Constrained Thermal Plant Control Based on the First Order Models

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Abstract
This article deals with the control design for a thermal plant with constrained input signal based on the first order time delayed models. The design follows the requirement of the fastest possible transient processes without overshoot. The piecewise constant (or slowly varying) disturbances are compensated by a windupless integral action added to the controllers output.
Keywords: thermal plant, constrained pole assignment controller, dead time, windupless controller design, disturbance reconstruction and compensation.

1 Introduction

The PI controllers are known as the basic and most frequently used instruments in the industrial automation and control. In the early stages that go back yet before the World War I. they were usually combined in many different ways with sensors and actuators in one device. But in the period after the World War I. they were already produced in the form we know them today. According to their functional simplicity one could expect that all basic problems relevant for their utilization have already been clarified many decades ago. However, in fact, contrary is the case. Up to date, practically at each conference oriented to the control systems design, new papers devoted to an "optimal" design or tuning of PI controllers can be found. So, already this single moment is enough to indicate a till now hidden aspects necessary for a reliable controller design. The key factors for this „inflation" were shown by Huba et al. (1998) as **the distribution of the dynamical terms within the control loop** in combination with the **control signal saturation**. This leads to the existence of two different dynamical classes of PI-controllers. And only one dynamical class can be identified as identical with the classical linear solution. In that paper also the new ways to generally acceptable windupless PI-solutions were proposed that were further extended to the PID control by Huba (2003).

In this paper, several practical steps of the new windupless PI controllers design are demonstrated by controlling thermal plant used in education.

2 Model of the Thermal Plant

Thermal plant consists of heating X (light bulb), temperature sensor, fan and control electronics. The diagram of the thermal plant is shown in Fig.1. Table Tab.1 contains data of the input-output characteristic of the plant, the measured dependency is shown in Fig.2.

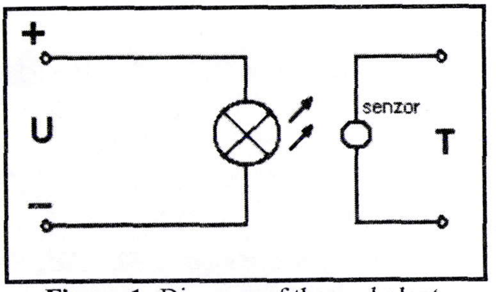

Figure 1: Diagram of thermal plant.

Table 1: Measured input-output characteristic values of thermal plant

T [°C]	u [V]
23	**-2.2**
25	**-2.28**
27	**-2.32**
29	**-2.34**
30	**-2.38**
31	**-2.42**
33	**-2.44**
35	**-2.46**

The input-output steady state characteristic was approximated by a 2nd degree polynomial computed by the least square method. The result is described by equation

$$T = 67.0031u^2 + 268.2099u + 288.7043 \quad (1)$$

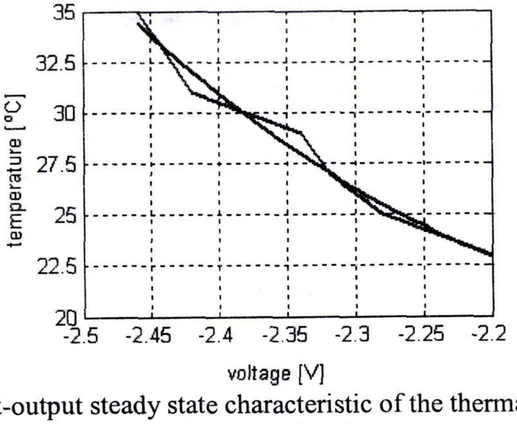

Figure 2: The input-output steady state characteristic of the thermal plant (broken line) and its approximation.

Fig. 3 shows a step response of the thermal system. The step of the input voltage started from -2.28 V to -2.44 V. By drawing tangent to the step response we got approximation by single integrator + dead time. Note that the system behavior corresponds at the beginning to the exponential one of the 1st order static system, but later it turns to behavior similar to the 1st order integral system. Neither in the case of a downward step the temperature transient is simple. First it drops down and then it grows up slowly. This does not correspond to the 1st order integral system and this characteristic of the system we could model as disturbance acting always in one direction and causing a non-symmetry in behavior. The other possibility would be to describe it by time-variant parameters of the system.

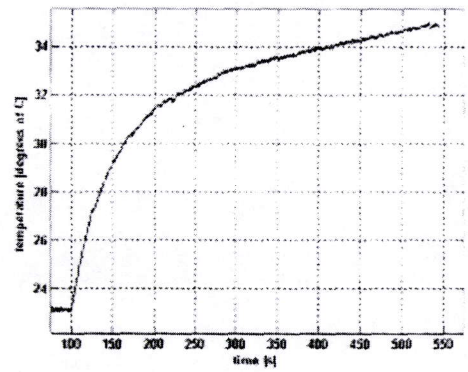

Figure 3: Step response of the thermal plant.

The I_1T_d approximation of the response corresponding to an upward step is shown in Fig. 4. The acquired model is:

$$F(s) = \frac{K_s}{s} e^{-T_d s}; \quad T_d = 1\,s;\ K_s = -1{,}1165 \qquad (2)$$

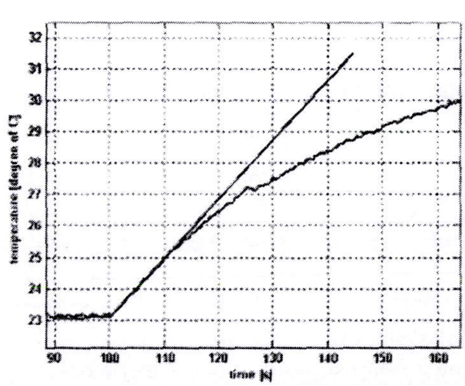

Figure.4: I_1T_d approximation of the upward step response of the thermal plant.

3 Constrained Control of the 1st Order Systems

3.1 Controller Design

Consider the stable 1st order system described by transfer function (3a).

$$S(s) = \frac{K_s}{s + a} \tag{3a}$$

Its discrete-time transfer function can be expressed as

$$S(z) = \frac{K_a}{z - D} \tag{3b}$$

whereby

$$D = e^{-aT}; \qquad T \text{ is sampling time}$$
$$K_a = K_s(1 - e^{-aT})/a, \quad a \neq 0 \tag{4}$$
$$K_a = K_s T, \quad a = 0$$

At first, let us use just the P-controller with the gain K_R. The pole of the continuous closed control loop is given by (5a)

$$\alpha = -K_R . K_s + a \tag{5a}$$

or, in discrete case, by (5b)

$$\lambda = D - K_R . K_a \tag{5b}$$

If we require the closed loop to have a chosen pole α or λ, we have to adjust the continuous controller gain K_R as

$$K_R = -\frac{a + \alpha}{K_s} \tag{6a}$$

and in discrete-time case according to

$$K_R = \frac{D - \lambda}{K_a} \tag{6b}$$

Theoretically, the control process has no overshoot even if K_R approaches very high values.

In practice we can find that increasing the value of K_R an overshoot appears. This fact is caused by the unmodelled delays, which are involved in almost every real system. If we do not change the control loop structure, the parasitic delays need to be compensated in a "passive" way - by tuning the value of K_R. So, the aim of a passive compensation of the time delays is to ensure the fastest possible transient processes without oscillations or overshooting that might occur in the case of real delayed system. This is achieved by assigning the closed loop pole value denoted as the *equivalent* pole. The equivalent pole is a number, which, after substituting into formulas (6a)-(6b), ensures the optimal value of K_R for the loop with delayed system. The value of equivalent pole depends on the nature of the plant (static or integral one) and in the discrete-time case also on the ratio of the sampling period T and the value of the transport delay T_d. Tables 2 and 3 summarizes formulas to compute the value of equivalent pole for particular cases.

Table 2: Formulas for computing the value of the equivalent pole in dependence on T_d/T ratio for the time delayed integrator ($a=0$)

	α or λ
$T_d \le T$	$1-\dfrac{T}{(\sqrt{T_d}+\sqrt{T})^2}$
$T_d = nT$ $n=1, 2, \dots$	$1-\dfrac{n^n}{(n+1)^{n+1}}$
$T \to 0$	$-\dfrac{1}{T_d e}$

Table 3: Formulas for computing the value of the equivalent pole in dependence on T_d/T ratio for the static plant ($a \ne 0$)

	α or λ
$T_d \le T$	$e^{-aT} - \dfrac{e^{aT_d} e^{aT-1}(1-e^{-aT})\left(1+\sqrt{\dfrac{1-e^{-aT}}{1-e^{-aT_d}}}\right)^2}{aT}$
$T_d = nT$; $n=1, 2, \dots$	$e^{-aT} - \dfrac{e^{-aT_d}(1-e^{-aT})n^n}{(e^{-aT}-1)(n+1)^{n+1}}$
$T \to 0$	$-\dfrac{e^{-aT_d}+aT_d e}{T_d e}$

whereby T_d - transport delay
T - sampling time

In order to achieve the zero steady state error, the above-mentioned P-controller can be supplemented with the feed-forward control with the gain a/K_s.

In an ideal case such a control structure ensures the zero steady state error, but it does not compensate disturbances and does not work satisfactorily if the plant is nonlinear, or not exactly identified. In such a case we need to add an I-action to the control structure.

3.2 Windupless I- action

The traditional I-action used e.g. in the parallel PI controller should guarantee the zero steady state error also in presence of piece-vise constant disturbances. This is achieved by generating a signal that is counteracting the disturbance related to the process input. The problem of the parallel I-action is, however, caused by the fact that the integration process starts in direction given by the sign of the control error, not by the sign of the disturbance!

The above-mentioned shortness of the classical I-action can be avoided also without adding the anti-wind-up blocks into the control loop by using reconstruction based disturbance compensation. While the known input-output behavior may be improved by using static feed-forward control, the unknown disturbances are reconstructed by means of the reconstruction of the actual plant input using the inverse plant transfer function filtered by the 1st order filter, from which the equally filtered controller output is subtracted. The discrete-time and continuous control structures are shown in Fig. 5 - 6.

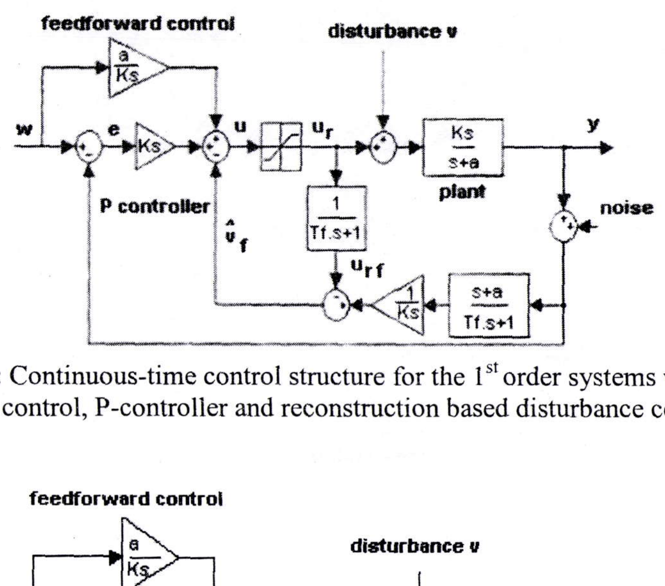

Figure 5: Continuous-time control structure for the 1st order systems with static feedforward control, P-controller and reconstruction based disturbance compensation.

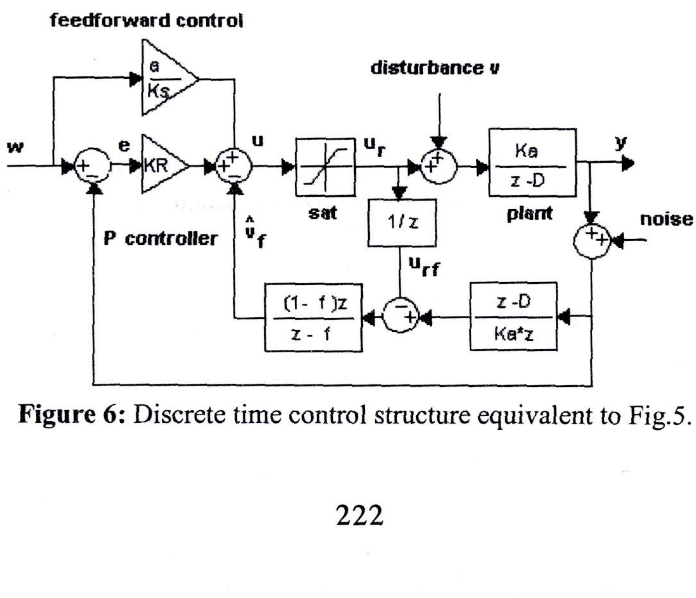

Figure 6: Discrete time control structure equivalent to Fig.5.

Whereby

$S(s) = \frac{K_R}{s+a}$ — S(s) is the system transfer function

$D = e^{-aT}$ — T is sampling time

$$K_a = K_S(1-e^{-aT})/a \quad a \neq 0$$
$$K_a = K_S T \qquad a = 0 \tag{7}$$

K_R — P-controller gain

T_f, α_f, λ_f — Filter time constant or the pole of the filter (continuous and discrete)

In the case when using the I_1T_d plant approximation, when the parameter $a = 0$, the static feed-forward control may be omitted (it gives a zero signal).

This reconstruction based disturbance compensation ensures that the additional signal $-v_f$ will actuate against disturbance - with a short transient determined by the filter time constant T_f.

Let us derive optimal values of parameters K_R and T_f. For simplicity consider integral system with $a = 0$, $D = 1$, $K_a = K_sT$ and the dead time equal to integer multiple of the sampling period nT. Corresponding characteristic polynomial of the closed control loop is

$$P(z) = (z-1)^2 z^n + K(z-\lambda_f) + (1-\lambda_f)(z-1)$$
$$K = K_s K_R T \tag{8}$$

The requirement of equally fast transient process results in finding a triple real pole when one can write system of equations

$$P(z_0) = 0; \quad \left[\frac{dP(z)}{dz}\right]_{z=z0} = 0$$
$$\left[\frac{d^2P(z)}{dz^2}\right]_{z=z0} = 0 \tag{9}$$

Obtained solutions for K and λ_f are complex, so we approximate them by taking their real parts. The optimal parameters of continuous-time controller can be achieved as limits for $T \to 0$:

$$\alpha_f = \lim_{T\to 0}\frac{1}{T}\ln\lambda_f = -\frac{\left(\sqrt{2}-1\right)e^{\sqrt{2}-2}}{T_d} \approx -\frac{0.231}{T_d} \tag{10}$$

$$T_f = -\frac{1}{\alpha_f} \approx 4.337T_d \tag{11}$$

$$K_R = \lim_{T\to 0}\frac{K}{K_sT} \approx \frac{0.231}{K_sT_d} = \frac{1}{4.337K_sT_d} \tag{12}$$

Solving the same problem in the continuous-time domain, one gets the optimal value K_R that is equal to (12), but the value of the filter time constant is

$$T_f = 2.91 T_d \tag{13}$$

Because these values are just approximations of optimal complex values, they do not guarantee the originally planned triple closed loop dominant pole

$$s_0 = -0.423 / T_d \tag{14}$$

but for (11-12) they give the poles

$$s_1 = -0.166 / T_d \ ; \ s_{2,3} = (-0.786 \pm j0.441) / T_d \tag{15}$$

and for (12-13) poles

$$s_1 = -0.19 / T_d \ ; \ s_{2,3} = (-0.631 \pm j0.762) / T_d \tag{16}$$

While the first couple of controller parameters (11-12) is simple to remember by the relations

$$K_R = 1/(cT_d K_s) \ ; \ T_f = cT_d \tag{17}$$

the second couple gives a bit higher dynamics. In the following experiments we prefer the simplicity of tuning and use the controller parameters corresponding to (17) with $c = 4.337$.

Let us notice that the optimal parameters for static system with a short dead time are roughly given as (11) and (12) multiplied by e^{-aT_d}

$$T_f = -\frac{e^{-aT_d}}{\alpha_f} \approx 4.337.T_d e^{-aT_d} \tag{18}$$

$$K_R \approx \frac{0.231e^{-aT_d}}{K_s T_d} = \frac{e^{-aT_d}}{4.337 K_s T_d} \tag{19}$$

3.3 Experimental Results

The quasi-continuous control with controller parameters set according to (11) and (12) (using the structure of Fig. 6) results into the responses shown in Fig.7. The sampling period was set to T=0,1 s.

In the case of a downward step an undershooting of the output signal below the desired value occurs. This is due to the already mentioned non-symmetry of behavior. The other problem is caused by the fact that in the flow processes the identified dead time depends on the associated flow. In means that increasing the flow the identified dead time decreases. So, when we tuned the controller by raising the value of K_R 5x to -1.032, while the value T_f remained unchanged, the overshooting reasonably decreased. The corresponding control result can be seen in Fig.8. So, a more precise tuning requires analysis of the process for the whole range of possible inputs.

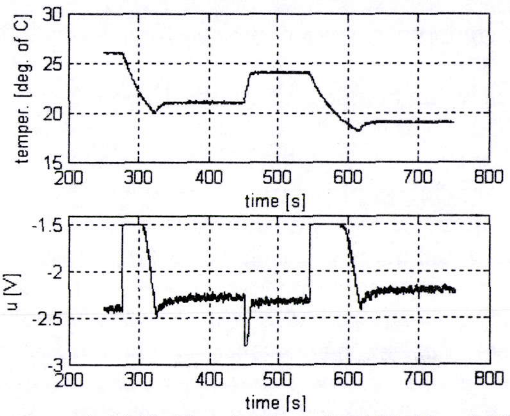

Figure 7: K_s = -1.1165, T_d = 1, K_R = -0.2065, T_f = 4.3370; T=0,1 s

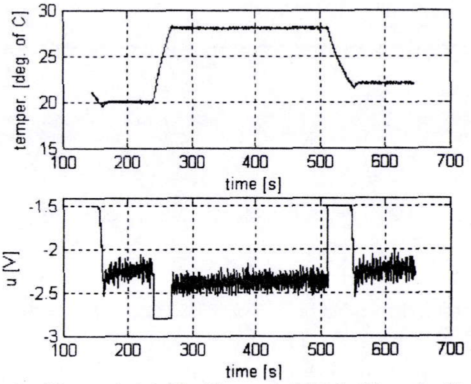

Figure 8: K_s = -1.1165, K_R = -1.0326, T_d = 1, T_f = 4.3370

4 Conclusion

In this paper we presented effective approach to the control design of the thermal plant with constrained input based on the 1st order models. The proposed structure respects the control signal constraints and, moreover, it enables to achieve monotonous transient processes with short settling time. The designed I-action responds fast and its implementation is windupless. The structure is robust and can be applied to a broad range of systems with the dominant 1st order dynamics (and not only to systems with clearly linear behavior). Results obtained from experiments on real plant are better than those obtained by traditional PI controllers supplemented with standard anti-wind-up structures. The computation of control is simple as contrasted to the most of alternative constrained control design approaches (e.g. the constrained predictive control, see e.g. Scheffer-Dutra et al. (2002)).

One segment of the new solution derived for the 1st order systems - the structure of the PI-controller with a feedback from the saturation output - was mentioned already by several papers (e.g. Åström and Hägglund (1995), Kothare et al. (1994)). It is implemented by several producers of industrial controllers. However, as a fraction of the physically motivated approach the up to now used solution did not yield the optimal possible properties and also did not have a sound theoretical background.

Of course, by modifying the linear PI-controllers with different anti-windup structures one can get many solutions and it is far beyond the possibilities of a single paper to test all of them. But, we believe that the optimal solutions to constrained systems cannot be simply achieved by modifying the linear solutions on a heuristic basis (or by the trial-error procedures). So, we prefer solutions motivated by clear physical interpretation. Not those motivated by the mathematical properties (linearity), or simply by the traditional habits of practice.

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