# Cellular Automata with Anticipation: Some new Research Problems

Alexander Makarenko
Institute for Applied System Analysis at
National Technical University of Ukraine (KPI),
37 Pobedy Awenue, 03056, Kyiv- 56, Ukraine,
makalex@i.com.ua

**Abstract**
Recently some investigations on model of cellular automata (game 'Life') with anticipation ('LifeA') had been developed with accounting anticipatory properties, which indicate some interesting types of behavior of such cellular automata. One of new aspect is appearance of multiple solutions of cellular automata. The considering of such solutions follows to attemps of remembering and refreshing some concepts considered by D. M. Dubois, and collecting a list of some existing issues for further detailed considerations and implementation of such concepts, especially of hyperincursion.
**Keywords**: cellular automata, anticipation, hyperincursion, multivaluednes.

## 1. Introduction

Cellular automata (CA) is a well known class of models which uniteы a regular structure of space (space devided space on the cells); simple rules for the cells state changes with time and presumable complex behavior (Toffoli&Margolus, 1991; Illiacinski, 2001; Collision, 2002; Delorme&Mazoyer, 1999) and many papers in journals 'Physica D' and 'Theoretical Computer Science'). Cellular automata has a long period of development since the investigations by J.Neumann and S.Ulam . Many interesting ideas in the field of cellular automata had been proposed. One of the most important is relation with computational processes.
Since the beginning of 90 – th in the works by D. M. Dubois - see (Dubois, 1992a,b; 1997) the idea of strong anticipation had been introduced :
" Definition of an incursive discrete strong anticipatory system:
an incursive discrete system is a system which computes its current state at time t, as a function of its states at past times , … , t-3, t-2, t-1, present time, t , and even its states at future times t+1, t+2, t+3, …

$$x(t+1) = A(\ldots, x(t-2), x(t-1), x(t), x(t+1), x(t+2), \ldots, p) \qquad (1)$$

where the variable x at future times t+1, t+2, .... is computed in using the equation itself.
….

Definition of an incursive discrete weak anticipatory system:
an incursive discrete system is a system which computes its current state at time t, as a function of its states at past times , ... , t-3, t-2, t-1, present time, t , and even its predicted states at future times t+1, t+2, t+3, ...

$$x(t+1) = A(\ldots, x(t-2), x(t-1), x(t), x^*(t+1), x^*(t+2), \ldots, p) \quad (2)$$

where the variable x* at future times t+1, t+2, .... are computed in using the predictive model of the system" (Dubois, 2001, p. 447).
Some first examples of such CA were 'fractal machines' (Dubois, 1997, 1998). Then many objects with anticipation had have been investigated – see the papers of D. M. Dubois, especially in International Journal of Computing Anticipatory System and in Proceedings of CASYS conferences in Proceedings series of American Institute of Physics (CHAOS, 2008).
Special case of incursion had been found when the possibility of many solutions exists:
"In the same way, the hyperincursion is an extension of the hyper recursion in which several different solutions can be generated at each time step" (Dubois, 1997, p.98).
Some objects of CA with hyperincursion had been investigated and many ideas related to hyperincursion had been proposed: hyperincursion CA, anticipation logic, memory element, hyperincursion field (Dubois, 1997, 1998).
Some of proposed objects had been considered in detailes but some issues are still the topics of further investigations and implementations. It seems that for further development of concept more concrete examples and models should be considered, especially in case of distributed systems. Recently one new example of CA with anticipation had been investigated numerically – namely the modification of well – known CA game 'Life' by J.Conway (Goldengorin et al, 2007, 2008).
Here we pose a shot description of game "Life" (the detail and formal description see in any books on cellular automata (Toffoli&Margolus, 1991; Illiacinski, 2001; Collision, 2002; Delorme&Mazoyer, 1999; Wolfram, 2002).
Let $Z_2$ be a two – dimensional grid; S is a finite set of states of elements (cells) of the grid, $s_i$ from S is a state of *i*-th cell from $Z_2$. Local rule updating the state of a cell *i* from $Z_2$ is a predefined transform $T_i$ , that turns the present state of a cell to the state at next moment of time.
The rules of state change are local. This means that the state of *i*-th cell depends on the states of cells from its local neighborhood $N_i$.
In "classical" Conway's game "Life" every cell has only two states (denoted as "0" and "1", S={0,1}); a neighbourhood used in local rules is a Moore's neighbourhood consisting of 8 neighbours.
In a majority of "Life" descriptions cell states of "0" and "1" are treated according to the biological interpretation as "dead" and "living" cell. Thus, cells' state change may be defined via the following rule:
1. If a living cell has two or three living neighbours then it stays alive at the next moment of time; otherwise it dies.

2. If a dead cell has exactly three living neighbors then it turns to a "living" state at the next moment of time.

The anticipation of some kind may be introduced to the rules of the game. We have realized numerically one modification when the two rules above have been changes by the accounting the number of living neigbours in Moore's neighbourhood at the next moment of time. Remark that such type of anticipation accounting correlates with the structure of neural nertwork rules.

The choise of variant of game 'Life' with anticipation is determinated by importance of the game 'Life' for theory of computation. Moreover a lot of theoretical and practical investigations of game 'Life' exist with interpretations in automata vtheory, physics, biology (Toffoly&Morgolus, 1991; Illiachinski, 2001; Collision, 2002).

Just first computational investigations of considered variant of game 'Life' with anticipation follows to conclusion on multivalued solutions possibilities that to the realization of hyperincursion. That is in such case many values of cell's state is possible. Remark that for CA sometimes very useful is to describes the behaviour of the model in terms of 'configurations'. The configuration is the collection of states of CA cells at given time moment. So the case of hyperincursion corresponds to the possibilities of many configurations existing at the same time. Here at the Fig. 1 we schematically represent the presumable case of many configurations (the results of compute investigation on configurations for one modification of game 'Life' with anticipation see in (Goldengorin at al, 2008)).

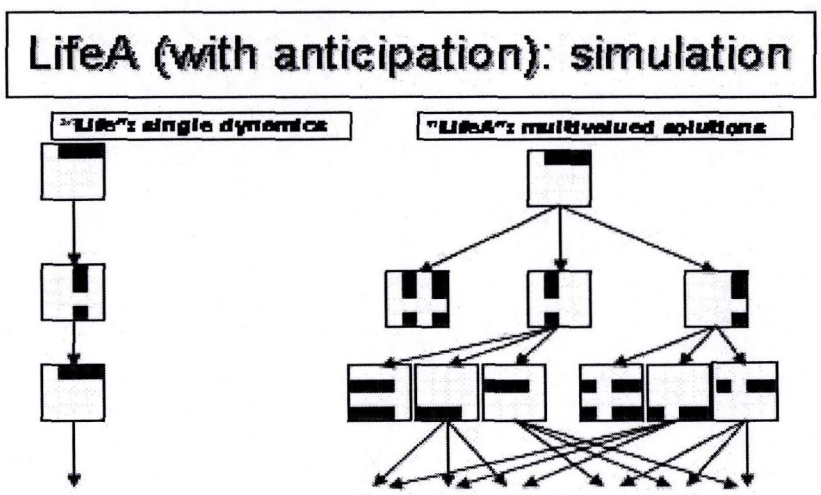


**Figure 1.** Set of all possible configurations in the computations with the periodical boundary conditions for 16 – cells example of game "Life" with anticipation.

The black squares have the state "1" and all white cells (places) correspond to the state "0". The arrors display the transitions with time from one configuration to others on discrete time steps from initial configuration (top) to next time steps. At the left side of the picture we display the dynamics of classical game "Life" with the same initial conditions.
Namely the presumable existing of such multivaluednes and at the same time importance of CA game 'Life' allows to pose the tasks of remembering some ideas from earlear works by D. M. Dubois on hyperincursion and posing some questions on implementation of such ideas in CA and in real systems in computation, physics, biology.
So below in given paper we try to remember and describe some possibilities to reconsidering old and posing research problem (or at least some presumable associations).
So, the basic distinctive feature is presumable multiplicity of solutions and configurations at every instant of time. Possible consequences of these facts and the problems of reconstruction of automata theory are considered in next sections of paper.

## 2. Some New Possibilities in Old Problems

At it was pointed out in previous section, the accounting of anticipation in cellular automata follows to presumable multivaluedness of solutions. In this section we pose discussion related to presumable generalizations in some fields of knowledge: informatics, physics, biology, consciousness investigations, decision theory etc. Here we propose only the list of possible topics because of large number of presumable fields of development (we hope to implement the details of some of the topics in further investigations).
Remark that recently the investigations of possibilities of multivaluedness is very interesting problems in science (as for hyperincursion as for other types of multivaluedness).

### 2.1 Informatics and Cybernetics

2.1.1. Theory of Automata.

Since the investigations by A.Turing, J. von Neumann classical automata theory had been developed. One of the main result is the classical definition of Turing machine (see for example (Aho et all, 1976) as 5 - tuple: M (Q, I, O, T, F) where I is the set of in input symbols, O is the set of output symbols, Q is the set of states of machine, T - the transition rules for state, F is the collection of rules for automata output generation. As the generalization we may propose possible multivaleadness of all components in definition. Remark that more far generalizations are possible, including self-reference.
So the old and customary notions and definitions from machine and automata theory need the re-examination and reformulation. First such notion is connected with computation. According Turing – Cherch thesis the physically relevant solutions of

equations may be realized by Turing machine. Accepting generalized machine follows to the generalized Turing – Cherch thesis and extending of accepted physical solutions. Just the basic notion of classical algorithm (see for example definitions and history in (Maltcev, 1986)) may receive the reformulation in our case.
Also in classical automata theory different description of automata and machines are equivalent (by tables, by graphs of states and their transitions, by logical calculus, by operators). So the generalization of Turing machine will follows to foundations of new such objects. Remember the comments of D. M. Dubois (Dubois, 2001) on such issues. Remark that the notions of automata and machines may receiver further generalization by formulation it in the frames of category and functors theory.

### 2.1.2 Multivalued Computers

Having at hand theoretical concepts of multivalued automata theory the problem of searching principles for multivalued computer may be posed. In such case all elements of computer should be reconsidered: realization of logical rules, algorithms, the results of operating, time in the computer, parallel and concurrent processes etc. The second problems is physical realization of such M – computers. As physical elements may serve existing multivalued elements or systems with hyperincursion (Dubois, 1998), other physical phenomena with multiple solutions (voltage ark, broken fronts of waves in inviscid vater etc.). Also we should remark the approach for building computation on the base of interaction of specific disturcances (patterns) of cellular automata (including game "Life" example) (Collision, 2002). Patterns in hyperincursive automata are the candidates for generalization of such computations to multivalued cases. Also veryt prospective are quantum mechanical systems.

### 2.1.3 Discrete and Digital World

One of the interesting ideas related to classical CA is the idea to consider the Universe as computer, or as the result of computational processes, or as giant cellular automata (K.Zuse, L.Penrose, D.Finkelstein, S.Wolfram, J.Shmidhuber, M.Tegmark; see also many references in physical literature). Following the idea of minimal length in quantum mechanics (Plank length) the space (or space – time) may be considered as discrete object (composed from elementary cells). So the idea on Universe as cellular automata is rather natural. In correlation with automata theory is the idea that Universe is computer (or automata of special kind). Sometimes it named Digital Physics. Some examples of transitions from differential models to the discrete type models with anticipation had been proposed by D. M. Dubois.
According the new possibilities (multivaluedness) we can propose to further implementations of the idea of Universe as multivalued CA (M – Universe; or multivalued computer; automata, etc.). It seems that at first such ideas had been proposed by D. M. Dubois (Dubois, 1992, 1997) but anyway further investigations are important. All schemes of single – valued approach still are valued but the main change

is in the interpretation of solutions and result. Remark here that if remember some interpretations of quantum mechanics (see below).

2.1.4. Quantum Mechanical Analogies

As it was remarked earlier (Makarenko, 2003, 2005) multivaluedness in models of social systems with anticipation allows considering some relations to some interpretations of quantum mechanics. The behavior of such systems reminiscent the pattern of the world in Everett interpretation of quantum mechanics (Everett, 1957). So in case of CA models with anticipation the presumable multivaluedness may follow to the uncertainty of system output in time and really to quantum – mechanical analogies in classical system. It seems that the first correlation between anticipatory systems and quantum mechanics had been proposed by D. M. Dubois (Dubois, 1997). But such point of view (accounting of anticipation) may help in some open question in quantum mechanics. First of all in Everett interpretation the possible multivaluedness (existence of many worlds) is postulated. Namely anticipation at each moment of time and space point may lead to such possibility. The degree of uncertainty may depend on the horizon of anticipation and from environment condition and may be reduced to zero in transition to many – particle classical systems. The second still open problem is the measurement and collapse of wave function.

2.1.5. Consciousness and Life Sciences.

It is well known that classical cellular automata have as backgrounds biology and life science (Illliachinski, 2001; Toffoli&Margoolis, 1991; Wolfram, 2002). Moreover now the ideas of theory of CA have revised impact on the development of life sciences. So it is natural to consider the possible utility of generalized cellular automata for the life science.
As the first important problem we would like to remark the problem of consciousness. Understanding of consciousness nature and its manifestations is important (may be just crucial) for all recent theoretical sciences: informational technologies and artificial intelligence, computer science, medicine, evolution theory, bioinformatics, quantum mechanics, nanoscience etc. (Penrose, 1996, Santoli, 2001, Hammeroff, 1987). Remark that the origin of consciousness is one of the main problems in investigations of essence, definitions and origin of life (Hammeroff, 1987, Penrose, 1996, Bedau, 1998). Many investigations had been proposed with presumable solutions just as critics of proposed approach. Say by B.Eckdahl the inability of classical computers for intelligence is argumented by logical considerations. One of the approaches to consciousness consists in relation of consciousness with ability to make choose in environment by some mechanisms (different according different authors) (M.Pitkanen, P.Marcer, C.Deutch, S.Santoly, S.Hammeroff). Frequently life derives (or connects) with ability to have consciousness. It is assumed that the properties of anticipating systems (including CA with anticipating) are well adjusted to such concepts of consciousness (May be at first such idea considered by D. M. Dubois). In case of

hyperincursion we also have many possibilities and choose events at each time moment for all elements. It may have some surprising consequences for many systems. As the first example we remember the ideas of cellular automata architecture at the level of microtubules of living cell (neuron in the brain) (Hammeroff, 1987). Remark that Hammeroff and co-authors had supported that microtubules has own consciousness (S.Hammeroff, et al, 1996). But accepting the anticipatory properties of microtubules elements at such level we may consider the anticipatory CA architecture and then we can accept the hypothesis on consciousness. At second in accordance with subsection 2.1.2 on automata theory the logic of CA with anticipation may be non – usual (one of the first examples see in (Dubois, 1998)). So the automata with anticipation may consider as the candidate for artifial intelligence which may be corresponded to natural intelligence. Thirdly much more natural systems may fall in the class of systems with consciousness. For example it is presumable that the MultiUniverse (M – Universe, Universe with anticipation) may have the consciousness property. Also our models of society with anticipation (Makarenko, 2003) may follows to the conclusion of possibility of collective intelligence of society.

2.1.6. Statistical Mechanics

Remark that discussed results on cellular automata with anticipation may be useful for considering other branch of theoretical physics – namely statistical physics. The notion of statistical ensembles and number of possible configurations on the surface of constant energy is one of the milestones of statistical mechanics and calculation of Boltzman entropy. But the the origin of ensemble of systems is interesting question. As we remarked in the investigation of 'Life' game with anticipation, collection of cells stats in CA named configuration. In CA with anticipation presumably many configurations can exist at each moment of time. As the hypotheses we may pose the idea that one of the sources of ensembles (possible configurations) may be anticipatory property.

2.1.7. Decision – Making

Essential component of CA with anticipation is the process of choosing single component of solution. At first the problem of many pathes and of choise of single part in some anticipating systems had been considered at (Dubois, 1992, 1997). So it is necessary to make more detailed investigations of these processes. Moreover searching of appropriate framework for representing and investigation such objects is necessary. The general framework should represent as as the multivaluedness as the choising mechanism. That is useful mathematical structures should be defined for investigation of nonlinear distributed (discrete and continuous) systems. (Remark that some correspondence can be found between discrete and continuous system at the lattices). In the case of hypericursion we have the structure of solutions with possible branching of solutions at each element and each moment of time. It requires further mathematical investigations of models with multiple branching. The choosing process may

corresponds to application of special choosing operator. Remark that in the existing theories of decision – making the theories usually are based or on probabilities (measures) arguments or on decision – trees technique. In our case the corresponding probabilities may be derived from the history of the system behavior and the structure of choosing operator will be the result of learning processes vithing the system. It is interesting that the branching structures of CA solutions with anticipatory remember the branching space – time structures (see N.Belnap). May be this is also one more argument to the analogies between CA with anticipation and quantum systems. The future adequate formalization of anticipatory systems with choose operation may be useful for considering still weakly formalized society processes (such as economical, political, social psychology etc.).

## 3. Conclusions

Thus, in the proposed paper some presumable consequences of numerical investigation of one example of anticipation accounting in cellular automata game 'Life" are considered. As one of important properties it should be noted of appearance of the multi - valued solutions (the realization of hyperincursion). The results of numerical investigations follows to needs of remembering and reconsidering some earlear issues on anticipation and hyperincursion from the papers by D. M. Dubois: possible generalization of Turing machine, anticipative logic, fractal machines, consciousness, algorithms, computation processes. Of course proposed considerationss may looks like the remarks, comments and list of intentions. It seems that all of issues may require a separate and thorough formulation and investigationse. But some of them are more or less evident and useful: 1) further investigation of new modifications of CA with anticipation with the goal to better understand the hyperincursion propery and their realization; 2) strict mathematical formalization and investigation of proposed models as mathematical objects; 3) more detail investigations on the choise rules and related topics. Firther implementation of concepts of anticipation, hyperincursion and multivaluadness may be very interesting to the problems of consciousness, life, quantum mechanics, statistical mechanics and decision – making theory.

**Acknowledgements**
The author acknowledges the University of Groningen for financial support for preparing some materials for paper and access to electronic library of RuG.

## References

ACRI, (2006) Cellular Automata. Proceedings of 7th Int. Conf. ACRI2006, Perpignian, France, LNCS 4173, Springer.

Aho a., Hopcroft J., Ullman J. (1976) The design and analysis of computer algorithms. Addison – Wesley, Reading, Mass.

Bedau M. (1998) Four Puzzles about Life, Artificial Life, vol. 4. pp.125-140.

CHAOS (2008) CHAOS – Centre for Hyperincursion and Anticipation in Ordered Systems http://www.ulg.ac.be/mathgen/CHAOS

Collision – Based Computing. (2002) Ed. Andrew Adamatzky. Springer – Verlag.

Delorme M., Mazoyer J. (1999) Cellular Automata: A Parallel Model. Springer – Verlag.

Dubois D. (1992) The Fractal Machine, Press Universitaires de Liege.

Dubois Daniel (1997). Generation of fractals from incursive automata, digital diffusion and wave equation systems. BioSystems, 43, pp. 97-114.

Dubois D. (1998) Introduction to Computing Anticipatory Systems. International Journal of Computing Anticipatory Systems, 2, 3-14.

Dubois Daniel (2001). Incursive and hyperincursive systems, fractal machine and anticipatory logic. Computing Anticipatory Systems: CASYS 2000 – Fourth International Conference. Published by the American Institute of Physics, AIP Conference Proceedings 573, pp. 437 – 451.

Dubois Daniel, Belly M. (2000) Generating self-symmetrical fractals by hyperincursion automata and multiple reduction copy machine. International Journal of Computing Anticipatory Systems. vol.6, pp. 95 – 115.

Everett H. (1957) "Relative State" Formulation of Quantum Mechanics. Reviews of Modern Physics, vol. 29, n.3. Pp.454 - 465.

Feynman R. (1986) Quantum mechanical computers. Foundation of Physics, Vol.16, n.6. pp. 507- 531.

Goldengorin B., Makarenko A., Smilianec N. (2006) Some applications and prospects of cellular automata in traffic problems Cellular Automata, Proceed. Int. Conf. ACRI'06, LNCS 4173, Springer. pp.532-537

Goldengorin B., Krushinskiy D., Makarenko A., Smilianec N. (2007) Cellular Automata with Anticipation. Examples and Presumable Applications. Abstract BOOK of CASYS'07, Liege, Belgium, 06 – 11 August 2007. 1 p.

Goldengorin B., Krushinskiy D., Makarenko A., Smilianec N. (2008) Cellular Automata with Anticipation. Examples and Presumable Applications. Proceedings of CASYS'07, International Journal of Computing Anticipatory Systems. 13 p. (Submitted)

Hammeroff S. (1987) Ultiate computing Biomolecular Consciousness and Nano Technology. Elsevier.

Hammeroff S., Penrose C. (1996) Orchestrated Objective Reduction of Quantum Coherence in Brain Microtubules: The "Orch OR" Model for Consciousness. Mathematics and Computer Simulation. Vol.40. pp.453- 480.

Illachinski A. (2001) Cellular Automata. A Discrete Universe. World Scientific Publishing, Singapoure.

Makarenko A. (2002) Anticipating in modeling of large social systems - neuronets with internal structure and multivaluedness. International .Journal of .Computing Anticipatory Systems. vol. 13, pp. 77 - 92..

Makarenko A. (20040 Anticipatory agents, scenarios approach in decision- making and some quantum – mechanical analogies. International Journal of Computing Anticipatory Systems, 15, 217 - 225.

Maltcev A. (1986) Algorithms and recursive functions. Moscow, Nauka.
Marcer P. (2001) Quantum millennium, quantum universe, quantum biosphere, quantum man - or what physicists can teach biologists, and biology, physics. International Journal of Computing Anticipatory Systems. Vol.10. pp.249- 264.
John von Neumann (1966) Theory of self-reproducing automata, Ed. A.Burks, Univ. of Illinoice, Urbana and London.
Penrose R. (1989) The Emperor's New Mind. Concerning Computers, Mind and The Laws of Physics. Oxford Univ Press.
Santoli S. (2001) Mesophysical Anticipatory Cell Behavior – Enroute to Quantum and Classical Nanobiology. Int. J. Computing Anticipatory Systems. vol. 10. p.382 – 401
Toffoli T., Margolis N. Cellular automata computation. Moscow, Mir, 1991.
Wolfram S. (2002) New kind of science. Wolfram Media Inc., USA.