Topological Geometrodynamics: an Overall View

M. Pitkänen
Puutarhurinkatu 9, 10960, Hanko, Finland.
matpitka@luukku.com-http://www.helsinki.fi/ matpitka/.

Abstract
A brief summary of the basic ideas of Topological Geometrodynamics (TGD) is given with a special emphasis on the most recent mathematical developments. There are several first principle approaches to quantum TGD. The basic dynamical objects are light-like 3-surfaces which means a generalization of the conformal symmetries of super string models and implies a parton level formulation of quantum TGD as an almost topological quantum field theory with discrete symplectic fusion algebras playing a key role in the formulation of theory. The notion of finite measurement resolution realized in terms of inclusions of hyperfinite factors of type II_1 can be also taken as a fundamental dynamical principle. Associativity - in both number theoretical sense and in the sense of conformal field theories - and number theoretical universality stating that the scattering amplitudes make sense in both real and p-adic number fields define further principles of this kind. All these approaches lead to the notion of number theoretical braid meaning discretization at space-time level. The notion of operad is tailor made for formulating the notion of finite measurement resolution and rather precise formulation of generalized Feynman diagrammatics emerges in this framework.
Keywords: fusion algebras, hyperfinite factor, p-adic numbers, braids, operads.

1 Introduction

Last years have meant a rapid development in the understanding of the conceptual and mathematical structure of Topological Geometrodynamics (TGD) [6].

a) In zero energy ontology S-matrix is replaced with -in general non-unitary- M-matrix identified as time-like entanglement coefficients between positive and negative energy parts of zero energy states having interpretation as "complex square root" of density matrix (ch. *Construction of Quantum Theory: S-matrix* of [2]).

b) Hyper-finite factors of type II_1 emerge naturally through Clifford algebra of the "world of classical worlds" and allow a formulation of quantum measurement theory with a finite measurement resolution (ch. *Construction of Quantum Theory: S-matrix* of [2]).

c) A generalization of the notion of imbedding space $M^4 \times CP_2$ emerges from the requirement that the choice of quantization axes has a geometric correlate also at the level of imbedding space (ch. *Construction of Quantum Theory: S-matrix* of [2]). The physical implication is the identification of dark matter in terms of a hierarchy of phases with quantized values of Planck constant having arbitrarily large values.

d) Number theoretical braid has become a key concept in the formulation of quantum TGD. Besides super-conformal symmetries number theoretical universality meaning fusion of real and p-adic physics to single coherent whole force this notion. Number theoretical braids realize the notion of finite measurement resolution at space-time level. Also symplectic fusion algebras imply this notion.

e) M-matrix can be regarded as a representation of a genuine quantum state in zero energy ontology: quantum state represents the laws of physics in its structure. The notion of finite measurement realized in terms of inclusions of HFFs requires that M-matrix corresponds to a Connes tensor product which means almost uniqueness of M-matrix.

f) A category theoretical interpretation of M-matrix as a functor is possible. The super-canonical invariance of quantum TGD can be formulated in terms of infinite hierarchy of discrete symplectic fusion algebras (ch. *Construction of Quantum Theory: S-matrix* of [2]) forming an operad. This gives powerful constraints on discretized versions of generalized Feynman diagrams realized in terms of number theoretic braids with braiding S-matrices assignable with the lines of diagram and n-points functions of conformal field theory associated with the vertices. The notion of operad [25] -originally developed for quite different mathematical purposes- turns out to be tailor made for formulating the notion of finite measurement resolution and coupling constant evolution.

2 Basic Ideas

Because of page limitations the following summary of quantum TGD is far from complete and I must refer to [6] fora more detailed discussion.

2.1 Two Manners to End up with TGD

One can end up with TGD (for overall view see [6]) as a solution of energy problem of general relativity by assuming that space-times are representable as 4-surfaces in certain higher-dimensional space-time allowing Poincare group as isometries. TGD results also as a generalization of string model obtained by replacing strings with light-like 3-surfaces representing partons. The choice $H = M^4 \times CP_2$ leads to a geometrization of elementary particle quantum numbers and classical fields if one accepts the topological explanation of family replication phenomenon of elementary fermions based on genus of partonic 2-surface (ch. *Elementary Particle Vacuum Functionals* of [5]).

Simple topological considerations lead to the notion of many-sheeted space-time and a general vision about quantum TGD. In particular, already classical considerations involving only the induced gauge field concept strongly suggests fractality meaning infinite hierarchy of copies of standard model physics in arbitrarily long length and time scales [5].

The huge vacuum degeneracy of Kähler action implying 4-dimensional spin glass degeneracy for non-vacuum extremals (ch. *Basic Extremals of Kähler Action* of [4]) means a failure of quantization methods based on path integrals and canonical quantization and leaves the generalization of the notion of Wheeler's super-space the only viable road to quantum theory. By quantum-classical correspondence 4-D spin glass degeneracy has an interpretation in terms of quantum critical fluctuations possible in all length and time scales so that macroscopic and -temporal quantum coherence are predicted: the implementation of this seems to require a generalization of the ordinary quantum theory.

2.2 p-Adic Mass Calculations and Generalization of Number Concept

The success of p-adic mass calculations based on p-adic thermodynamics (see the first part of [5]) motivates the generalization of the notion of number achieved by gluing reals and various algebraic extensions of p-adic number fields together along common algebraic numbers. This implies also a generalization of the notion of the imbedding space, and it becomes possible to speak about real and p-adic space-time sheets whose intersection consists of a discrete set of algebraic points belonging to the algebraic extension of the p-adic numbers considered.

Mass calculations demonstrate that primes $p \simeq 2^k$, k integer are in special role. In particular, primes and powers of prime as values of k are preferred. Mersenne primes and the ordinary primes associated with Gaussian Mersennes $(1+i)^k - 1$ as their norm seem to be of special importance. The first principle explanation for the powers of two comes from the formulation of the quantum TGD in terms of zero energy ontology [6] predicting that fundamental time scales come as power of two multiples of a fundamental time scale and are identifiable as secondary p-adic time scales. For electron this time scale is .1 seconds and corresponds to a fundamental biorhythm.

p-Adic physics is interpreted as physics of cognition and intentionality with p-adic space sheets defining the correlates of cognition or the "mind stuff" of Descartes [7]. The hierarchy of p-adic number fields and their algebraic extensions defines cognitive hierarchies with 2-adic numbers at the lowest level. One important implication of the fact that p-adically very small distances correspond to very large real distances is that the purely local p-adic physics implies long range correlations of real physics manifesting as p-adic fractality (ch. *TGD as a Generalized Number Theory: p-Adicization Program* of [3]). This justifies p-adic mass calculations, and seems to imply that cognition and intentionality are present already at elementary particle level.

2.3 Physical States as Classical Spinor Fields in the World of Classical worlds

Generalizing Wheeler's super-space approach, quantum states are identified as modes of classical spinor fields in the "world of classical worlds" (call it CH) consisting of light-like 3-surfaces of H. More precisely: $CH = \cup_{CD} CH_{CD}$, where CD refers to a causal diamond formed by the interactions of future directed lightcone and past directed light-cones in its future. Light-like 3-surface has dual interpretations as random light-like orbit of a partonic 2-surface or a basic dynamical unit with the assumption of light-likeness possibly justified as a gauge choice allowed by the 4-D general coordinate invariance. The failure of the strict classical determinism implies however that fundamental objects are 3-dimensional in discrete sense.

The condition that the world of classical worlds allows Kähler geometry is highly non-trivial and the simpler example of loop space geometry [10] suggests that the existence of an infinite-dimensional isometry group, naturally identifiable as canonical transformations of $\delta H_{\pm}$, is a necessary prerequisite. Configuration space would decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical variables essential in quantum measurement theory (without zero modes space-time sheet of electron would be metrically equivalent with that of galaxy as a point of CH) (ch. *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I,II* of [1]).

General coordinate invariance is achieved if space-time surface is identified as a preferred extremal of Kähler action, which is thus analogous to Bohr orbit so that semiclassical quantum theory emerges already at the level of configuration space geometry.

The only free parameter of the theory is Kähler coupling strength α_K associated with the exponent of the Kähler action defining the vacuum functional of the theory [6]. This parameter is completely analogous to temperature and the requirement of quantum criticality fixes the value of α_K as an analog of critical temperature. Physical considerations allow to determine the value of α_K rather precisely (ch. *Is it Possible to Understand Coupling Constant Evolution at Space-Time Level?* of [2]). The fundamental approach to quantum dynamics assuming light-like 3-D surfaces identified as partons are the basic geometric objects.

2.4 Magic Properties of 3-D Light-like Surfaces and Generalization of Super-conformal Symmetries

The very special conformal properties of both boundary $\delta M^4_{\pm}$ of 4-D light-cone and of light-like partonic 3-surfaces X^3 imply a generalization and extension of the super-conformal symmetries of super-string models to 3-D context (ch. *Construction of Quantum Theory: Symmetries* of [2]). Both the Virasoro algebras associated with the light-like coordinate r and to the complex coordinate z transversal to it define super-conformal algebras so that the structure of conformal symmetries is much richer than in string models.

a) The canonical (symplectic/contact) transformations of $\delta M^4_\pm \times CP_2$ give rise to an infinite-dimensional symplectic algebra having naturally a structure of Kac-Moody type algebra with respect to the light-like coordinate of $\delta M^4_\pm = S^2 \times R_+$ with finite-dimensional Lie group G being replaced with the symplectic group. The conformal transformations of S^2 localized with respect to the light like coordinate act as conformal symmetries analogous to those of string models. The super-canonical algebra, call it SC, made local with respect to partonic 2-surface can be regarded as a Kac-Moody algebra associated with an infinite-dimensional Lie algebra.

b) The light-likeness of partonic 3-surfaces is respected by conformal transformations of H made local with respect to the partonic 3-surface and gives to a generalization of bosonic Kac-Moody algebra, call it KM. Also now the longitudinal and transversal Virasoro algebras emerge. Generalized coset construction means that the differences of the generators of these Super Virasoro algebras annihilate physical states: the interpretation is in terms of Equivalence Principle since the four-momenta associated with two representations must be identical and thus have interpretation as gravitational and inertial four-momenta.

c) Fermionic Kac-Moody algebras act as algebras of left and right handed spinor rotations in M^4 and CP_2 degrees of freedom. Also the modified Dirac operator allows super-conformal symmetries as gauge symmetries of its generalized eigen modes.

d) The presence of symplectic algebra as the analog of Lie group defining Kac-Moody algebra leads to a generalization of conformal field theory to what might be called symplecto-conformal field theory defined in the sets of points associated with the number theoretic braids (ch. *Category Theory and Quantum TGD* of [2]). Symplectic fusion algebras are commutative and associative modulo braiding and are generated by nilpotent elements ($x^2 = 0$). It is possible to construct an infinite hierarchy of these algebras and they form a structure known as operad [25] in category theory. The basic structure of the operad codes for the coupling constant evolution and there are excellent hopes of constructing explicitly the generalized Feynman graphs in terms of n-point functions of conformal field theories restricted to points of number theoretical braids.

2.5 Quantum TGD as Almost Topological Quantum Field Theory at Parton Level

The light-likeness of partonic 3-surfaces fixes the partonic quantum dynamics uniquely and Chern-Simons action for the induced Kähler gauge potential of CP_2 determines the classical dynamics of partonic 3-surfaces (ch. *Configuration Space Spinor Structure* of [1]). For the extremals of C-S action the CP_2 projection of surface is at most 2-dimensional.

The modified Dirac action obtained as its super-symmetric counterpart fixes the dynamics of the second quantized free fermionic fields in terms of which configuration space gamma matrices and configuration space spinors can be constructed.

The essential difference to the ordinary massless Dirac action is that induced gamma matrices are replaced by the contractions of the canonical momentum densities of Chern-Simons action with imbedding space gamma matrices so that modified Dirac action is consistent with the symmetries of Chern-Simonas action. Fermionic statistics is geometrized in terms of spinor geometry of WCW since gamma matrices are linear combinations of fermionic oscillator operators identifiable also as super-canonical generators (ch. *Configuration Space Spinor Structure* of [1]). Only the light-likeness property involving the notion of induced metric breaks the topological QFT property of the theory so that the theory is as close to a physically trivial theory as it can be.

The resulting super-conformal symmetry involves super-canonical algebra (SC)and super Kac-Moody algebra (SKM) (ch. *Construction of Quantum Theory: Symmetries* of [2]). There are considerable differences as compared to string models.

a) Super generators carry fermion number, no sparticles are predicted (at least super Poincare invariance is not obtained), SKM algebra and corresponding Virasoro algebra associated with light-like coordinates of X^3 and $\delta M^4_\pm$ do not annihilate physical states which justifies p-adic thermodynamics used in p-adic mass calculations, and mass squared is p-adic thermal expectation of conformal weight.

b) One can generalize coset construction for SC and SKM algebras and Equivalence Principle generalizes to the condition that the actions of SC and SKM generators on physical states are identical.

c) The conformal weights and eigenvalues of modified Dirac operator are complex and one can assign to them zeta function and Dirac determinant identified as exponent of Kähler function for the space-time surface associated with light-like 3-surface (ch. *Construction of Quantum Theory: S-matrix* of [2]). This however requires the notion of number theoretic braid.

2.6 Physics as Generalized Number Theory

Number theoretical vision involves three different aspects: number theoretical universality, classical number fields, and the notion of infinite prime.

2.6.1 Number Theoretical Universality

Number theoretical universality leads to the hypothesis that S-matrix elements must be algebraic numbers (ch. *Physics as a Generalized Number Theory: p-Adicization program* of [3]) or rather, continuable from algebraic number fields to real number field and various p-adic number fields. This can be achieved if the definition of S-matrix elements involves only the data associated with number theoretic braids.

The partonic vertices appearing in S-matrix elements should be expressible in terms of N-point functions of almost topological $N = 4$ super-conformal field theory [21, 22] but with the p-adically questionable N-fold integrals over string replaced with sums over the strands of a braid: spin chain type string discretization could be in question (ch. *Physics as a Generalized Number Theory: p-Adicization program* of [3]).

2.6.2 Classical Number Fields

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Imbedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces, and by conformal invariance one-dimensional structures are basic objects. The lowest level corresponds to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [23]) are involved (ch. *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [3]) and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ as number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

Space-time surfaces correspond to preferred extremals of Kähler action. Number theoretical vision allows to settle the question what "preferred" should mean. The tangent space of $X^4(X^3)$ must contain preferred 2-D subspace of $M^2 \subset M^4$ as tangent plane at each point identifiable physically as the plane of un-physical polarizations. This fixes the boundary conditions at X^3. This condition is necessary for "number theoretic compactification" stating that space-time surfaces can be equivalently regarded as surfaces in $M^4 \times CP_2$ or in M^8 regarded as the 8-D subspace of complexified octonions with Minkowskian signature of natural metric (hyper-octonions) (ch. *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [3]) in which space-time surfaces are hyper-quaternionic surfaces of M^8 in the sense that the tangent space at each point is hyper-quaternionic sub-space of M^8. Mathematically M^2 corresponds to hyper-complex and thus commutative sub-space of both M^8 and $M^4 \times CP_2$. The choice of the preferred M^2 is also involved with the generalization of the imbedding space concept to realize the hierarchy of Planck constants (ch. *Does TGD Predict the Spectrum of Planck Constants?* of [2]). The conjecture which remains to be proven is that hyper-quaternionic surfaces of M^8 containing preferred M^2 in their tangent space correspond to preferred extremal of Kähler action when interpreted as surfaces in $M^4 \times CP_2$.

It would not be surprising if the uniqueness of infinite-dimensional Kähler geometric existence - fixing also physics completely - would require that the isometries and holonomies of imbedding space defining standard model symmetries correspond to a group having a number theoretic interpretation and that $X^4 \subset M^4 \times CP_2$ would be preferred by its number theoretic interpretation (ch. *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of

[3]).

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. The powerful number theoretic formulation using the notion of hyper-octonionic conformal field [6] with dynamics dictated by associativity implies large number of "must-be-trues" inspired by quantum TGD.

2.6.3 The Hierarchy of Infinite Primes

The hierarchy of infinite primes (ch. *TGD as a Generalized Number Theory: Infinite Primes* of [3]) - and of integers and rationals- represents the third aspect of the number theoretic vision. This notion was the first mathematical discovery stimulated by TGD inspired theory of consciousness [7]. The construction recipe is equivalent with a repeated second quantization of a super-symmetric arithmetic quantum field theory with bosons and fermions labeled by primes such that the many particle states of previous level become the elementary particles of new level. The hierarchy of space-time sheets with many particle states of space-time sheet becoming elementary particles at the next level of hierarchy and also the hierarchy of n:th order logics are also possible correlates for this hierarchy. For instance, the description of proton as an elementary fermion would be in a well defined sense exact in TGD Universe.

This construction leads also to a number theoretic generalization of space-time point since given real number has infinitely rich number theoretical structure not visible at the level of the real norm of the number a due to the existence of real units expressible in terms of ratios of infinite integers. This number theoretical anatomy suggest kind of number theoretical Brahman=Atman principle stating that the set consisting of number theoretic variants of single point of the imbedding space (equivalent in real sense) is able to represent the points of the world of classical worlds or maybe even quantum states of the Universe [6]. Also a formulation in terms of number theoretic holography is possible.

2.7 The Properties of Infinite-dimensional Clifford Algebras as a Key to the Understanding of the Theory

Infinite-dimensional Clifford algebra of CH can be regarded as a canonical example of a von Neumann algebra known as a hyper-finite factor of type II_1 [11, 12, 19](shortly HFF) characterized by the defining condition that the trace of infinite-dimensional unit matrix equals to unity: $Tr(Id) = 1$. In TGD framework the most obvious implication is the absence of fermionic normal ordering infinities whereas the absence of bosonic divergences is guaranteed by the basic properties of the configuration space Kähler geometry, in particular the non-locality of the Kähler function as a functional of 3-surface.

The special properties of this algebra, which are very closely related to braid and knot invariants [17], quantum groups[18, 19], non-commutative geometry [13], spin chains [16], integrable models, topological quantum field theories [14], conformal field theories, and at the level of concrete physics to anyons [20], generate several new insights and ideas about the structure of quantum TGD.

2.8 Quantum Measurement Theory with Finite Measurement Resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ [15, 19] of HFFs lead to quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ (ch. *Construction of Quantum Theory: S-matrix* of [2]). Quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ interpreted as $\mathcal{N}$-module creates physical states modulo measurement resolution. Complex rays of the state space resulting in the ordinary state function reduction are replaced by $\mathcal{N}$-rays and the notions of unitarity, hermiticity, and eigenvalue generalize (ch. *Construction of Quantum Theory: S-matrix* of [2]).

Non-commutative physics would be interpreted in terms of a finite measurement resolution rather than something emerging below Planck length scale. An important implication is that a finite measurement sequence can never completely reduce quantum entanglement so that entire universe would necessarily be an organic whole.

At the level of conscious experience, the entanglement below measurement resolution would give rise to a pool of shared and fused mental images giving rise to "stereo consciousness" (say stereovision) [7] so that contents of consciousness would not be something completely private as usually believed. Also fuzzy logic emerges naturally since ordinary spinors are replaced by quantum spinors for which the discrete spectrum of the eigenvalues of the moduli of its spinor components can be interpreted as probabilities that corresponding belief is true is universal (ch. *Construction of Quantum Theory: S-matrix* of [2]).

2.9 The Notion of Number Theoretic braid

The notion of number theoretic braid has gradually become basic notion of quantum TGD. One ends up to the notion of number theoretic braid in several manners.

a) The notion of finite measurement resolution can be also described in terms of quantum groups. The transition $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ interpreted in terms of finite measurement resolution should have space-time counterpart. The simplest guess is that the functions, in particular the generalized eigen values, appearing in the generalized eigen modes of the modified Dirac operator D become non-commutative. This would be due the non-commutativity of some H coordinates, most naturally the complex coordinates associated with the geodesic spheres of CP_2 and $\delta M^4_+ = S^2 \times R_+$. Stringy picture would suggest that these complex coordinates become effectively quantum fields. Their appearance in the generalized eigenvalues for the modified

Dirac operator would reduce the anti-commutativity of the induced spinor fields along 1-dimensional number theoretic string to anti-commutativity at the points of the number theoretic braid only. The difficulties related to general coordinate invariance would be avoided by the special character of the quantized H-coordinates. The outcome is that the points of geodesic spheres of light-cone boundary and CP_2 become non-commuting fields and number theoretic braids correspond to the points for which the coordinates commute.

b) Number theoretical universality requires that M-matrix should be constructed using only data from a discrete set of points of space-time surface which as imbedding space points have algebraic coordinate values in preferred coordinate system (guaranteed by isometries of H for given CD). M-matrix at these points for algebraic values of various wave functions (such as plane wave factors) must be algebraic.

c) Vacuum functional emerges as an appropriately defined Dirac determinant. Dirac determinant is defined as a discrete product of the generalized eigenvalues (functions) of the modified Dirac operator at points defined by number theoretic braids.

d) Number theoretic compactification allows a unique identification of the number theoretic braids. The points of space-time surface of M^8 appearing in n-point functions must be associative: this restricts the points to hyper-quaternionic subspace M^4 of M^8. The intersection $X^4 \cap M^4$ is a discrete set in the generic situation. The highly non-trivial boundary condition is that the ends of the partonic 3-surfaces at boundaries CDs belong to $M^4 \subset M^8$ and are thus associative 2-surfaces. The condition that the points of X^2 appearing in n-point function commute implies discretization and the light-like orbits of points define number theoretical braid strands.

e) Symplectic fusion algebras are defined only in discrete case and one can construct geometric representations for the algebras in terms of number theoretical braids (ch. *Category Theory and Quantum TGD* of [2]).

2.10 Zero Energy Ontology

Zero energy ontology (ch. *Construction of Quantum Theory: S-matrix* of [2]) states that all physical states have vanishing net quantum numbers and decompose to positive and negative energy components. A precise formulation of zero energy ontology is in terms of the "world of classical worlds" concept (WCW). WCW decomposes into a union of sub-WCW:s associated with causal diamonds of $M^4 \times P_2$ consisting of intersections of future and past directed light-cones. Each CD breaks the full Poincare symmetry but at the level of entire structure Poincare symmetry is exact.

Positive and negative energy parts of the zero energy states are assignable to the 2-surfaces at the upper and lower light-like boundaries of the CD at the ends of light-like 3-surfaces. CDs contain CDs within CDs and insertion of zero energy states to either positive or negative energy state has interpretation in terms of improved measurement resolution bringing new quantum processes in daylight. A geometric realization of coupling constant evolution in terms of the resolution scale emerges.

The diamond like geometry of CD implies that the temporal distances between tips of CD come naturally as powers of two identifiable and are identifiable secondary p-adic time scales so that p-adic length scale hypothesis follows naturally. Also and answer to the long standing question about the precise relationship between geometric time and experienced time interpreted as sequence of quantum jumps emerges from this formulation [9].

There are arguments supporting the belief that the U-matrix characterizing the unitary process associated with quantum jump is rather trivial from the point of view of particle physics (ch. *Construction of Quantum Theory: S-matrix* of [2]). U matrix would be however very relevant for understanding of p-adic-to-real transitions serving as correlates for the transformation of intentions to actions. The almost triviality for real-real transitions and reduction of U matrix to tensor product of U-matrices associated with positive and negative energy parts of the zero energy state would explain why the usual positive energy ontology (having clockwork universe as its extreme version) is so good an approximation. The properties of U-matrix would also explain why sensory perceptions seem to be about reality rather than the change of reality.

A more natural identification of the TGD counterpart for the particle physics S-matrix is as entanglement coefficients for time like entanglement between positive and negative energy states which at space-time level are located at the boundaries of future and past light-cones defining CD (ch. *Construction of Quantum Theory: S-matrix* of [2]). There is no need to pose unitarity condition anymore: hence the term M-matrix. One can identify M-matrix as a "complex square root" of density matrix which can be written as a product of real and diagonal square-root of density matrix and unitary S-matrix so that a unification of thermodynamics and quantum theory emerges. This hypothesis makes sense even when density matrix is unit matrix thanks to the condition $Tr(SS^{\dagger}) = Tr(Id) = 1$ holding true for HFFs.

2.11 Generalization of Imbedding Space Concept and Hierarchy of Planck Constants

Quantum classical correspondence suggests that Jones inclusions [15] have space-time correlates (chapters *Was von Neumann Right After All?* and *Does TGD Predict the Spectrum of Planck Constants?* of [2]. There is a canonical hierarchy of Jones inclusions labeled by finite subgroups of SU(2)[19].

This forces a generalization of the imbedding space to a book like structure obtained by gluing an infinite number of copies of H regarded as singular $G_a \times G_b$ coverings of H and bundles over $H/G_a \times G_b$, where $G_a \times G_b$ is a subgroup of $SU(2) \times SU(2) \subset SL(2,C) \times SU(3)$. Gluing occurs along a factor for which the group is same.

The groups in question define in a natural manner the direction of quantization axes for for various isometry charges and this hierarchy seems to be an essential element of quantum measurement theory. Ordinary Planck constant, as opposed to Planck constants $\hbar_a = n_a\hbar_0$ and $\hbar_b = n_b\hbar_0$ appearing in the commutation relations of symmetry algebras assignable to M^4 and CP_2, is naturally quantized as $\hbar = (n_a/n_b)\hbar_0$, where n_i is the order of maximal cyclic subgroup of G_i. What is also important is that $(n_a/n_b)^2$ appear as a scaling factor of M^4 metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to the functional integral over WCW does not mean vanishing of radiative corrections. G_a and G_b could correspond directly to observable symmetries of visible matter induced by the underlying dark matter (ch. *Construction of Quantum Theory: S-matrix* of [2]).

The particles at different pages of the book like structure cannot appear in the same vertex of Feynman diagram and are dark relative to each other in this restricted sense. This weaker notion of darkness seems to be consistent with what is known about dark matter, and leads to a detailed model for bio-systems in terms of dark matter explaining large number of observed anomalies [8].

A further fascinating possibility is that the observed indications for Bohr orbit quantization of planetary orbits [26] could have interpretation in terms of gigantic Planck constant for underlying dark matter (chs. *TGD and Astrophysics* and *Quantum Astrophysics* of [4]) so that quantum coherence would be possible in astrophysical scales manifesting itself in many manners: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.

3 About the Construction of M-matrix

The following summarizes the general principles behind the construction of M-matrix.

3.1 M-matrix as a Functor

Almost topological QFT property of quantum allow to interpret M-matrix as a functor from the category of generalized Feynman cobordisms to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces (this generalizes the idea of John Baez [24]).

a) Feynman cobordism is a generalized Feynman diagram having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. Incoming partons of the generalized Feynman diagram are at the light-like boundaries boundaries of CD. Lines correspond to light-like 3-surfaces and vertices to 2-D partonic two surfaces at the boundaries of sub-CD. In TGD framework stringy vertices describe the interference of waves propagating via different paths represented by the legs of the stringy diagram. To the lines of generalized Feynman diagrams one an assign braiding S-matrix and to the vertices n-point functions of a conformal field theory for the fundamental fermions. The earlier hypothesis that braid strands emerging in given vertex replicate is not necessary and is also inconsistent with the symplectic fusion rules. The strands of braids just end to the the partonic two surface and n-point function of the conformal field theory determines the vertex.

b) The addition of smaller CD inside CD corresponds to a zero energy insertion adding to a positive or negative energy part of state a zero energy state in a shorter time scale. Locally dynamical objects are 2-D partonic surfaces at light-cone boundaries but the possibility to add zero energy insertions means discretized 3-dimensionality. One obtains also discretized four-dimensionality and there is a dimensional hierarchy starting from number theoretical braids. This picture about coupling constant evolution can be formulated in terms of a generalization of the notion of planar operad (ch. *Construction of Quantum Theory: S-matrix* of [2]).

c) There is a functional integral over the small deformations of Feynman cobordisms corresponding to maxima of Kähler function. Functor property generalizes the unitary condition for S-matrix part of M-matric and allows also thermal M-matrices which seem to be unavoidable since imbedding space degrees of freedom give rise to a factor of type I with $Tr(Id) = \infty$ (for factors of type II_1 one has $Tr(Id) = 1$). Hence thermodynamics becomes a natural part of quantum theory. The most general identification of the time like entanglement coefficients would be as a "square root" of density matrix thus satisfying the condition $\rho^+ = SS^\dagger$, $\rho^- = SS^\dagger$, $Tr(\rho^\pm) = 1$. $\rho^\pm$ has interpretation as density matrix for positive/negative energy states. Physical intuition suggest that M can be written as a product of universal unitary matrix S and square root of state dependent density matrix.

e) One might hope that all complications related to what happens for *space-like* 3-surfaces could be eliminated by quantum classical correspondence stating that space-time view about particle reaction is only a space-time correlate for what happens in quantum fluctuating degrees of freedom associated with partonic 2-surfaces. This is the case only in non-perturbative phase since the arguments of n-point function appear as continuous moduli of Kähler function. In non-perturbative phases the dependence of the maximum of Kähler function on the arguments of n-point function cannot be neglected and Kähler function becomes the key to the understanding of these effects including formation of bound states and color confinement. There are excellent number theoretical reasons to hope that the functional integral over small deformations can be carried out exactly.

3.2 Symplectic Fusion Algebra and Feynman Operad

Super-canonical transformations (ch. *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [1]) distinguish quantum TGD from string model and the resulting symplectic fusion algebras realize algebraically the absence of ultraviolet divergences predicted by very general arguments (Kähler function is non-local functional of 3-surface and Gaussian and metric determinants cancel each other). Algebraically this corresponds to the nilpotency ($x^2 = 0$) of the generating elements of the necessarily discrete symplectic fusion algebras forming an infinite hierarchy labeled by all possible trees. The addition of a tree at the end of given branch means improvement of measurement resolution replacing single strand of braid with a braid. Symplecto-conformal fusion algebras form an operad like structure meaning the existence of homomorphisms between the original algebra and that obtained by improving measurement resolution. These homomorphisms characterize algebraically what happens as the measurement resolution is increased or reduced locally. Symplectic transformations of $\delta M^4_\pm \times CP_2$ act as gauge symmetries mapping to each other equivalent triangulations of the sphere defining representations of symplectic fusion algebras.

The structure constants of the symplectic fusion algebra are products of three roots of unity (this realizes number theoretical universality) and have physical interpretation as plane wave factors. Also the representations of color quantum numbers in this manner are possible. Momentum conservation modulo measurement resolution means that the product of the phase factors associated with the lines arriving to the vertex equals to unity. At the continuum limit exact momentum conservation emerges and usual conformal N-point functions represent vertices.

3.3 Quantum S-matrix

The description of finite measurement resolution in terms of Jones inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum S-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that the full M-matrix can be expressed as S-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum M-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the S-matrix.

Quantum S-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

4 Conclusions

These three decades have taught me how slow the progress in physics really is and how small the contribution of individual is bound to be. I have been able to identify these visions about basic principles and perhaps even demonstrate that M-matrix exists and is unique to a high degree. There is still a long way to concrete Feynman rules although the understanding has increased dramatically also in this respect. I can only hope that some young mathematically oriented colleague becomes convinced that this strange jungle of ideas that I call TGD might provide inspiration if not anything else.

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