} - a^{1-b}}{(N-N_0)^{1/2}} = (b-1)q(N-N_0)^{1/2} + W, \tag{23}$$

where W is normally distributed with mean zero and standard deviation $(b\text{–}1)\sigma$ independent of N. Thus if b is known the estimator of $(b\text{–}1)q$ is just the least-squares estimator of the coefficient in Equation (23) and the estimate of $(b\text{–}1)\sigma$ is just the estimate of the variance of the regression. It remains to determine what to do about b.

Given the data describing a single crack, say a sequence $\{(a_i, N_i)\}_{i=1}^{n}$, it is easy to construct a log-likelihood using the density given by (22) and estimate the parameters b, q and σ by maximum likelihood. The log-likelihood is

$$L(b,q,\sigma;\{(a_i,N_i)\} = -b\sum_{i=1}^{n}\ln a_i - n\ln\sigma - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{a_0^{1-b} - a_i^{1-b} - (b-1)q(N_i - N_0)}{(b-1)\sigma(N_i - N_0)^{1/2}}\right)^2. \quad (24)$$

Inspection shows that this differs from the standard least-squares equation only in the term $-b\Sigma\ln a$, where the subscript i has been dropped. The likelihood estimators are obtained by solving the equations

$$\mathrm{d}L/\mathrm{d}b = 0;\quad \mathrm{d}L/\mathrm{d}q = 0;\quad \mathrm{d}L/\mathrm{d}\sigma = 0. \quad (25)$$

In this case the equations have no closed solution. However, it is easy to see that the estimators for q and σ given b are the usual least-squares estimators for the coefficients in (23) conditioned on b,

$$\hat{q}(b) = \frac{1}{b-1}\left(na_0^{1-b} - \sum_{i=1}^{n} a_i^{1-b}\right)\left(\sum_{i=1}^{n}(N_i - N_0)\right)^{-1}, \quad (26)$$

$$\hat{\sigma}^2(b) = \frac{1}{n(b-1)^2}\sum_{i=1}^{n}\frac{[a_0^{1-b} - a_i^{1-b} - \hat{q}(b)(b-1)(N_i - N_0)]^2}{N_i - N_0}, \quad (27)$$

and on substituting these back in the log-likelihood gives a function of b alone,

$$L(b) = -b\sum_{i=1}^{n}\ln a_i - n\ln[\hat{\sigma}(b)] - n/2. \quad (28)$$

Thus the technique is to search for the value of b that maximizes $L(b)$ by estimating q and σ as functions of b and substituting in $L(b)$. In this study a simple golden-section search worked very effectively.

6.3 Pooling Data

When several experiments have been performed it is possible to combine the log-likelihoods from each experiment to give estimators of the parameters of interest. Suppose that several experiments have been performed. Each experiment is labelled with j, j runs from 1 to m, and yields n_j observations. The data are then a set of sequences $\{(a_{jk}, N_{jk})\}$, with j=1, …, m, k=1, …, n_j. The log-likelihood for the whole set of experiments is simply the sum of the log-likelihoods for the individual cracks; writing $L_j(b_j, q_j, \sigma_j)$ for the log-likelihood for the jth crack gives

$$L_j(b_j,q_j,\sigma_j;\{(a_{jk},N_{jk})\}) = -b_j\sum_{k=1}^{n_j}\ln a_{jk} - n_j\ln\sigma_j$$

$$-\frac{1}{2}\sum_{k=1}^{n_j}\left(\frac{a_0^{1-b_j} - a_k^{1-b_j} - (b_j-1)q_j(N_k - N_0)}{(b_j-1)\sigma_j(N - N_0)^{1/2}}\right)^2 \tag{29}$$

and

$$L = \sum_{j=1}^{m} L_j(b_j,q_j,\sigma_j;\{(a_{jk},N_{jk})\}). \tag{30}$$

The global log-likelihood can be used to investigate explicit parametric models for the parameters, or simply as a way to pool data. Estimation by maximum likelihood proceeds exactly as above; the q_j and σ_j are obtained as ordinary least-squares estimators from equations like (26) and (27), one for each crack, and substituted back into the log-likelihood to yield

$$L(b_1,b_2,...,b_m) = -\sum_{j=1}^{m} b_j\sum_{k=1}^{n_j}\ln a_{jk} - \sum_{j=1}^{m} n_j\ln[\hat{\sigma}_j(b_j)] - \frac{1}{2}\sum_{j=1}^{m} n_j. \tag{31}$$

When the cracks are all assumed to be independent with distinct parameters the estimators from the joint log-likelihood are precisely those obtained by estimating from each separately as outlined above.

If a common value of b is used and the q_j and σ_j are assumed to absorb most of the experimental variability, the joint log- likelihood reduces to

$$L(b) = -b\sum_{j=1}^{m}\sum_{k=1}^{n_j}\ln a_{jk} - \sum_{j=1}^{m} n_j\ln[\hat{\sigma}_j(b)] - \frac{1}{2}\sum_{j=1}^{m} n_j. \tag{32}$$

7 Stochastic Fatigue-Crack Growth Parameter and Lifetime Variability Models

These models allow one to describe the uncertainties in the fatigue-crack growth of the Paris-Erdogan law via crack growth equation with a stochastic noise dependent, in general, on the crack size, and parameters modelled as random variables in order to characterize the random properties, which seem to vary both from specimen to specimen and during crack growth (see Figure 4). In other words, the stochastic fatigue-crack growth parameter and propagation lifetime variability model (with respect to the parameters B and Q, modelled as random variables, and the stochastic noise V dependent, in general, on the crack size a) may be given, for example, as

$$N - N_0 = \frac{1}{(B-1)Q}\left(\frac{1}{a_0^{B-1}} - \frac{1}{a^{B-1}}\right) + V. \tag{33}$$

7.1 Crack Exceedance Probability

In this case, the probability that crack size $a(N)$ will exceed any given (say, maximum allowable) crack size $a^{\bullet}$ can be derived and expressed as

$$\Pr\{a(N) > a^{\bullet}\} = \mathrm{E}\left\{\exp\left[-\left(\frac{[a(N_0)]^{-(b-1)} - [a^{\bullet}]^{-(b-1)}}{\sigma(b-1)(N - N_0 + V)}\right)^{\delta}\right]\right\}. \tag{34}$$

8 Example

Let us assume that a fatigue-sensitive component has been found cracked on n=10 aircraft within a warranty period. Here a fleet of ten aircraft have all been inspected.

Table 1: Inspection results.

Aircraft	Flight hours (N_i)	Crack size (mm) (a_i)
1	3000	1
2	2300	0.5
3	2200	1
4	2000	2
5	1500	0.8
6	1500	1.5
7	1300	1
8	1100	1
9	1000	1
10	800	1

It is assumed that cracks start growing from the time the aircraft entered service. For typical aircraft metallic materials, an initial discontinuity size (a_0) found through quantitative fractography is approximately between 0.02 and 0.05 mm. Choosing a typical value for initial discontinuity state (e.g., 0.02 mm) is more conservative than choosing an extreme value (e.g., 0.05 mm). This implies that if the lead cracks can be attributed to unusually large initiating discontinuities then the available life increases.

We test a goodness of fit of the data of Table 1 with the Weibull fatigue-crack growth parameter variability model (see Case 1), where

$$\widehat{Q}_i(b) = \frac{a_0^{1-b} - a_i^{1-b}}{(b-1)(N_i - N_0)}, \quad i = 1(1)n, \tag{35}$$

with a common value of b, a_0=0.02, and N_0=0.

8.1 Goodness-of-Fit Testing

We assess the statistical significance of departures from the Weibull model by performing empirical distribution function goodness-of-fit test. We use the S statistic [7]. For complete datasets, the S statistic is given by

$$S(b)=\frac{\sum_{i=[n/2]+1}^{n-1}\left(\frac{\ln(\hat{Q}_{i+1}(b)/\hat{Q}_i(b))}{M_i}\right)}{\sum_{i=1}^{n-1}\left(\frac{\ln(\hat{Q}_{i+1}(b)/\hat{Q}_i(b))}{M_i}\right)}=0.43, \tag{36}$$

where $[n/2]$ is a largest integer $\leq n/2$, $\hat{Q}_i$ is the ith order statistic, the values of M_i are given in Table 13 [7]. The rejection region for the α level of significance is $\{S>S_{n;1-\alpha}\}$. The percentage points for $S_{n;1-\alpha}$ were given in [7]. The value of b is one that minimizes $S(b)$. For this example, $b = 0.87$ and

$$S=0.43 < S_{n=10;\ 1-\alpha=0.95}=0.69. \tag{37}$$

Thus, there is not evidence to rule out the Weibull model. Using the relationship (4), the inspection results can be extrapolated from the expected initial crack size, a_0, to the time of the next inspection when the maximum allowable crack size is equal to $a^{\bullet}$=10 mm as presented in Table 2.

Table 2: Predicted next inspection results.

Aircraft	Maximum allowable crack size $a^{\bullet}$ (mm)	Next inspection time (flight hours)
1	10	5626
2	10	5503
3	10	4126
4	10	3033
5	10	3030
6	10	2477
7	10	2438
8	10	2063
9	10	1875
10	10	1500

9 Conclusion

The authors hope that this work will stimulate further investigation using the approaches on specific applications to see whether obtained results with it are feasible for realistic applications.

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