# Riemann Hypothesis

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**Abstract**
A novel approach to a proof of the Riemann Hypothesis (that all the zeros of the Zeta function lie on the line $x = ½$) is presented. It is based on the universal nilpotent computational rewrite system (NUCRS), derived in the World Scientific book *Zero to Infinity*, from a single nilpotent Dirac operator (Rowlands, 2007), establishing an entirely novel semantic computational foundation simultaneously for both mathematics and quantum physics. Tangible evidence is that the Zeta function is known to represent a quantum system and that the criterion of nilpotence corresponds to (a) Pauli exclusion with unique fermion states spin ½ and (b) an infinite rewrite alphabet that also corresponds to the infinite roots of −1, of which the nilpotent generalization of Dirac's famous quantum mechanical equation is the universal computational order code.
**Keywords:** the Riemann Hypothesis, the nilpotent computational rewrite system, quantum mechanics, the nilpotent Dirac operator, the infinite roots of −1.

## 1 Introduction: The Criterion of NUCRS Nilpotence

Physics at its most fundamental level is entirely concerned with fermions and their interactions gauge bosons being generated by such interactions. In the NUCRS nilpotent formalism (Rowlands 2007) the only requirement for defining the entire quantum mechanical apparatus relating to physics i.e. a fermion and its interactions, is to specify its creation operator in the form $(\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, where $\boldsymbol{k}, \boldsymbol{i}, \boldsymbol{j}$ are quaternion units and $\mathbf{p}$ takes the form of a multivariate or quaternion-like vector. Wavefunction, phase factor, amplitude, spinor structure, vacuum, and quantum mechanical equation are then automatic consequences of the initial definition and do not represent independent information.

Thus there exists in relation to each fermion state space spin $x = ½$, a zero $0_n$ specified by the NUCRS nilpotent criterion $X_n \neq 0$, $X_n^2 = 0_n = 0$ and as fermion state nilpotence also tells us, each such state n is subject to the Pauli exclusion principle and so is unique, i.e. $X_n \neq X_m$ unless $m = n$, and $X_n$, n = 1, 2, 3 ... corresponds to the NUCRS infinite rewrite alphabet. Thus since all the above information is derivable from a single nilpotent Dirac operator, and Diaz and Rowlands (2005, 2006) have shown the NUCRS alphabet corresponds to the infinite square roots of −1, there must exists a mapping $\zeta$ in the complex plane $x + iy$ in respect of the fermion state space corresponding to the line spin $x = ½$ such that all the distinct nilpotent zeros $0_n$ are a point property of this line, where these n zeros, each uniquely distinct, are distinguished from one another by a phase factor $\theta_n$, which quantum theoretically corresponds to a (boson) gauge invariant phase and is calculable in principle from the nilpotent criterion $X_n$ to yield $\theta_n$ which the criterion says is unique. Thus this unknown function $\zeta$ constitutes a unique analytic conformal mapping to the integers $n$, which we can hypothesize is the Riemann Zeta function $\zeta$ and is such that the Riemann Hypothesis follows from the argument above. (The constraint of nilpotency allows the description of each fermion to be reduced to a single phase factor so that the relativistic quantum mechanical system in question, can be completely represented by a single operator, from which calculation then proceeds.)

Thus in the NUCRS this Zeta function $\zeta(n)$ may originate from the definition of the fermion as a point-like particle, with 0 dimension, and, in fact, the only point-like concept in nature, space, for example, being definable only in terms of extension. Here, we use the kissing number as a way of describing the most efficient packing of space using identical spheres (see next section) or, in dimensions greater than 3, hyperspheres. In any dimensionality of space, it is the number of equally-sized spheres or hyperspheres that can simultaneously touch any central one. Now, according to the Minkowski-Hlawaka theorem, lattices must exist in $n$ dimensions with hypersphere packing densities greater than or equal to $\zeta(n) / 2^{n-1}$. However, if we define $\zeta(n)$, for complex $n = a + bi$, by the summation,

$$\zeta(n)=\sum_{t=1}^{\infty}\frac{1}{t^n} \tag{1}$$

over integer values of $t$ from 1 to $\infty$, then the Riemann hypothesis projects an infinite number of values of 0 for $\zeta(n)$, when $a = ½$, and $b$ seemingly random[1], which is suggestive of fermions with spin ½ and effectively random energy and momentum. Perhaps this means that, to take space to the limit of fermionic point-singularity (the only physical thing that can conceivably be defined in this way), with the complexity introduced by fermion spin, and zero minimum packing, we need to generate an infinite number of possible values of $\zeta(n) = 0$ by generating an infinite number of almost random possible states of energy and momentum.[1]

## 2 An independent proof that the algebraic function, that NUCRS determines, is unique

In Conway (1976), Alling (1988) where a universal computational rewrite mechanism is the basis for the non standard analysis over the surreal number fields, it is shown that any such field must possess a unique birth order field automorphism or birthordering. For example, Conway constructs the birthordering of the field of all the numbers great and small including the transfinite and the infinitesimal, from their empty set using an order operation which distinguishes L left from R right. It is notable that this birthordering shows that the number 2 and its inverse ½ play a unique role, and that for Conway to prove that all such numbers constitutes the desired universally embedding totally ordered mathematical field, that he has to arbitrarily introduce the

1 It is known that $\zeta$ concerns the distribution of the primes and that these 'play a game of chance' (Kac, 1959).

number $i$ the root of $-1$ by means of rings in order to include the algebraic and transcendental numbers as logical extensions to sums, products, and inverses in his proof. That is to say, the NUCRS is the unique birthorder field automorphism of the algebraic theory of universal nilpotent quantum mechanics (NQM), of which by Deutsch (1985) the integers and computation over integers, i.e. universal Turing computation are both universal sub-fields.[2]

It follows from the fact that the NUCRS specifies computation in terms of a semantics with a universal grammar, that, as independently proved by Deutsch (1985), universal Turing computation which guarantees in principle that any computation will be well formed logically so that the syntax of the language of the computation is correct, is necessarily a subset of universal quantum computation, for which the NUCRS defines the necessary additional semantic logic, because additionally it specifies a universal grammar by means of its infinite alphabet.

## 3 The computer universality of quantum physics

The uniqueness of every fermion state by Pauli exclusion supplies the basis for the canonical labelling(of quantum physics) essential to the theory of the universal quantum computer used by Deutsch in his 1985 paper, Quantum Theory, the Church Turing Principle and the universal quantum computer. That is to say it is Pauli exclusion, which is defined by nilpotence, and applies to all fermions whether free or interacting, which provides us with a new universal quantum principle of calculation, as has been shown by Rowlands in respect to NUCRS, where each such state possesses a complementary vacuum state that of the remainder of universe itself. That is to say that Pauli exclusion, which determines the behaviour of any fermion, whether free or interacting is a universal semantic computational property of quantum physics.

The extra constraint of nilpotency allows the description of each fermion to be reduced a single phase factor so that the relativistic quantum mechanical system in question, can be completely represented by a single operator, from which calculation then proceeds. This happens even though the complementary vacuum state of each fermion state shows that all the fermion states are totally entangled states of the universe as whole. The precise nature of this single computational operator/ Hamiltonian representing this universal entanglement of all the fermions can therefore be postulated. For there exists a unique NUCRS function known to represent a quantum mechanical system, where such fermion states represented in terms of their common property of spin = ½ are each specified by a unique phase factor. Could this be the Riemann Zeta function? In particular, it should have time reversal asymmetry, because the universal nilpotent computational rewrite process with its universal grammar constitutes a dissipative quantum emergent process, from 'zero' / the empty set as the

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2 Conway's generator for the birthordering of the numbers (which Knuth called 'surreal') begins from the empty set/state of no numbers, where it is necessary to assign the first number symbol i.e. the empty set in relation to itself as the value zero and where the symbol zero in relation to itself is not a number, but something more general what Conway calls a game.

totally degenerate set of all the topological states 0, to the non-degenerate set symbolically represented as $\theta_n$, $n = 1, 2, \ldots,$. where n increases without limit. That is to say, each new fermion state which by Pauli exclusion is unique must define an entirely novel state of matter and so will, in relation to the dissipative quantum model of thermodynamics, the Quantum Carnot Engine a generalization of its classical thermodynamic counterpart, be what Scully et al (2003) has called a phaseonium, where this retains a small amount of coherence/entanglement phase $\theta_n$. This process of fermion state emergence thus physically provides the unique field birthorder automorphism or birthordering, referred to above. That it is the Zeta function is also an hypothesis of Berry (1986) who asserts that the imaginary parts of the non trivial zeros of the Riemann Zeta function are the eigenvalues of some still unknown self adjoint Hamiltonian operator with time reversal asymmetry of which phase space trajectories are chaotic, and where, in view of the NUCRS's nilpotent nature $0_n$, it too must be quantum chaotic.

## 4 Universal nilpotent fermion supersymmetry, as realized in 3D space and the complex unit plane

This goes towards the proof that the NUCRS concerns a conformal mapping in 2 dimensions. A way of representing the totally degenerate states $0_n$ cited above is therefore to think of $0_n$ as the origin P of a unit sphere in three dimensions from which the unit vectors from this origin to the surface of the sphere emerge, so that each such ray may be said to represent a unique fermion state relative to some arbitrary fixed ray, in accordance with the fact each fermion quantum mechanical state vector is only defined up to such an arbitrary fixed phase, and only relative gauge invariant phases are measurable. That is, we can think of every P in three dimensional space as modelling the quantum mechanical ray space, so that, every P has this quantum mechanical property. Thus, the 3-dimensionality of space, the fact of which is now experimentally well established, plays, as Rowlands shows in respect to the NUCRS, a central defining role in relation to quantum physics, but only in the case where nilpotence applies so that it is defined by a single unique relative phase factor. (Except in one instance which is chosen to be the arbitrary fixed phase, and in relation to any quantum measurement process can be envisaged as exemplifying the measurement standard against which any measurement must be made.)

### 4.1 The symmetries of 3D space as a central feature of the NUCRS nilpotent quantum mechanical machine order code and grammar, as for example they appertain to quantum holographic encoding / decoding in relation to for example Synthetic Aperture Radars (SARs) and Magnetic Resonance Imaging (MRI)

Consideration of the set of radial vectors to the surface of a unit sphere centre P, as defining each neighbourhood in 3D space, shows that any such vector through the centre P of the sphere and the plane perpendicular to it, are unique and correspond to the symmetries, $U(1)$, $SU(2)$, in relation to the symmetry of the sphere $SU(3)$. Thus each P defining the ray space cited above (Figure 1) where the fundamental quantum mechanical spectral theorem of Hilbert and Von Neumann applies, in 3 dimensions is such that $U(1)$, $SU(2)$ and $SU(3)$ are respectively the symmetry groups, now known to determine the quantizations of the elementary particles of the electromagnetic, weak and strong forces, and $U(1).SU(2)$ is the symmetry of the electroweak force (the photon with its vector bosons) combined in just the right way. Thus (subject to the proviso that every P can indeed be considered as a Lie topological neighbourhood, where the symmetries are those of Lie groups together with their Lie algebras, as defined by their smooth tangent spaces of lines and planes) the NUCRS grammatical sub-alphabet of P, which must differentiate lexicographically between the three axes of space in order to describe three distinct orthogonal spatial axes, may be labelled 'electron', 'muon' and 'tau', in good accord with the Pauli exclusion principle for spin ½ (fermion) anti-commutative states, which applies to each and every unique unit vector and its two perpendicular plane axes at P.

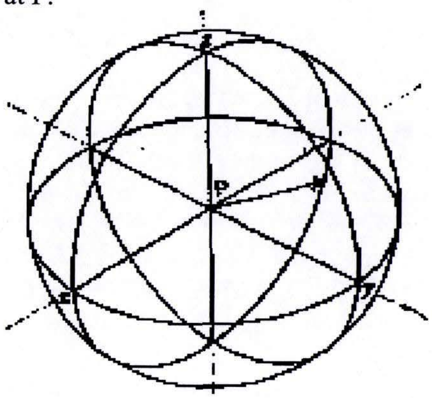


**Figure 1: 3D** ray space at point P.

That is to say, in NUCRS where the elementary particles predicted are exclusively those of the Standard Model (and so exclude gravitons!) and 3D space corresponds, as described above, to a quantum field, the particles with spin ½ known as the neutrinos take on, as a new testable hypothesis of their behaviour, an unexplored role in respect to the axes of 3D space. And remarkably, in respect of this newly proposed '3D spatial quantum field', there also exists a Lie symmetry, as was known to Weyl in 1928, that of the 3D Heisenberg Lie Group $G$, in the form of its nilpotent Lie algebra $g$, which defines its Heisenberg uncertainty, in terms known as a Robertson relation. However as both $g$ and $G$ have Lie duals, there exist corresponding (Lie) exponential differentiable mappings with differentiable inverses, so that Heisenberg uncertainty can, in this case, be used as the actual means to compute geometrically. As, for example, the description of $U(1, C)$ signal processing in the form of quantum holographic encoding / decoding in Magnetic Resonance Imaging (MRI) machines proves (Schempp, 1988, 2006)

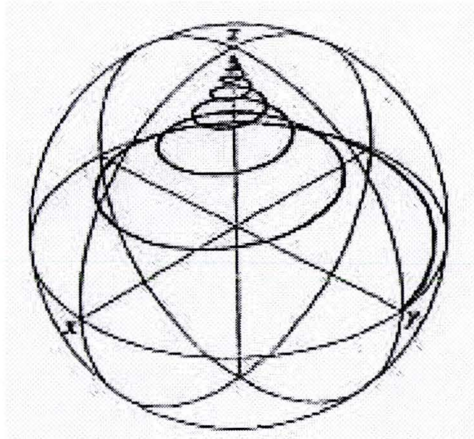


**Figure 2:** Quantum wave collapse and re-expansion.

Figure 2 illustrates how in actual 3D space, the encoding/decoding Fourier transform action (in accord with the Heisenberg uncertainty principle defined by *g* the Lie algebra of *G*) actual happens in MRI. It shows the 'frequency induced signal' *U*(1,C) described by the Heisenberg helix of *G* off resonance losing amplitude (*z* axis) i.e. thermodynamically decaying due to a transverse relaxation effect, but remarkably simultaneously regaining energy due to longitudinal relaxation, so as to embed the *U*(1) signal in the complex plane $C = (x + iy)$, where it can be described as a phase difference. For, with respect to *G* and *g*, *G* represented as the (3 line) matrix

$$\begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix},$$

is such that ($x$ – path difference, $y$ – phase difference) form a Fourier duality encoding pair $(x, y) = (x + iy)$ so embedding the complex plane *C* in *G*, and the infinitesimal Lie generators of g are $\{P, Q, Z\}$, where $[P, Q] = Z$; $[P, Z] = 0$; $[Q, Z] = 0$, are represented by the (3 line) matrices

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad Z = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

having Lie exponential diffeomorphisms

$$\exp P = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad \exp Q = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad \exp Z = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

are the components of the automorphism of $G$, able to describe the inverse Fourier transform action of decoding, i.e. they give rise to the identity exp $(xP + yQ + (z - \frac{1}{2}xy)Z) = G$, where the conformal complex mode coordinates $T = \frac{1}{2}(P + iQ)$ and $T^* = \frac{1}{2}(P - iQ)$ permit through the linear Schrödinger representation $U$ of $G$, a quantum mechanical description at the photon / massless boson level in terms of the creation/ annihilation operators $a = U(T)$ and $a^* = U(T^*)$ of an emitter / absorber model.

Similarly, Rowlands and Diaz (2006) have shown that the NUCRS infinite alphabet determined by the nilpotents $X_n^2 = 0$, $X_n \neq 0$ corresponds to the infinite square roots of $-1$, $n = 1, 2, \ldots$. Thus the symbol $i$ (i.e.'$\sqrt{-1}$') in NUCRS may stand for the whole infinite alphabet, embedding it in a particular complex plane $C$ on a line $i = \sqrt{-1}$, where each letter of the alphabet located on this line denotes a (fermion) anti-commutative state and is unique, so locating this line at a point of an x axis in a particular complex plane at x = spin ½.. This immediately suggests, that the Riemann Zeta function in relation to the famous Riemann Hypothesis represents this NUCRS mapping in the complex plane, so that through the NUCRS alphabet, it describes the wholly entangled nature of the NUCRS Worldview, in terms of the Zeta function's zeros, where by the Riemann Hypothesis, these are non trivial i.e. nilpotent and are all located on the line $x = \frac{1}{2}$; a fact as already cited in line with Berry's hypothesis, that the imaginary parts of the non trivial zeros of the Riemann Zeta function are the eigenvalues of some still unknown self adjoint Hamiltonian operator with time reversal asymmetry of which phase space trajectories are chaotic. It is in agreement with the fact the each letter of NUCRS infinite alphabet has the same 'nilpotent' formulation $X_n^2 = 0$ at all levels of its rewrite structure and so is self similar of fractal dimension 2 and therefore embeddable in the complex plane $C$; i.e. can be mapped in a conformal fashion onto the open unit disc (which is the geometry of $i = \sqrt{-1}$). It thus corresponds to the universal fractal attractor of the Golden number (Figure 3) and relates to the wave behaviour seen at the boundary of the Mandelbrot set.

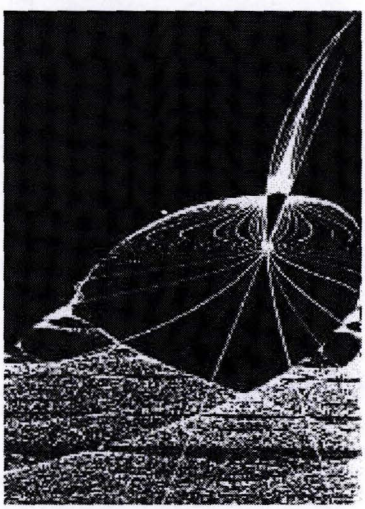

**Figure 3:** The universal Golden attractor.

As the harmonic analysis on the 3D Heisenberg Lie group *G* discovered and applied by Schempp (1986) to the description and control of MRI and Synthetic Aperture Radars, proves is indeed the case. As do – the field symmetry *U*(1, *C*) in terms of the polarization wave property of electromagnetic signals in relation, for example, to the quantum mechanical Aharonov-Bohm effect; the helices of RNA / DNA (see Hill and Rowlands, 2007) and the representation the Riemann sphere of the complex plane by stereographic projection, where the pole is the point at infinity called in perspective the 'vanishing point', which in art is a very late discovery of the nature of human perception (Figure 4).

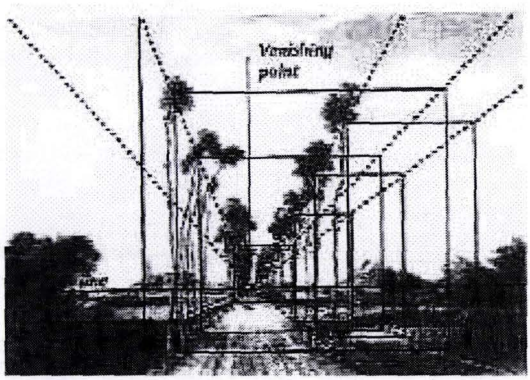

**Figure 4:** Perspective with vanishing point / pole.

In further support of this NUCRS hypothesis, is Trell's paper (2007) (based on Lie's 1871 Norwegian language thesis concerning real 3D space, foundational to his more general famous transformation methodology), where Trell uses the 'cubit' symmetry structure of the eight cubes about the point P of the neighbourhood with its eightfold way cubit metric scaling of signature +++, –++, +–+,++–, +—,–+–, —+, — to calculate the relative masses of all the families of the elementary particles, in good accord with their experimentally determined values, and their quantizations as previously calculated by Rowlands (2006). Mathematically these properties are, of course, those to be expected from Lie transformational theory, where bi-linearisation takes place via the calculation of invariants – a fact that would be explained if these invariants are all physical properties of universal standing wave or a soliton.

The very highly detailed paper by Hill and Rowlands in relation to DNA / RNA biological systems and their geometric structures, Nature's Code (2007), is written from the single nilpotent Dirac operator perspective of Rowlands for the NUCRS universal grammar and infinite alphabet. It fully underpins and extends that from the earlier complementary perspective of the nilpotent Heisenberg Lie group *G* which predicts, for example, the structure of DNA as a semantic wave bio-computer (Gariaev et al, 2002, 2001; Marcer and Schempp 1986) where the two Heisenberg helices of *G* and its dual *G*' and their base-pairing *SU*(2) hologram planes are the basis for the *U*(1, *C*) holographic helical signal processing taking place and the semiotics of the genetic code.

Hill and Rowlands' very comprehensive treatment fully substantiates the NUCRS hypothesis that the DNA / RNA genetic code is indeed a rewrite at a higher level of molecular complexity further extending the NUCRS universal alphabet – and where the DNA is known to include the encoding of the human brain (Marcer and Rowlands 2007).

## 5 Conclusion

It seems from the evidence above that quantum entanglement which in the NUCRS may be represented by $i$ the square root of $-1$ symbolizing the entire NUCRS universal alphabet is itself computer universal and may be mathematically represented by Riemann's Zeta function. Similarly 3D space should no longer be considered as just a plenum within which matter moves, but as a quantum field such as Einstein describes in general relativity, where, it has been said, 'matter bends space and space shapes matter'. An example would be changes in scaling resulting from nearby massive objects that can be expected from Trell's research to subject $U(1)$ signals to lensing effects. Indeed as shown in *Zero to Infinity* (Rowlands, 2007), this is, in fact, the actual means by which the dichotomy between nilpotent quantum mechanics and general relativity is resolved thermodynamically.

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