Synchronization - The Font of Physical Structure

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Abstract
The computational operation called synchronization, vital for realizing multi-process systems, is described in terms of a Clifford algebra over {-1,0,1}. This provides a two-way bridge between the worlds of computation and quantum mechanics, and casts new light on such matters as quantum non-determinism, mechanism and causality, the explicit structure of particles (including dark matter), and the like. We dub this the *synchronizational* model of quantum mechanics. Oppositely, we show how to represent any computation - sequential or concurrent - in these algebraic terms, thus providing a novel and powerful *physically-oriented* mathematics for computer science and allied disciplines.
Keywords synchronization, exclusion, mechanism, causality, non-determinism, emergent, quantum, combinatorial, distributed.

1 Introduction

Synchronization is unique among the instructions routinely executed by contemporary computers, in that unlike all the others, it is by definition *transparent* to the computation executing it. This is so because synchronization addresses the interaction *between* sequential programs, which interaction must not affect the correct operation of the individual interacting programs themselves. A typical use of synchronization is to assure that program processes P_1 and P_2 exclude each other in their access to some shared entity, eg. a printer, a disk or memory block, an I/O port, etc. Operating systems, real-time systems, and the internet would be literally impossible to construct without synchronization instructions.

A primitive synchronizer T consists of a notional internal binary flag - *Open* or *Closed* - that can be changed by two operations: *Wait* and *Signal*, denoted hereafter by W and S. The restriction to binary behavior implies no loss of generality. A synchronizer must supply the following behavior:

	A *Signal* sets T to *Open*, and passes the *Signalling* process;
S_{out}	Successive *Signals* are the same as a single *Signal*;
$\uparrow$	A *Wait* on *Closed* T fails, ie. the *Waiter* is not passed thru;
$W_{in} \rightarrow \mathrm{T} \rightarrow W_{out}$	A *Wait* on *Open* T sets T to *Closed*, and passes the *Waiter*;
$\uparrow$	Simultaneous *Waits* on the same $T \rightsquigarrow$ max one *Waiter* passes;
S_{in}	Simultaneous *Signals* on the same T = a single *Signal*.

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In the above diagram, *Waits* enter from the left and exit to the right; similarly, *Signals* enter from the bottom and exit at the top. The exclusion of processes over (say) a printer is realized by placing the use of the printer on the W_{out} leg, and thereafter directing the process to perform a corresponding S_{in} before exiting entirely; this arrangement guarantees that processes will use the printer serially (otherwise, output from different processes would be meaninglessly interleaved on the paper record, which is why synchronization is necessary in the first place). More complex examples can be found in any good operating system textbook.

Implicit in such arrangements is the requirement that synchronization be *transparent* to the participating processes: it would be unacceptable for the correct operation of a program to be dependent on whether it "really" waited to acquire some resource because some other process(es) happened to be present. Hence, no information in the Shannon sense is conveyed between two processes via the act of synchronization. Rather, synchronization induces/enforces a phase shift at the inter-process level. This phase shift is expressed in the non-deterministic ordering of the processes as they pass through the synchronizer.

Mathematically, a synchronizer establishes a partial order on the events W and S, such that a *Wait* never succeeds unless it has been preceded by a *Signal.* Physically, this ordering is tantamount to imputing a causal relationship between the S and the subsequent W. Thus one would expect that a mathematical treatment of the synchronization mechanism will cast new light on such matters as causality and it's quantum cousin, non-determinism. This expectation is grandly satisfied, as will become clear.[1]

The analysis of synchronization presented here approaches the issues via a Clifford algebra $\mathcal{G}$ whose generators, the 1-vectors $\{a, b, c, ..., x, y, z\}$, represent the boundary[2] of the system or entity in question *vis a vis* its surround. These vectors will take their values from $\mathbb{Z}_3 = \{0, 1, 2\} = \{0, 1, -1\}$. In practice, a 1-vector will have a magnitude ± 1; zero, on the other hand, denoting the exclusion of these values, $x + (-x) = 0$, implies the interpretation "can/does not occur". One can think of the 1-vector x as a one bit "sensor", with $x = +1$ denoting the current existence, in the surround, of whatever x senses, and $x = -1$ denoting oppositely that whatever x senses does not currently exist in the surround.

These definitions imply the following:

- for $x \neq 0$, $|x| = 1$
- $1 + 1 = -1$, whence
- $X + X + X = 0$ for any expression X in the algebra

[1]But no discussion here of the uncertainty principle; the Event Window mechanism of [1] is however my basis for understanding it.

[2]*Boundary* in the homological sense; this will not be elaborated further here.

The algebra's '+' operation denotes concurrent existence, understood as the opposite of "enforced mutual exclusion". This makes sense because co-existence is implicitly mutual and commutative: $x + y$ and $y + x$ both mean that x and y co-exist. The use of $+$ to represent such a formal "parallel composition" of processes is common in the CS theoretical literature (though the use of vector algebras, as here, is not).
[*Some asides*:

0. Notation: lower case letters $\{a, b, c, ..., x, y, z\}$ denote ordinary 1-vectors; upper case letters $\{A, B, C, ..., X, Y, Z\}$ denote arbitrary expressions ("multi-vectors") in the algebra; the product xy is a 2-vector, xyz is a 3-vector, etc. Expressions written with $\{x, y, z\}$ are generic forms, with x, y, z chosen from $\{a, b, c, ...\}$ without duplication and with arbitrary sign (modulo local context in the case of ambiguity). Thus $x + yz + xyz$ can represent $a - bc - abc$, $-b + ac - abc$, etc; in that the algebra is exceedingly symmetric, it is common that expressions having the same form also have the same algebraic properties. Nested parenthesized expressions specify more complex computations.

1. The 1-vector generators of the algebra are the "logical bottom", so x cannot take on the superposed value ± 1; superposition enters the picture with the algebra's anti-commutative product.

2. The choice of $\mathbb{Z}_3 = \{0, 1, -1\}$ removes the ambiguity present in $\mathbb{Z}_2 = \{0, 1\}$, where zero wears two hats: the opposite of one, and Void. In $\mathbb{Z}_3$, the opposite of +1 is -1 and vice versa; zero (Void) is a meaningless value for a vector, and occurs only as the result of sums (ie. multi-party computations).

3. The restriction to $\mathbb{Z}_3$, disallowing such expressions as $x + 2y + 3z$, is not viewed as such, since this expression can be re-written as $(x + y + z) + (y + z) + z$, which contains the same information regarding bottom-line existence, "ground states" so to speak; the algebra's distributivity and associativity guarantee that the only effects that will be missed are those that encounter cancellation mod 3, ie. coefficients larger than unity function merely to express larger amplitudes. Clearly, expansion to $\mathbb{Z}_n, n$ prime, is desirable at some point, but as will be seen, fundamental outlines appear when the picture is unconfused by matters of multiplicity.

End asides]

Besides co-existence, the other thing that can happen with two processes is that they interact, ie. they "operate" on each other. For this we use the algebra's anti-commutative multiplication:

- $xy = -yx$ for distinct 1-vectors x, y. *[The canonical ordering is alphabetical.]*
- $xx = +1$

The anti-commutative property applies only to 1-vectors; in general, $XY \neq -YX$, though simple non-commutativity is common. Application of the above rules for addition and multiplication yields the fact that $(xy)(xy) = -1$, that is, xy is a representation of $i = \sqrt{-1}$. Thus the algebra implicitly incorporates all the felicities of complex numbers, and indeed, exhibits a plethora of i's, whence such products are often called pseudo-scalars, ... and spinors. The 2-vector xy expresses XNOR (same/different) compactly, and other expressions in $\mathcal{G}_2$ express logical AND and OR. [An m-vector expresses an m-ary XNOR, whence our use of the term *distinction-space* for $\mathcal{G}$'s space.]

Finally, the algebra is associative and distributive as usual:

- $X + Y + Z = (X + Y) + Z = X + (Y + Z)$
- $XYZ = (XY)Z = X(YZ)$
- $X(Y + Z) = XY + XZ$ and $(Y + Z)X = YX + ZX$

This very simple Clifford algebra over $\mathbb{Z}_3 = \{0, 1, -1\}$ is remarkably expressive, containing

- Idempotents, $XX = X$, eg. $(-1 + x)$, $(-1 + x + y + xy) = -(1 - y)(1 - x)$
- Nilpotents, $XX = 0$, eg. $x + xy$, $x + y + z$, $xy + xz + yz$
- Bell and Magic operators, cf. entanglement [2].

Given this algebraic apparatus, *computational processes are represented directly and literally by the expressions of the algebra.* Sums express concurrent activity; this is a formal addition: subtraction $X - Y$ is understood as addition of the negative: $X + (-Y)$. Products express *action.*

The following general properties of Clifford algebras should be noted:

- The full specification of a Clifford algebra is $\mathcal{G}(p, q)$, where p is the number of generators that square to +1, and q the number that square to -1. Our algebra is thus $\mathcal{G}(n, 0) = \mathcal{G}_n$. The Pauli algebra, which spans quantum mechanics, is isomorphic to $\mathcal{G}_3$.
- The set $\{1, x, y, z, xy, xz, yz, xyz\}$ forms an ortho-normal basis for a $2^3 = 8$ dimensional space; similarly, n generators produce a space of 2^n dimensions. These spaces express abstract *distinctions* [1], and *must* not be confused with relativity's 3+1 space, which latter I insist must be constructed from the former.
- Theorem: For any expression X in the algebra, X has no inverse iff X has an idempotent factor.

Quantum mechanics having been mentioned, the reader may have noticed that nothing has been said of probability distributions and the like: our $\mathbb{Z}_3$ algebra is finite and discrete, quite unlike the continuous $[-1 : +1]$ space of correlations. On the other hand, concurrent computational systems are *inherently* non-deterministic, and it is argued that the present computational view, via its discrete and finite combinatorics, pierces the current source-less probabilistic skin over the actual goings-on. Rather, calculations in the algebra yield unique, concrete outcomes (whose combinatorics give the statistics). It will become clear that in the end, although mechanism and causality endure, *one must give up determinism.* Period.[3]

It is of course the author's hope that the present approach will ultimately translate into a novel *computational* physical theory. Being computational, such a theory is necessarily *constructive*, and hence can supply the (non-material, information-based synchronization) *mechanism* whose lack has for so long hampered our understanding of the quantum world. The story begins with ordinary sequential processes.

2 Sequential Systems

A sequential program, unrolled into its future, forms a system consisting of a single process, namely itself - there is no talk of other processes: even if they're present, any synchronization is transparent, and any interference oblique and unrecognized. The single most important property of a process is that it is a *sequence*: the order in which its events take place is crucial, *defining* in effect what the process does. As will be seen, it is similarly crucial not to confuse the three concepts of ordering/sequence, determinism, and causality, as was done in the early years of quantum mechanics.

Let X, Y, Z be arbitrary expressions in the algebra, and consider the process XYZ, which states the process "do Z, then do Y, then do X", that is, we *always* operate on the left. If any of X, Y, or Z has an inverse, we could algebraically manipulate XYZ to produce some other order. This will not do! Rather, to enforce sequence, we will require that none of X, Y, Z has an inverse. In physical terms, this means that they are irreversible and *time-like*, and we will intend these three terms interchangeably, as well as their opposites: possessing an inverse = reversible (which expresses wave-like activity) = *space-like.* [Again, this is *not* physical 3-D space, just space-like rotations]

Taking this reasoning further, if X, Y, Z have any reversible factors, they can all be moved to (say) the end of the sequence, leaving the sequence to consist of only irreversible factors with an final reversible postlude. Because the single reversible factor can be placed anywhere, choose to exclude it entirely from consideration without loss of generality. Therefore, a sequential program is represented by a product of irreversible factors, namely idempotents [ie. $SS = S$], whose order therefore cannot be changed.

For example, suppressing *much* detail, the generic sequential program DoA; DoB; DoC would translate to the sequence *(-1+DoC)(-1+DoB)(-1+DoA)*, where it is assumed that *(-1+DoX)* is idempotent. However distant this may seem from an actual implementation, it captures the fact that ordinary computation *is* fundamentally

[3]Note that as a result, the "many worlds" interpretation of QM is obviated.

irreversible at each step. Furthermore, an idempotent operator can, on closer examination, be seen to contain the germ of the concepts of memory and its reading and writing. To see this, take M to stand for a 1-bit memory. Then $M^2 = 1$ models its persistence independent of its content, $(-1 + M)(\mathrm{M})$ models a memory Write via the inversion of M (the actual memory), and, simultaneously, a (destructive) memory Read (the resulting +1) indicating that $M = \mathrm{M}$; cf. ***if***, below. This is easily expanded to multiple bits, and as well, captures the common special hardware synchronization instructions *test-and-set* and *swap*. So the DoX example is not all that far from computational reality after all.

So far, so good. To get a feel for how to use this algebraic representation of computation, analyzing the ***if-then-else*** construction is a good warming-up exercise. I will write ***if*** V ***then*** X ***else*** Y, where V, X, Y are arbitrary expressions representing arbitrary computations. For simplicity and with no loss of generality, take $V = a$, a 1-vector ("sensor").

"***if*** a" implies a probing of the current state of a: is it +1 (so do X), or is it -1 (so do Y).

Given that the only relevant states of a are ± 1, the next question is how to ascertain which of these obtains? Clearly, said ascertaining requires *measuring* a, where again idempotent operators play the central role. Consider the following identities:

$$\begin{array}{lll} (1+a) = (1+a)(a) & (-1+a) = (-1+a)(-a) & (-1+a) = (1-a)(1-a) \\ (1-a) = (1-a)(-a) & (-1-a) = (-1-a)(a) & (-1-a) = (1+a)(1+a) \end{array}$$

Taking $P = (1 + a) = (1 + a)(a)$ as an example, multiply P's rhs out to get $a + aa$, whence we see that the +1 in the lhs can be seen as the product of a with itself. It follows, and this is the key point, that if the a we have in hand - in the rhs's "$(1 + a)$" factor - has the same sign as the a we probe - the rhs's "(a)" factor - then the sign of the scalar will be +1, whereas if the a we probe is actually $-a$, then the sign of the scalar will be -1. This also applies if P, oppositely, specifies "$(-a)$" and we find "$-a$" ($\Rightarrow$ +1), or we find "+a" ($\Rightarrow$ -1). Finally, take *just* the scalar value from $(-1 \pm a)(\pm a)$ to complete the measurement (one can only actually measure scalars ... like a meter reading).

This is the basic act of measurement. Because $(1 + a)$ has no inverse, the act of measurement is irreversible, in accordance with contemporary understanding of the equivalence of energy and (Shannon) information. Furthermore, successive measurements using the idempotent form yield no new information, in that $PP = P$. [4]

So now we know how to do "***if*** a": we will write $(1 + a)(a)$ or suchlike, depending. The next issue is to choose the correct continuation depending on what the measurement on a produces.

[4] Actually, $(1 + a)$ is the square root ("sqert") of an idempotent, cf. the third column above, but this is unimportant for our present purposes.

The basic idea now is to arrange for the conjugate forms $(1 + a)$ and $(1 - a)$, whose product is zero, to collide on the unwanted branches of the ***if***, thus eliminating those continuations. A zero means the computation's future is empty, ie. it does not occur; generating a zero to eliminate an unwanted continuation is a key tool in the following.

Therefore, write the test in the ***if*** as a probe: $1+a$ or $1-a$, acting on the actual a, which can be plus or minus. The ***then*** and ***else*** branches apply respectively $1 + a$ or $1 - a$ to the result of the test, whence one of them should yield 0 (because conjugate) and the other the correct continuation based on the observed value of a. There are four possibilities (the | marks off visually (only) the shared ***if***-probe, rightmost because it occurs first): [5]

if	*probe*	***then***	*left branch | probe*	***else***	*right branch | probe*
1	$(1+a)(+a)$		$X(1+a)\,\|\,(1+a)(a)$ $= -X(1+a)$ ***yes***		$Y(1-a)\,\|\,(1+a)(a)$ $= 0$ ***yes***
2	$(1+a)(-a)$		$X(1+a)\,\|\,(1+a)(-a)$ $= X(1+a)$ ***no***		$Y(1-a)\,\|\,(1+a)(-a)$ $= 0$ ***yes***
3	$(1-a)(+a)$		$X(1+a)\,\|\,(1-a)(a)$ $= 0$ ***yes***		$Y(1-a)\,\|\,(1-a)(a)$ $= Y(1-a)$ ***no***
4	$(1-a)(-a)$		$X(1+a)\,\|\,(1-a)(-a)$ $= 0$ ***yes***		$Y(1-a)\,\|\,(1-a)(-a)$ $= -Y(1-a)$ ***yes***

In situation 1 above, we probe for $+a$ with $(1+a)$, and a is in fact $+a$; situation 2 has the same probe, but discovers $-a$; situation 3 probes for $-a$ but discovers $+a$; and situation 4 probes for $-a$ and discovers $-a$. Notice that if we consider all four possibilities concurrently (ie. Left + Right, 1 thru 4), we get zero: this situation (namely, a having both values simultaneously) cannot occur. So instead, combine 1&2 and 3&4 by subtraction to get the desired terms to double instead of cancel: 1-2 $= +X(1+a)$; 4-3 $= +Y(1-a)$, and move the |-cue to the right, eliminating the common probe-preface of the previous version:

1 minus 2: $\quad -X(1+a)\,|\,(\pm a)$

4 minus 3: $\quad -Y(1-a)\,|\,(\pm a)$

Finally, run 1-2 and 4-3 concurrently (ie. add), and factor out $(\pm a)$:

$$-X(1+a)\,|\,(\pm a) \; - \; Y(1-a)\,|\,(\pm a)$$
$$= [-X(1+a) - Y(1-a)]\,|\,(\pm a)$$

If a=+1 then the Y term drops out leaving $+X$; and if a=-1 then the X term drops out, leaving $+Y$. Just as we wanted! Push the minus-signs into the parentheses:

$$= [X(-1-a) + Y(-1+a)]\,|\,(\pm a)$$

[5]The ***yes*** and ***no*** indicate desired outcomes.

and we see that doing ***if-then-else*** necessarily invokes observation, ie. idempotents, not sqerts, consistent with thermodynamic and quantum measurement theory. The form also makes good computational sense when multiplied out:

$$= X(-1-a)(\pm a) + Y(-1+a)(\pm a)$$

which transparently describes two independent processes X and Y, each independently and concurrently testing for its own condition, only one of which will succeed.

NB: if one tries simultaneously to measure with $1+a$ *and* $1-a$, one gets (summing) an inversion (1+1 = -1), but no knowledge of a, in accordance with quantum measurement theory: if one is to get information, one must specify *exactly* what it is one is looking for ... $+a$ or $-a$, and this *cannot* be finessed.

3 Synchronization in the Algebra

Having warmed up with *if-then-else*, we now tackle synchronization's *Wait* and *Signal*. From the introduction, the required behavior is

a. A *Signal* sets T to *Open*, and passes the *Signalling* process thru;
b. Successive *Signals* are the same as a single *Signal*;
c. A *Wait* on *Closed* T fails, ie. the *Waiting* process is not passed thru;
d. A *Wait* on *Open* T sets T to *Closed*, and passes the *Waiting* process thru;
e. Simultaneous *Waits* on the same T result in max one *Wait*er passing thru;
f. Simultaneous *Signals* on the same T are the same as a single *Signal*.

Items e and f refer to situations where there is competition between multiple *Waiters* and/or *Signallers*; this complication will be deferred for the moment.

The first step comes from item b, which in effect says $SS = S$, ie. S must be *idempotent*.

Item d says that WT must succeed if T is *Open*. Therefore initialize T to *Open*, which we can do via item a by setting $T = S$. Item d then reads $WT = WS$, which must be non-zero to succeed. [6]

Item c in effect says (together with item a) that successive *Waits* without an intervening *Signal* must fail. That is, $WW = 0$, so W must be nilpotent. So now we know the shapes of both W and S, and very specific ones at that.[7]

These considerations imply that a sequence like $SSWSWSST = SWSWST = SWSWS$, and any sequence with consecutive W's yields zero, eg. $WWSWST = 0$.

[6] Initializing T to W (ie. T is initially *Closed*) doesn't work: $WT = WW = 0$, whence SWT also yields zero, which it shouldn't. Initializing T to 1 (which is idempotent) is indiscriminate - *any* W will succeed.

[7] I am embarassed at how easily this (finally!) goes, considering the time spent considering the problem. My big mistake was thinking that $WW = W$, ie. that successive unsuccessful *Waits* are a no-op, just like successive *Signals*; the error is that the point-of-view must be from ***inside*** T, whereas the $WW = W$ view, endemic in the computational world, is from outside T.

Process-wise (see figure just below), there is process P_1, which after a sequence of arbitrary irreversible operations X issues the signal S, creating a so-called 'synchronization token'; and then there is process P_2 which after a sequence of Y's consumes this token by *Wait*ing on it, whereafter P_2 continues, executing Z's (read right-to-left: things begin on the right!):

P_1 : $...X\,X\,X\,S\,X\,X\,X...\hookleftarrow$

$\downarrow$ $X, Y, Z$are arbitrary irreversible actions

P_2 : $...Z\,Z\,Z\,W\,Y\,Y\,Y...\hookleftarrow$

Despite the visually implicit timeline in the above two sequences, the $Wait$ in P_2 can occur any time 'before', 'simultaneously with', or 'after' the *Signal* in P_1, but unless the *Wait* occurs 'after' the *Signal*, process P_2 is logically halted at the W. Whichever of these circumstances obtains, the ultimate result is a logically and physically seamless transition from P_1's $SXXX$ to P_2's $ZZZW$. *This sequence too is a process*, process P_3:

P_3 : $...ZZZWSXXX...$

The fact that W *must* be nilpotent means that 'whenever' the WS mating actually occurs, it is just as though P_3 occurred seamlessly. An example: when one absorbs a photon in the retina, at that very instant one is exactly connected with the state that generated the S - even if the star that generated the photon has 'long since' disappeared. [8]

P_1 and P_2 are *classical*, in that we imagine them to be deterministic - good old-fashioned Newtonian / Einsteinian processes. [We might think of the state preparations preceding an actual quantum experiment, which are classical.] P_3, on the other hand, is non-deterministic, because it was precisely $>> P_2 <<$'s *Wait* that succeeded, leading to the Z's. If however it had happened that some P_4's *Wait* occurred ahead of P_2's, P_3's continuation would be entirely different.

This *emergent* non-determinism is old news in computer science, though it is most often noted in the form of unwanted *values* (cf. the interleaved printer output example earlier), rather than the entirely proper non-deterministic *ordering* induced by the serialization as just described.[9] In both cases - *order* or *value* non-determinism - the root is the *asynchrony* of the interaction of two independent processes. Said a bit differently, *if* one is to use *process* as a conceptual primitive, *then* one necesarily must accept into the bargain the consequent, unavoidable emergent non-determinism born of the asynchronous interaction of these same processes.[10] Both non-deterministic values and non-deterministic order are produced by asynchrony. I therefore advance the claim that asynchrony is the very source of QM's non-determinism.

[8] It's pretty limited time travel tho - you only get the single bit of information that the photon carries ... not much of a view!

[9] Both are the source of the most difficult bugs, because they are namely not repeatable; cf. Ullman's fine novel, "The Bug".

[10] It is the *necessity* for exclusion, at *every* step, that dictates that processes be discrete, cf. Planck's constant.

Order-non-determinism forms the *coarse-grained* skeleton of physical non-determinism. Suppose now that one has guaranteed that only a *particular* Wait-continuation will match a given Signal, so *order* is out of the picture. One still doesn't know what one will get from the measurement, cf. *if-then-else*'s measurement earlier. So within the *order*-skeleton is a second, finer-grained source of non-determinism, *value* non-determinism, induced by the measurements encapsulated in the Signals. For example, the idempotent $-1 + xy + xz$ expresses a value-changing intrusion into the entity $xy + xz$, which in principle "lives its own (reversible) life" both prior to and subsequent to the measurement.

Popping up conceptually, imagine now P_3's form as it evolves into its future. Its sequence of Z's is just shorthand for an arbitrary sequence of idempotents, for example $(1+a)(1+b)...(1+r)$. Being idempotents, each of them can act as a *Signal* to some matching *Wait* 'out there'. [It is important that they are idempotents, because this means that the event that the *Wait* is dependent on has actually physically occurred.] Ultimately, if every idempotent in P_3 triggers a *Wait*, and all those *Waits*' continuations do the same, the universe will be populated entirely by utterly non-deterministic processes that look like $(WS)(WS)(WS)...(WS)$ - these W's and S's being notionally distinct. In fact, we see that our classical view of P_1 and P_2 as deterministic processes puts them in an improbable and miniscule minority - namely that minority inhabiting/forming classical 3+1 space-time (plus all ordinary sequential computer programs).

Finally, consider the issue of competition from multiple *Signals* and/or *Waits* for a given synchronizer. Taking the case of multiple *identical Signals*, we can consider combining them concurrently (addition) or as interacting (multiplication). This yields $S + S = -S$ and $SS = S$, so both possibilities yield the same result, S, in that a sign difference is irrelevant in the present context.

Similarly for identical *Waits*, the same reasoning yields $W + W = -W$ and $WW = 0$. Thinking in physical terms, the choice between the two ways of combining can be made in terms of energy. If the event S is such that it is appropriate that it trigger multiple continuations (like a race-starting pistol shot), then additive combination of *Waits* is the choice. If on the other hand it is appropriate that there be only one continuation arising from a *Wait*, then construing the collision of *Waits* as an actual interaction yields $WW = 0$ and no continuation for either *Waiter*, which is entirely acceptable, computationally speaking. Lacking further knowledge or insight, it seems best to assume the worst and define competing *Waits* multiplicatively: $WW = 0$.

There is one more variant, namely that the multiple *Waits* and/or *Signals* are not identical. It is a fact that for a given $T(= S)$, several different W's will yield acceptable continuations, and similarly for S's. The sum $W_1 + W_2$ need not be nilpotent, and $S_1 + S_2$ need not be idempotent; nor their products (though they're always irreversible). Given that what happens is therefore dependent on more details than the nilpotent or idempotent properties provide per se, this case requires careful algebraic investigation, leading presumably to the symmetries that define conservation laws. Howsoever, the discussion below ignores this complication, but should not be misleading for that. We will therefore assume that for a given T, all *Signals* are identical, and all *Waits* are identical, with their combination in the case of competition as described in the preceding two paragraphs.

4 Structures Induced by Synchronization

I argued above that the processes under examination are in fact all of the form $(WS)*$, where the * indicates one or more repetitions, and the W's and S's are notionally distinct (ie. not identical, but not competing). Thus the processes $(WS)*$ are 'sentences' whose 'words' are the various possible juxtapostions of the 'phoneme' W to the 'phoneme' S, each such word being a primitive causal act.

With this in mind, the algebra seems to imply that *any* nilpotent W will work with *any* idempotent S, but although many W/S pairs are 'compatible', this is not always so. For example, in $\mathcal{G}_3$:

$$S = -1 + a + b + c + ab + ac, \qquad T = S$$
$$W = a + b + c$$
$$WT = 0$$

That is, $WT = WS = 0$. Physically, the process ("P_3") simply ends; this collision of phonemes might describe an annihilation, but we can at least say that this particular WS pair produces no future - the computation simply ends. The physical interpretation then is that this particular S will not enable a process requiring this particular W as a pre-condition. For example, given that $a + b + c$ is a photon, and if this were true of all 8 photon sign variants (which it isn't), this would mean that the condition established by S is unaffected by electro-magnetism.

$W = a + b + c$ and $S = -1 + d = T$ turn out to produce $SWT = SWS = 0$. The correct interpretation would here seem that the interaction WS negates the further existence of T - it can no longer be *Signalled.* Whatever the interpretation, it is clear that SWS expresses a one-shot event.

Here is a 'compatible' solution in $\mathcal{G}_3$:

W = a+b+c	*WW = 0*
S = -1+b	*SS = S (b is arbitrary - could also be a or c)*
T = S	*Synchronizer T is initially open.*
ST = SS = -1+b	*ST = SS = S, and T is still open.*
WT = WS = 1-a-b-c+ab-bc	*T is now closed...*
SWST = SWS = 1-b	*Ie. SWS = -S*
WSWST = WSWS =-1+a+b+c-ab+bc	*Ie. WSW = -W*

Notice here that, unlike the two preceding examples, this WS pair cycles indefinitely between $\pm S$ and $\pm W$, what I called 'compatible'. Note that the synchronizing occurs independently (as it were) of the signs of S and W: the synchronizing relationship is one of *orthogonality*, whereas sign differences are 180^o apart, ie. *same* dimension. The cyclicity reflects the *external* view of T that it cycles between being *Open* and *Closed*, and as well that the virtual synchronization token created by S and consumed by W is continually conserved.

5 Stepping Back - Implications

The lesson of these examples is that the mathematics itself - representing actual computational cum physical processes - imposes restrictions - a grammar - on what can happen. It tells us that only certain WS combinations produce on-going processes. Given that WS pairs express causal events, and hence$(WS)^*$ is a causal (though non-deterministic) process, such processes represent the real world of irreversibility, energy expenditure, and entropy creation. These processes are what we see when we experience the world around us (even though we constantly try to fit them into a deterministic, classical framework).

Do the processes described by $(WS)^*$ exhaust the realm of causal events? By the preceding analysis, a sequence of irreversible actions represents what we traditionally mean by 'causality'. Consider the simplest such sequence: $(1+y)(1+x)$. Recalling that we always operate on the left, one would say that the action $(1+x)$ *caused* $(1+y)$, in that $(1+x)$ establishes [11] the pre-condition for $(1+y)$ to occur. Observing that $(1+x) = x(1+x)$, however, we see that $(1+y)(1+x) = (x-xy)(1+x)$, where $(x-xy)$ is nilpotent. Since this same trick can be used ad libitum on a longer such sequence, we see that *any* such even causal sequence can be expressed in $(WS)^*$ form; in the odd case, one S is left, so the final result is $S(WS)^*$. Furthermore, it can be shown that the $\mathcal{G}_3$ idempotents $-1+xy+xz$ and $-1+x+(y+z)+-x(y+z)$ are time-like boundaries of the same such sequences, and this consideration generalizes to higher-level sequences and idempotents. I therefore claim that the form WS is *the* causal atom, and there are no others.[12] Note however that WS is time-like, which one associates with causality and entropy, versus *change* in general, which can also be reversible (ie. wave-like, eg. the quantum potential).

This said, the fundamental issue is, to what extent there exists, for every idempotent, a nilpotent partner. The corresponding statement in ordinary vector spaces is the Jordan Normal Form theorem ("spectral decomposition"), which states that the set

$$\{p_1, p_2, p_3, ..., p_n, p_{n+1}, q_{1,n+1}, q_{2,n+1}, ..., p_r, q_{1,r}, ..., q_{s,r}\}$$

where the p_i are idempotents, and the $q_{j,k}$ are nilpotents such that $q_{j,k}^m = 0, m > 1$, constitutes a basis for the vector space. The generalization to Clifford algebras is apparently an open question [6]. Related aspects are whether for any X there exists a corresponding Y such that $XY = -YX$; and the theorem cited earlier

[11] *Somehow...* the story is tellingly vague; one could ask, "What prevents writing (1+y) *plus* (1+x) here?"

[12] Which conclusion the boson/fermion distinction also implicitly encodes.

(X irreversible *iff* X contains an idempotent factor). Regarding the latter, because nilpotents are also irreversible, it implies that for any nilpotent, there exists a corrsponding idempotent, but not necessarily ($n > 0$) vice versa. Finally, in the case where $n > 0$ (which occurs first in $\mathcal{G}_4$), what is the physical interpretation of an idempotent without a matching nilpotent, since the computational interpretation would be that there exist states p_i that have no continuation, but rather just sit there?[13]

Given that the algebra reflects the quantum world (though not in the usual terms), it does not seem unreasonable to try now to connect a little more explicitly to the physics. Of course, most of the following hypotheses are probably at least partially wrong, and yet probably also partially right. [14]

Since the algebra in any particular case is finite, we can *mechanically* generate all its idempotents S and nilpotents W and directly calculate which pairs produce what processes. This list should then be an exhaustive catalog of what can happen, and by implication, of the 'particles' that are possible. The Appendix therefore exhibits a complete list of the nilpotents and idempotents of $\mathcal{G}_3$. The nilpotent forms (bottom row = totals) are:

1	2	3	4	5
$x + xy$	$x + y + (x + y)z$	$x + y + z$	$x + y + x + xyz(x + y + z)$	$xy + xz + yz$
	$= x + y + xz + yz$		$= x + y + z + xy - xz + yz$	
24	24	8	16	8

Column 2 consists of particular pairs (namely those that form a nilpotent) from column 1:

$$(x + xz) + (y + yz) = (x + yz) + (y + xz)$$

If we instead take triples from column 1, we get column 4, which is itself formed from particular pairs from columns 3 and 5. Thus both sets emergently exhibit pairs with the form $x + yz$, either in two's or in three's. Also, the cube roots of -1 are identical to column 1 with a scalar component: $-1+x+xy$, which (in multiplicative combination) express a transition from $x + y$ to $-x - y$; added together in pairs or triples with the scalars cancelling again yields pairs $(x + yz) + (z + xy)$ and triples $(x + yz) + (z + xy) + (y + xz)$. The three 'singlets' $(x + yz)$, and pairs and triples thereof, are all boundaries of xyz, the top element of $\mathcal{G}_3$ (which is isomorphic to the Pauli algebra).

Shifting from nilpotents to idempotents, the criterion for $-1 + X$ to be idempotent is that $X^2 = 1$, that is, X is unitary, and thus a persisting entity. That is, X is a *particle*. So, extracting from the list in the Appendix, the particles specified by our analysis are:

[13] I speculate that a larger algebra will always contain such a nilpotent.

[14] I'm sure some friendly physicist will be pleased to point out any errors, which would be most welcome!

Count : Of		
6:6	x	3 families of 2
24:24	$x+y+xy$	3 families of 8
12:12	$xy+xz$	3 families of 4
48:96	$x+y+z+xy+xz$	3 families of 16

Noting that the form $x+yz$ does not exist *alone* in either $\{W\}$ or $\{S\}$ - in that it *emerges in pairs* from $\mathcal{G}_2$ forms - causes me to see so-called quark confinement, and thus to believe that the form $x+yz$ is the basic quarkish atom. The number 48 is also characteristic of this family. I therefore advance the hypothesis that among these various forms with $x+yz$ and their precursors are to be found the quarks, gluons, hadrons, and mesons of the standard model of QM; the Appendix offers further details.

The appearance of photons with $\mathcal{G}_3$ invokes the physics of electro-magnetism, so one can reasonably infer that the nilpotents and idempotents of $\mathcal{G}_2$ will reflect the physics of this simpler level. Similarly, this reasoning opens the interesting possibility that higher-level nilpotents and idempotents (ie. $\mathcal{G}_n, n>3$) will throw light on the mechanism of gravity and more. The hierarchy of algebras $\mathcal{G}_i \rightarrow \mathcal{G}_{i+1}$ presents a natural and elegant path to unification, though neither attribute guarantees success. In this connection, cf. the discussion above of Jordan's theorem, we see how the present approach brings us directly to a super-symmetric theory, where the idempotents (or rather, their unitary components) are the fermions, and the nilpotents the associated bosons.

Howsoever, what might we elucidate regarding dark matter? We know that it is 'dark' because it does not interact with electro-magnetism, so $W=x+y+z$ must yield $WT=WS=0$. On the other hand, it *does* interact with gravity, so the right combination of W and S must yield non-zero continuations. I hold the view [1] that 3+1 space-time (and the gravity that shapes it) cannot emerge before a fourth level of complexity, ie. $\mathcal{G}_4$. A weighty argument for this view is that just as superposition and spin $\frac{1}{2}$ emerge in $\mathcal{G}_2$ and *exhaust* the information-carrying capacity of that level; and that further structure (namely charge) can therefore first emerge in $\mathcal{G}_3$, which exhausts *its* information-carrying capacity; so similarly, gravity can first emerge in $\mathcal{G}_4$.

Thus, we seek level 4 (or higher) nilpotents and idempotents that can mediate our putative gravitational interaction. Consider the following table of powers of n-vectors:

level n	n-vector	$(n\text{-vector})^2$	
0	1	+1	scalars
1	x	+1	vectors
2	xy	-1	spinors, quaternions
3	xyz	-1	volume, charge
4	$wxyz$	+1	EPR, ?mass
5	$vwxyz$	+1	3+1 space-time?

Clearly, the pattern $++--++--...$ is that of powers of $i = \sqrt{-1}$, hence the 4-cycle. Many algebraic properties repeat mod 4 - for example 1-vectors and 5-vectors (with no shared variables) both anti-commute. More to the point, the mod 4 cycling of the algebra means that $\mathcal{G}_4$ is implicitly and inherently scalar-like ($\mathcal{G}_0$), and mass is a scalar quantity. Also noteworthy about 4- and 5-vectors is that they both square to $+1$, indicating a non-polar form of interaction, as opposed to the -1 of 2- and 3-vectors, indicating the polarity characteristic of electro-magnetism. So $\mathcal{G}_4$ and $\mathcal{G}_5$ are likely candidates on this score as well.

Unfortunately, $\mathcal{G}_4$ contains $3^{16} \approx 45$ million different expressions, discouraging for the exhaustive search that produced the $\mathcal{G}_3$ table in the Appendix. The $\mathcal{G}_4$ nilpotent $a+b+c+d+abcd(a+b+c+d)$, obtained by analogy, produces ambiguous results; the $\mathcal{G}_5$ version is not nilpotent. So although we are stymied at this point, this approach is both promising and pointed.

6 Conclusions

The overall approach described above, of applying vector algebra to computation qua computation, seems to have been this author's path alone [1]. This is perhaps not surprising, since computation as commonly understood, ie. ordinary sequential programs and systems thereof, is dominated by an automata-theoretic view that leaves little room for a physics of computation.[15] Nevertheless, whatever theoretical view is taken, the constant fact is that computation is, at bottom, about *mechanism*. As such, any computation-based theory is fundamentally *constructive* - at every stage, it must be specified what does what to what, and how. For this reason, any physics of computation is non-redundant: every statement in the theory must correspond 1-to-1 to the reality it describes. At the same time, computation's way of describing processes is independent of its way of realizing same: how an *Add* instruction is implemented has zero impact on its actual operation (aside from speed, which is logically irrelevant to this consideration).

In this context, quantum mechanics is famous for the inscrutability of its mechanism - after all, how can one have a finite mechanism that generates the unbounded information inherent in 'randomness'? Furthermore, careful analyses of the formalism of quantum mechanics have limited the scope of any hidden mechanism (cf. "hidden variables") quite severely. It is therefore noteworthy that the present analysis produces non-determinism as a phenomenon that *emerges* when one, tellingly, moves from the consideration of isolated deterministic processes (which isolation is implicitly, but unobviously, classical) to *interacting collections* of same. The inherent non-determinism of interacting computational processes is well-known in computer science, but connecting this solidly to physics, as here, is new.

[15]Eg. Penrose's analysis in *The Emperor's New Mind* - correct, but reaching a wrong conclusion: synchronization is namely the missing consideration in such analyses.

That the present novel characterization of synchronization - the key mechanism of multi-process systems - as the product of nilpotent and idempotent forms then generates what appears to be entire realms of insight - a unique primitive causal form and whole emergent families of explicit structure - is perhaps to be expected from such a foundational approach, but satisfying and encouraging nonetheless. One can even hope that more complex systems - molecular, biological, social - can be treated; the hierarchical aspect of the algebra should be especially helpful here. Less rosily, the description above suffers greatly from the absence of both a group-theoretic anatomy and concrete input from physics; hopefully others will be encouraged by the results so far to contribute.

Finally, the once obscure but now familiar type-setting term *font* denotes the physical, re-usable form underlying actual printed letters. The various forms that W, S, and WS can take are indeed the font that Nature uses to write out physical processes.

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Appendix

$\mathcal{G}_3$ contains $3^8 = 6561$ expressions in all, including 0 and 1; in all there are 81 nilpotents (including 0); and 92 idempotents (including 0 and 1). The identification of these forms with the various particles below is tentative.

$\mathcal{G}_3$ Nilpotents

$x + xy$:	weak vector bosons							
8	$a + ab$	$-a + ab$	$a - ab$	$-a - ab$	$a + ac$	$-a + ac$	$a - ac$	$-a - ac$
8	$b - ab$	$-b - ab$	$b + ab$	$-b + ab$	$b + bc$	$-b + bc$	$b - bc$	$-b - bc$
8	$c - bc$	$-c - bc$	$c + bc$	$-c + bc$	$c - ac$	$-c - ac$	$c + ac$	$-c + ac$
$= 24$								

$x + y + xz + yz = (x + yz) + (y + xz)$:	massless mesons [16]			
4	$a + b + ac + bc$	$-a - b + ac + bc$	$-a + b + ac - bc$	$a - b + ac - bc$
4	$-a - b - ac - bc$	$a + b - ac - bc$	$a - b - ac + bc$	$-a + b - ac + bc$
4	$-a + c + ab + bc$	$a - c + ab + bc$	$a + c - ab + bc$	$-a - c - ab + bc$
4	$a - c - ab - bc$	$-a + c - ab - bc$	$-a - c + ab - bc$	$a + c + ab - bc$
4	$b + c + ab + ac$	$-b - c + ab + ac$	-$b + c - ab + ac$	$b - c - ab + ac$
4	$-b - c - ab - ac$	$b + c - ab - ac$	$b - c + ab - ac$	$-b + c + ab - ac$
$= 24$				

$x + y + z$:	photons			
4	$a + b + c$	$-a + b + c$	$a - b + c$	$-a - b + c$
4	$a + b - c$	$-a + b - c$	$a - b - c$	$-a - b - c$
$= 8$				

$xy + xz + yz$:	quaternions (in distinction space)			
4	$ab + ac + bc$	$-ab + ac + bc$	$ab - ac + bc$	$-ab - ac + bc$
4	$ab + ac - bc$	$-ab + ac - bc$	$ab - ac - bc$	$-ab - ac - bc$
$= 8$				

$x + y + z + xyz(x + y + z)$:	gluons
4	$\pm(a + b + c) \pm (ab - ac + bc)$
4	$\pm(a - b + c) \pm (ab + ac + bc)$
4	$\pm(-a - b + c) \pm (ab + ac - bc)$
4	$\pm(-a + b + c) \pm (ab - ac - bc)$
$=$ **16**	

[16] Ie. taking x+yz as the quark form, mesons (massless or not) are indeed quark pairs.

$\mathcal{G}_3$ Idempotents

Count : Of		
2:2	$0, 1$	
6:6	$-1 + x$	3 families of 2
24:24	$-1 + x + y + xy$	3 families of 8: *neutrinos*
12:12	$-1 + xy + xz$	3 families of 4: *electrons* $= (-x - y - z)x$
48:96	$(-1 \pm x)(-x - y - z)$	3 families of 16: *protons*

Re the last line, the other 48 of the 96 are reversible; in fact, they are 40th roots of unity, w/ -1@^20. I speculate that the xyz rotation of the unitary component is a *neutron.*