Holographic Neural Technology, Systems and Applications

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Abstract

Described is a “neural“ operating system based on Holographic/Quantum Neural Technology (HNeT). The core of the HNeT technology applies Hilbert space operations in both the updating of cortical memory and generation of response recall, similar in form to the QM wave function. Within HNeT, information is represented by sets of complex scalars, leading to a natural predilection towards frequency domain representations of stimuli. Conversion of real valued information sets to frequency domain representation leads to a number of desirable qualities, such as orthogonalization of highly asymmetric or non-orthogonal pattern sets, a distributed representation of information, as well as an effective means for data reduction (i.e. Fourier quantization). Higher order frequency domain representations facilitate extraction of invariants that define discriminating features, often intractable using conventional pattern classification methods. This is performed utilizing a form of neural plasticity that scans the set of higher order harmonics for discovery of such invariants.

One of the most salient operational aspects of holographic/ quantum neural technology is the reduction in computational complexity over more traditional neural networks (NN). For instance, HNeT requires only binary cell structures in quite advanced application areas. Holographic/quantum neural technology also provides a dramatic increase in speed of learning and learning accuracy over traditional NN methods. The HNeT process facilitates real-time learning, in which large data sets may be learned to high accuracies following one training epoch.

The HNeT core processes have been extended considerably over the past few years to incorporate a number of auxiliary features. These features include application of higher order combinatorics for pre-process of input stimuli, the application and advanced control of neural plasticity, the use of cell assemblies that facilitate “super-cell” structures similar in form to neo-cortical assemblies, and unsupervised learning structures that facilitate hyperincursive and spatio-temporal learning paradigms, among others. Current work is directed towards structures that facilitate temporal accumulation of spatial patterns at the preprocess level, prior to entry into cortical cell structures. These accumulative structures possess certain analogous features to the thalamus, permitting synthetic neural systems to learn spatio-temporal patterns such as speech.

The HNeT system is biologically motivated, possessing an application programming interface (API) that allows the user to allocate specific cell types based on the granule, pyramidal, stellate, and Purkinje cells of the cerebellum and neo-cortex. This operating

International Journal of Computing Anticipatory Systems, Volume 7, 2000
Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-9600179-9-4

system permits the user to flexibly configure cell assemblies, and build cortical structures comprised of anywhere from 2 to several thousand cells.

Keywords: Holographic, Quantum, HNeT, Cortical, Cerebellar

1 Basic Concepts

The operation basis of HNeT presents a fundamental diversion from the standard connectionist models in the Artificial Neural System (ANS) field. Within the holographic/quantum neural model, the neuron cell follows a non-connectionist model whereby learning and subsequent recall of stimulus-response associations are performed within a *single cortical cell.* In effect, the operational features exhibited by the connectionist neural network is condensed into a single holographic/quantum neuron cell, and in virtually all cases the speed of learning and recall accuracy for this cell is dramatically improved over traditional neural network models.

The mathematical basis for HNeT permits such stimulus-response patterns to be learned or "mapped" within a matrix of complex scalars. The complex scalars stored within this matrix are referred to as the cell's *cortical memory elements.* The number of complex scalars contained within the matrix may be no larger than the number of fields within a single stimulus pattern. However, many associative stimulus-response patterns or "memories" may be stored onto the identically same matrix. This mechanism for holographic storage displays a capacity to achieve very high information densities, due to the fact that large numbers of associative patterns are superimposed. This aspect of superimposing associative information is referred to periodically as *enfolding* [1]. In addition to the aspect of enfolding, large numbers of associative patterns may be learned over a single training exposure (i.e. one epoch), providing the capability for "real-time" learning. The HNeT system generalizes in a manner that the mapping generated within the neuron cell achieves a smooth interpolation among all of the stimulus-response patterns learned by that cell.

A quantum analogy is derived from the *wave packet* equation found in quantum mechanics [2]. The wave packet relates to the superposition of energy waves of slightly differing angular momentum. This superposition of harmonics defined by the wave packet produces an energy envelope illustrated below, and is applied within quantum mechanics to describe the state of a photon, electron, proton and other particles.

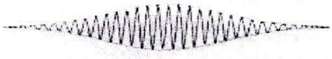

Fig. 1: Wave Packet Envelope for a Photon or Particle

The wave packet equation is shown below in both the discrete summation and standard Dirac notation:

$$\Psi_{x.t} = \sum_i C_i e^{i(k_i x - w_i t)} = \langle f \,|\, \psi \rangle \text{ integrated over wave number } (k_i) \qquad (1)$$

Operation of the holographic/quantum neuron cell within a (much simplified) linear framework follows a similar form as that shown above. Within the quantum neural context, the above inner product form describes the learning process that takes place within each cortical memory element X_n, of the neuron cell. Each neuron cell may possess many thousands of cortical memory elements.

$$X_n = \sum_t \lambda_{n.t}\gamma_t e^{i(\phi_t - \theta_{n.t})} = \langle S_n | R \rangle \text{ integrated over time } (t) \qquad (2)$$

where in this case $S_n = \left\{\lambda_{n.t} e^{i\theta_{n.t_1}}, \ldots\right\}, R = \left\{\gamma_{t_1} e^{i\phi_{t_1}}, \ldots\right\}$

The above operation superimposes or *enfolds* information pertaining to the complete set of stimulus-response patterns onto isolated complex scalars, each complex scalar representing a cortical memory element *X*. One cortical cell stores many scalars or cortical memory elements. For clarity, the S vector indicates the stimulus signal, and the *R* vector the response. The response recall process within holographic/ quantum neural cells follows a similar form, however, the inner product is performed over the cortical memory elements (indexed by *n*) rather than time.

$$R' = \Omega_S \langle S^* | X \rangle \quad \text{summed over cortical memory elements} (n) \qquad (3)$$

During the above recall operation, a new stimulus S^* is transformed simultaneously through the stimulus-response memories enfolded within the cell's cortical memory elements. This aspect of intrinsic parallel computation is analogous to the concept of "quantum parallelism".

1.1 Representation of Information

Current theories in computational neural dynamics follow from an idea known as the Hebb hypothesis [3]. That is, growth or learning mechanisms within the neuron cell are activated through coincident stimulation at both sides of the synaptic connection made between two cells. Recent neural theory has elaborated on this, leading to the current profusion of gradient-descent type algorithms, whose core aspects are largely built upon the real-valued inner product.

The holographic/quantum model described here presents a viewpoint that employs the mathematics of complex scalars in combination with a non-linear representation of stimulus information, plus a few other features. An element of information within the holographic/ quantum neural paradigm is represented by a complex scalar derived in most cases from a harmonic decomposition of the raw input pattern (i.e. Fourier, Gabor, wavelet). The processes outlined in this paper may be applied to input scalars of unit magnitude, although the process used within HNeT has been generalized to converge (learn) in an optimial manner for input scalars that vary from unity.

Some aspects governing the intra-cellular transmission of signals between biological neuron cells lend support to the complex scalar representation for information. Figure 1 illustrates a cell in which multiple input lines receive pulse modulated signals, and a similarly pulse modulated signal delivers the cortical cell's response output. In a practical sense, pulsed biochemical signals could facilitate single line (micro-tubule) transmission of complex scalar quantities mediated by the frequency and amplitude of the pulse. Far more important however are the operational features observed within a simulated cortical cell when complex scalars are applied.

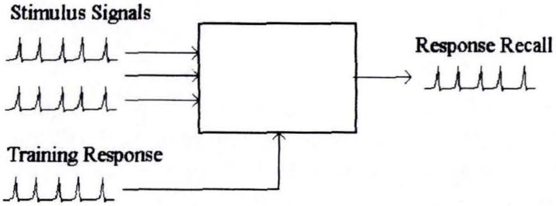

Fig. 2: Block Diagram of a Cortical Cell

The basic unit of information within the HNeT system may be represented by exponential form $\lambda e^{i\theta}$. Such quantities may be viewed to contain both a semantic content represented by the phase orientation (θ), and a level of confidence represented by magnitude (λ). In cases where a time to frequency transformation is applied as the preprocess operation, the phase angle orientation represents phase shift, and magnitude presents the power of the harmonic. In a similar vein, stimulus inputs of low power have less influence over the stimulus-response mappings learned by the cell, as well as less influence in the generation of the response signal during recall.

In the following discussions a simplifying assumption is made, this being that all stimulus and response scalars have unit magnitude. The memory generated within the cell for such stimulus-response associations are built up from *phase angle differences*. For instance, one element of a stimulus may be represented by phase orientation $e^{i\theta_j}$ and an associated response by orientation $e^{i\phi_k}$. The mathematical quantity that is generated for associating stimulus element j to response element k is obtained by the following phase conjugate operation $\overline{s_j} \cdot r_k$ producing the phase angle difference:

$$e^{i\theta_{diff}} = e^{\phi_k - \theta_j} \qquad (4)$$

The above quantity $e^{i\theta_{diff}}$ may be said to represent an indissoluble "quanta" of information. This information quanta stores the portion of an associative mapping that is learned by one cortical memory element for one pattern, this cortical memory element connecting one field of the stimulus to one field of the response. The mathematical properties of these information quanta provide the basis for the superposition of a large amounts of stimulus-response information onto the same sets of complex scalars. Learning capacity is largely based upon the number of cortical memory elements (i.e. complex scalars), and this learning capacity is directly proportional to the number of cortical memory elements stored within the cell. To provide a practical example, a cell with 100 memory elements is capable of learning and storing 100 stimulus-response associations derived from the Monte-Carlo method (and considerably more if the patterns reflect any coherent structure), 1000 memory elements can store 1000 stimulus-response patterns, and so on.

The HNeT cell displays a further aspect in its ability to respond to the space of unknown or unlearned stimuli through generation of a low magnitude in the generated response. The concept of an *enfolding of information*, and the corresponding increase in information density within the cortical memory elements are supported by rather fundamental properties that exist within the complex number domain. To illustrate, a complex scalar may define any point (*A*) in the Argand plane. This point may also describe any path from the complex origin to point A. However, the path actually encoded is given by:

$$A = \alpha_1 e^{i\vartheta_1} + \alpha_2 e^{i\vartheta_2} + \alpha_3 e^{i\vartheta_3} + \alpha_4 e^{i\vartheta_4} + \ldots \qquad (5)$$

The above scalar quantity defines one path within the set of all possible paths leading from the origin to point A. Within one cortical memory element of an HNeT cortical cell, each of the component scalars which define the encoded path leading to point A represent a single association "quanta" that has been learned in time.

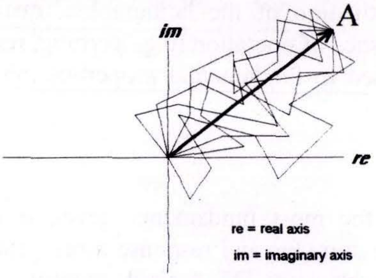

Fig. 3: Multiple Pathways Defining a Complex Scalar

The notion of enfolding results practicably from the ability to effect a determinable change upon those information quanta which make up the aggregate scalar *A*. Within the complex number domain, a multiplication operation imparts a phase angle rotation on the aggregate scalar. Solution of (5) shows that the complex multiply induces also an equivalent rotation on the set of quanta from which the aggregate scalar *A* is comprised. This is illustrated simply by multiplying the above complex summation with some arbitrary scalar $e^{i\delta}$, resulting in the following mathematical equality:

$$A \cdot e^{i\delta} = \alpha_1 e^{i(\vartheta_1+\delta)} + \alpha_2 e^{i(\vartheta_2+\delta)} + \alpha_3 e^{i(\vartheta_3+\delta)} + \alpha_4 e^{i(\vartheta_4+\delta)} + \ldots \qquad (6)$$

or diagramatically:

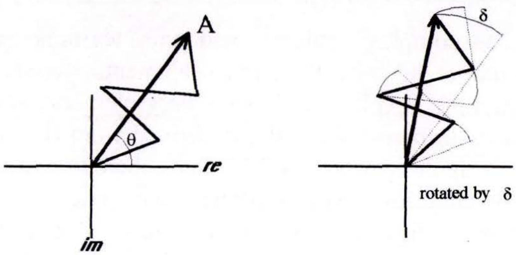

Fig. 4: Phase Rotation within Component Vectors

From an understanding of this fundamental aspect of rotational operators (i.e. the complex multiply), it is apparent that information can be manipulated and preserved within the aggregate complex scalars that comprise the cortical memory elements of a neuron cell. Such enfolded quanta, despite an apparent inaccessibility, may be modified in both their magnitude and phase allowing the embedded information (i.e. complex scalars shown on RHS of equation 5) to be manipulated in a deterministic manner.
The fact that this information (i.e. the quanta) can be manipulated, leads to the concept that information may be superimposed or *enfolded* onto the cell's cortical memory elements. Within the context of neural or learning systems, the desired objective is the re-expression of stimulus-response associations stored within the cell's cortical memory. Following investigation of the holographic/ quantum neural process, one realizes that the characteristics of operation (e.g. learning, response recall, properties of generalization) are attributed to fundamental properties inherent within Hilbert space mathematics.

1.2 Basic Learning

The learning process, at the most fundamental level, is performed using a vector product operation over the stimulus and response arrays, then accumulating the result within the cortical memory elements [*X*]. A single memory element maps one stimulus element *j* to an associated response element *k* by performing the following complex multiply and accumulate, applying phase conjugation to the stimuli:

$$x_{j.k} \mathrel{+}= \overline{s_j} \cdot r_k \tag{7}$$

The above complex product may be rewritten using complex exponential notation:

$$x_{j.k} \mathrel{+}= \lambda_j \, \gamma_k \, e^{i(\phi_k - \theta_j)} \tag{8}$$

One constructs a "neuron cell" using sets of stimulus and response elements extending over any given time frame, where the field index is indexed along the horizontal and time incremented along the vertical:

$$\text{element index} \rightarrow$$

$$[S] = \begin{array}{c} \text{t} \\ \text{i} \\ \text{m} \\ \text{e} \\ \downarrow \end{array} \begin{bmatrix} \lambda_{1.t_1} e^{i\theta_{1.t_1}} & \lambda_{2,t_1} e^{i\theta_{2.t_1}} & \lambda_{3.t_1} e^{i\theta_{3.t_1}} & \cdots \\ \lambda_{1.t_2} e^{i\theta_{1.t_2}} & \lambda_{2.t_2} e^{i\theta_{2.t_2}} & \lambda_{3.t_2} e^{i\theta_{3.t_2}} & \cdots \\ \lambda_{1.t_3} e^{i\theta_{1.t_3}} & \lambda_{2.t_3} e^{i\theta_{2.t_3}} & \lambda_{3.t_3} e^{i\theta_{3.t_3}} & \cdots \\ . & . & . & \cdots \end{bmatrix} \tag{9}$$

$$[R] = \begin{array}{c} \text{t} \\ \text{i} \\ \text{m} \\ \text{e} \\ \downarrow \end{array} \begin{bmatrix} \gamma_{1.t_1} e^{i\phi_{1.t_1}} & \gamma_{2.t_1} e^{i\phi_{2.t_1}} & \gamma_{3.t_1} e^{i\phi_{3.t_1}} & \cdots \\ \gamma_{1.t_2} e^{i\phi_{1.t_2}} & \gamma_{2.t_2} e^{i\phi_{2.t_2}} & \gamma_{3.t_2} e^{i\phi_{3.t_2}} & \cdots \\ \gamma_{1.t_3} e^{i\phi_{1.t_3}} & \gamma_{2.t_3} e^{i\phi_{2.t_3}} & \gamma_{3.t_3} e^{i\phi_{3.t_3}} & \cdots \\ . & . & . & \cdots \end{bmatrix} \tag{10}$$

The learning process for multiple patterns, as indicated above, may be represented in canonical form using the matrix operation:

$$[X] + = \overline{[S]}^{\mathrm{T}} \cdot [R] \tag{11}$$

Assuming only one element in the response field (i.e. one axon for the cell), the resulting cortical memory array [X] is presented as follows:

$$[X] = \begin{bmatrix} \sum_{T=t_1}^{t_N} \lambda_{1.T}\, \gamma_{1.T}\, e^{i(\phi_T - \theta_{1.T})} \\ \sum_{T=t_1}^{t_N} \lambda_{2.T}\, \gamma_{2.T}\, e^{i(\phi_T - \theta_{2.T})} \\ .. \\ .. \end{bmatrix} \tag{12}$$

The basic learning process given by (11) collapses the time axis in the solution of [*X*] while retaining information content. Information stored within the cell is preserved in the sense that one may regenerate a prior learned response upon exposure of the associated stimulus pattern to [*X*]. It should also be noted that this response recall has a greater level of determinism than that produced for linear search methods. This is due to the fact that the HNeT neural cell produces both a confidence (level of recognition), as well as a response expressed through phase angle orientation. Information density within the cell increases in an information theoretic sense, in that the number of mathematical operations required to do a response recall (i.e. the number of complex multiply and accumulates or CMACs), as well as the number of cortical memory elements, remain fixed and independent from the number of stimulus-response patterns stored within the cell.

Each of the above complex valued cortical memory elements, i.e. $x_j \in [X]$, have been generated from a set of stimulus-response associations represented in terms of phase angle differences (i.e. $\phi_t - \theta_{t,j}$). Through the equality expressed in (6), any subsequent exposure to a prior learned stimulus pattern induces a rotation or realignment of the associated complex valued quanta to the phase angle orientation of the trained response. This coherent interference of the associated quanta across all cortical memory elements produces a dominant response, which again coincides with the associated response. A residual "error" is also produced resulting from the destructive interference of remaining association quanta stored within the cell, this residual attaining a minimum contribution for orthogonal or nearly orthogonal associative pattern sets.

This basic learning method follows the principal of non-disturbance to a far higher degree than conventional neural methods, permitting large numbers of stimulus-response patterns to be learned on a low as one training epoch. Note that during response recall the initial stimulus patterns produce a relatively small recall error (10% or less of the analogue response range), again illustrating this principle of non-disturbance learning. Neural networks based on gradient descent methods exhibit far more limited non-disturbance learning capabilities, and very high numbers of training epochs are required (typically thousands to tens of thousands) to achieve learning convergence. In addition, at reasonably low pattern storage densities (i.e. 1000 patterns) gradient descent methods will break down (unable to produce measurable convergence).

The above presents the HNeT learning concept in its most primitive form. Aspects regarding the enhanced learning method employed within HNeT are summarized in a following section. This enhanced process operates in a manner whereby learning is influenced by prior learned memory, using an intracellular feedback mechanism to adjust the complex "weights". The enhanced learning process displays many additional capabilities, such as automatic control over attention, the capacity for reinforcement learning, increased storage densities, higher stability of operation, and a greater accuracy in the response recall. Further to this, non-linear aspects of the HNeT process are described.

1.3 Response Recall

Response recall operates in a manner whereby new stimulus patterns are transformed through the associative mappings stored (enfolded) within the cell's cortical memory [*X*]. During response recall, the transformation of new stimuli through each separate stimulus-response pattern learned by the cell is done in a concurrent or parallel manner. The following equations show the contribution of each prior learned stimulus-response pattern to the net response scalar *R*, issued during a response recall operation. In the event that the new stimulus pattern resembles one of the prior learned stimuli, associated information quanta undergo an alignment (become coherent or undergo constructive interference), and contribute the major portion of the generated response recall *R*. The operation performed for response recall assuming one response field may be illustrated by the following matrix inner product:

$$R = \Omega_S [S]^* \cdot [X] \qquad (13)$$

where $[S]^*$ is the new stimulus field exposed to the neuron cell for issuance of a response recall. This input stimulus may be represented by the following vector:

$$[S]^* = [\, \lambda_1^* e^{i\theta_1^*},\ \lambda_2^* e^{i\theta_2^*},\ \lambda_3^* e^{i\theta_3^*},\ \ldots \,] \qquad (14)$$

The normalization coefficient (Ω_s) in (13) is generally some function of the stimulus field. Stable convergence characteristics are produced even when applying the following simple relation:

$$\Omega_S = \frac{1}{\sum_{j=1}^{M} \lambda_j^*} \qquad (15)$$

The ability to *enfold* information may be illustrated by numerically deconvolving the vector components (i.e. quanta) embedded within the generated response values [*R*] in equation (13). Each generated response term is separated into constituent parts, whereby each part results from a stimulus [*S*]* transformation through one of the stimulus-response patterns previously learned by the cell (stored in [*X*]). Combining (14) for the new stimulus $[S]^*$, (12) for the cortical memory, and the response recall transform (13), the following solution for the generated response is obtained:

$$R = \Omega_S \sum_{j=1}^{M} \lambda_j^* e^{i\theta_j^*} \sum_{T=t_1}^{t_N} \lambda_{j,T}\, \gamma_T\, e^{i(\phi_T - \theta_{j,T})} \qquad (16)$$

Solution of the generated response *R* may be rewritten in the following equivalent form:

$$R = \Omega_S \sum_{T=t_1}^{t_N} \gamma_T\, e^{i\phi_T} \sum_{j=1}^{M} \lambda_j^*\, \lambda_{j,T}\, e^{i(\theta_j^* - \theta_{j,T})} \qquad (17)$$

The above separates the generated response into components that are summed over time ($T = t_1 \rightarrow t_N$). Each of these response components corresponds to the response recall for a new stimulus field [*S*]* as mapped through some previous stimulus-response association or "memory" learned at time *t*. These stimulus-response memories are indexed by time $t_1 \rightarrow t_N$ illustrated by matrices (9) and (10). Rewriting (17) into a form where each of these constituent response vectors are expressed over $T = t_1 \rightarrow t_N$ is shown as follows:

$$R = \Omega_S \left[\Lambda_{t_1} e^{i\zeta_{t1}} + \Lambda_{t_2} e^{i\zeta_{t2}} + \Lambda_{t_3} e^{i\zeta_{t3}} + \ldots \Lambda_{t_N} e^{i\zeta_{tN}} \right] \qquad (18)$$

Λ_t is the magnitude or confidence level for one component of the generate response, and ζ_t is the associated phase angle. Each separate component ($\Lambda_t e^{i\zeta_t}$) again corresponds to the response produced from a new stimulus $[S]^*$ transformed through one stimulus-response memory learned at time t. These component terms are given by the following relation, where for example the response contribution is evaluated for a stimulus-response learned at t_1:

$$\Lambda_{t_1} e^{i\zeta_{t_1}} = \gamma_{t_1} e^{i\phi_{t_1}} \sum_{j=1}^{M} \lambda_j^* \lambda_{j,t_1} e^{i(\theta_j^* - \theta_{j,t_1})} \tag{19}$$

Following from (19), the magnitude and phase of each of these component terms in R may be evaluated directly given the set of stimulus-response associations learned, thus:

$$\Lambda_{t_1} = \gamma_{t_1} \left[\left[\sum_{j=1}^{M} \lambda_j^* \lambda_{j,t_1} \cos(\theta_j^* - \theta_{j,t_1}) \right]^2 + \left[\sum_{j=1}^{M} \lambda_j^* \lambda_{j,t_1} \sin(\theta_j^* - \theta_{j,t_1}) \right]^2 \right]^{\frac{1}{2}} \tag{20}$$

$$\zeta_{t_1} = \tan^{-1} \left[\frac{\sum_{j=1}^{M} \lambda_j^* \lambda_{j,t_1} \sin(\theta_j^* - \theta_{j,t_1} + \phi_{t_1})}{\sum_{j=1}^{M} \lambda_j^* \lambda_{j,t_1} \cos(\theta_j^* - \theta_{j,t_1} + \phi_{t_1})} \right] \tag{21}$$

Each of the terms on the RHS of (18) contain a magnitude or confidence proportional to the proximity of the new stimulus pattern $[S]^*$ to the stimulus pattern learned at time t. In other words, the prior learned stimulus patterns displaying the greatest similarity to the new input pattern $[S]^*$ produce the more dominant response magnitude as measured by Λ_t. Figure 6 illustrates a vector summation of these response components, indicating the relative dominance of each. Referring back to the rotational transformation occurring on complex multiply operations presented earlier, the holographic/ quantum neural process relies on the property whereby a subset of the association quanta (as contained within $\Lambda_t e^{i\zeta_t}$ for $T = t_1$ in Figure 5) become aligned along the phase angle orientation corresponding to the associated response. This coherence, or constructive interference, results in generation of the dominant response. Conversely, the remaining terms in (18), and generated from a stimulus transform through disassociated memories (those learned at T =,/ t_1), produce a far smaller contribution do to non-coherence, or destructive interference. These disassociated response terms (i.e., for T =,/ t_1) follow a path characteristic of a random walk. The net contribution of the disassociated response terms produce a small residual error in the overall response recall (i.e. ϕ_{err}). The above random walk characteristics exhibited for the disassociated stimulus patterns, occur for data sets in which the phase orientations are reasonably uniform about 2π (i.e. symmetric or orthogonal). This important aspect of symmetry is discussed in further detail below. It is this random walk characteristic for disassociated response components that establishes the condition where multiple stimulus-response patterns may be enfolded onto, and subsequently recalled from, the neuron cell's cortical memory.

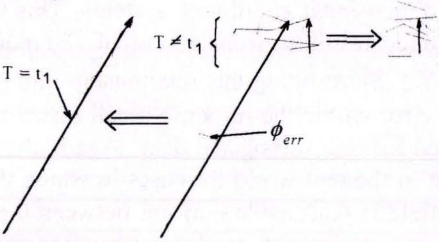

Fig 5: Summation of Convolved Response Terms

A simple limit theorem may be constructed to evaluate the properties observed for the holographic/quantum neural method by observing asymptotic conditions, as a stimulus input pattern approaches a prior learned stimulus loci. We may apply a limit value argument using a random error factor $\in_r$. This error factor diminishes as the information elements within the new stimulus field $[S]^*$ approach a stimulus pattern that was learned at time t_1. Establishing the following numerical relationship for each element within the stimulus pattern:

$$e^{i\theta_j} = e^{i(\theta_{j,t_1} + \in_r)} \qquad \text{over stimulus elements } j = 1 \text{ to N} \qquad (22)$$

where $\in_r$ is the random error quantity. As the phase elements within the stimulus field $[S]^*$ tend toward the stimulus pattern learned at t_1 $\in_r \rightarrow 0.0$ over all k. Substituting (22) into the magnitude and phase relationships for the response, (20) and (21), we find that:

$$\Lambda_{t_1} \rightarrow \gamma_{t_1} \frac{\sum_{j=1}^{N} \lambda_j^* \lambda_{j,t_1}}{\sum_{j=1}^{N} \lambda_j^*} \qquad (23)$$

assuming unit confidence values, i.e. $\lambda_j^*, \lambda_{j,t_1}, \gamma_{t_1} \rightarrow 1.0$ over all j, then $\Lambda_{t_1} \rightarrow 1.0$. Similarly, for the generated phase angle orientation, the approach $\zeta_{t_1} \rightarrow \phi_{t_1}^*$ is observed following a reduction of $\in_r$.

Memory obtained at t_1 presents the most dominant response component generated from the new stimulus $[S]^*$. Similarly, the generated response phase angle in the limit $(\in_r \rightarrow 0.0)$ approaches the prior learned analog response value for the associative pattern learned at time t_1 (i.e., $\phi_{t_1}^*$). The remainder of the component terms within the generated response (i.e. for T =/ t_1) characterizes a residual error, albeit deterministic (ϕ_{err} in Figure 5). The statistical variance on the magnitude for this error term is proportional to the total number of distinct and separate stimulus-response patterns learned by the cell. It is this apparent "fuzziness" which places a limit on the number of associative patterns which may be learned for any given level of recall accuracy. One may provide an upper estimate of recall "error" given the Brownian movement characteristics for multidimensional coordinate systems. This error may be estimated from the average magnitude resulting from a sum of N random walks each of unit magnitude (producing $\sqrt{N}$). Substituting this relationship into (18), one may obtain an estimate for the average error within the response recall assuming the characteristics of the random walk observed for near orthogonal data. Again, the characteristics required for random walk are met in the real world for cases in which the distribution of phase for each stimulus input field is reasonable uniform between 0 and 2π. A sum over the disassociated set of response components produce an average magnitude approaching:

$$\left|r_{T \neq t_1}\right| = \left|\Omega_S \sum_{T=t_2}^{N} \Lambda_T e^{i\tilde{\phi}_T}\right| \approx \sqrt{\frac{N-1}{M}} \qquad (24)$$

where: N is the number of stimulus-response patterns learned,
M is the number of cortical memory elements within the cell,
and $\gamma_t, \lambda_{j.t} \rightarrow 1.0$ over all j and t.

For purposes of evaluating general recall characteristics within the holographic/ quantum neural cell, one may derive an estimate for the average response recall error. This estimate assumes randomly generated phase angle orientations which, in one respect, have the advantage of producing highly symmetrical distributions. However, in another respect, random patterns may be used to evaluate the upper limit of storage capacity due to the fact that each pattern is functionally independent. In particular for stimulus patterns of low dimensionality, this forces the cell to learn a highly non-linear mapping topology. An upper limit on the recall error for one learning epoch may be approximated given the number of cortical memory elements of the neuron cell (M), and number of patterns learned (N):

$$\phi_{\text{error}} \approx \frac{1}{\pi\sqrt{8}} \tan^{-1}\left(\sqrt{\frac{N}{M}}\right) \qquad (25)$$

This residual response recall "error" is in fact deterministic, and increases in a square-root relation to the number of stimulus-response patterns learned. New stimulus patterns used in generation of response recall will invoke responses over the set of prior learned stimulus-response patterns, each of these component response terms (Λ_t in equation 18) issuing a proportionate degree of dominance or confidence within the generated response.

A series of enhancements on the basic learning procedure described in the previous section, are used within HNeT neuron cells to increase the recall accuracy well beyond that indicated by the error relation (25). For instance, one may eliminate the pattern storage restriction by increasing the dimensionality (size) of the input stimulus field, through generation of unique higher order scalars or "combinatorics" as a preprocess operation. Employing this process to expand the number of stimulus fields, a raw input field of say 10 elements may be increased to tens of thousands of higher order terms. The number of potential higher order combinatorics increases as a factorial of the product order. Random statistical tests may be used to validate that the relationship between pattern storage capacity and the number of cortical memory elements (using combinatorics) remains directly proportional.

1.4 Enhanced Learning

The operational characteristics of the holographic/quantum neural method may be enhanced significantly when learning is a function of stimulus-response "memories" previously absorbed by the cell. For the basic learning process, learning progresses independently from knowledge already accumulated, and no control is afforded over *attention*. Attention refers to the influence that prior learned memory exerts on the learning of new stimulus-response associations. Within the basic learning process, learning is very much sub-optimal, in the sense that many similar stimulus-response associations will tend to globally distort the mappings generated within the neuron cell's cortical memory. These asymmetries effect cortical memory such that response recall will tend toward a heavily reinforced response action. This undesirable characteristic or limitation is eliminated within the enhanced learning process used by HNeT cells. Additionally, magnitudes of the cortical memory elements and the response recall values remain stable and bounded irrespective of the number of stimulus-response patterns that are learned.

In application, the enhanced learning process incorporating attention, demonstrates the ability to construct a cell that generalizes to the point where it is capable of learning highly complex abstractions. An example of an abstraction is the visual concept of the human face. Through reinforcement learning, an HNeT neural cell with only a few hundred cortical memory elements is capable of learning many 10s of thousands of facial images, and generalizing to the point where a human face may be recognized as such, irrespective of ambient lighting, ethnicity, permutations in scale and rotation. In this respect, one can say that the cell has learned the "abstraction" of the visual form of the human face. A brief description of a face recognition system developed by AND is located at www.andcorporation. com/applications/htm. Another important aspect of the enhanced learning process is that, by contrast to the basic learning method, the enhanced process maps new stimulus exactly to the desired response on one learning exposure. In addition, the residual error within the response recall produced over large sets of stimulus-response patterns increases at a considerably slower rate than for the basic learning scheme.

The enhanced learning process may be subdivided into three stages of operation. The initial stage executes a response recall operation whereby a new stimulus [***S***] is transformed through the cell's cortical memory. The second stage evaluates the complex scalar difference (R_{diff}) between the generated and desired response values for this new stimulus. The final step performs a basic learning operation whereby the new stimulus pattern is mapped to the above scalar difference. Computationally, the enhanced learning operation requires two complex multiply and accumulate operations (CMAC's) per cortical memory element. By contrast, the basic learning process and the response recall operations require only one CMAC operation per cortical memory element. In this respect, both the learning and recall operations display high computational efficiency, requiring few and quite simple numeric operations per "synaptic connection". The operational steps for the enhanced learning process are illustrated below:

1) Process a stimulus pattern through the neuron cell to produce a response recall value R', i.e.:

$$R' = \Omega_S [\boldsymbol{S}] \cdot [X] \tag{26}$$

2) A complex scalar difference R_{diff} between the above generated response and the desired response R for this current stimulus-response association is evaluated, as follows:

$$R_{diff} = R - R' \tag{27}$$

3) Cortical memory is updated using the new stimulus [***S***] and the above difference scalar. The result of the following matrix product operation is added into the existing cortical memory; this operation follows the canonical form presented in (13):

$$[X] + = [\boldsymbol{S}]^{\mathrm{T}} \cdot R_{diff} \tag{28}$$

This process yields the learning process whereby a new stimulus pattern is mapped exactly to the desired response (R) following one learning exposure, and irrespective of prior learned memory. A general formulation for this learning procedure may be derived by combining steps (26) to (28), neglecting cross product terms in the matrix solution:

$$[\mathrm{X}] = [\boldsymbol{S}]^{\mathrm{T}} \cdot \left(R - \Omega_S [\boldsymbol{S}] \cdot [X]\right) \tag{29}$$

or re-expressing in an approximate form:

$$[\mathrm{X}] + = \underbrace{[\boldsymbol{S}]^{\mathrm{T.}} \cdot R}_{\text{basic encoding}} - \underbrace{[\mathbf{H}] \cdot [X]}_{\text{enhancement term}} \tag{30}$$

where [**H**] represents a Hermitian expansion of [***S***], that is:

$$[\mathbf{H}] = \frac{1}{c}[\boldsymbol{S}]^{\mathrm{T}} \cdot [\boldsymbol{S}] \tag{31}$$

The optimal mapping is achieved at the point where $[X]$ converges to a stable locus defined at:

$$[X] = [\mathbf{H}]^{-1} \cdot [X]_{basic} \tag{32}$$

where $[X]_{basic}$ is the correlation set produced within the basic learning scheme presented by (11). A nearly precise mapping over large sets of stimulus-response patterns may be achieved with relatively few reinforcement learning trials, using the enhanced learning process as presented by (29). This matrix solution presents a general form of the iterative learning process and represents one reinforcement learning trial over the entire suite of stimulus-response associations contained in $[\boldsymbol{S}]$ and $[R]$. For cases where the stimulus patterns are symmetric and the number of patterns learned is equivalent to the number of cortical memory elements stored by the cell, the analog error produced during response recall achieves low values (i.e. 4-6% absolute analog scale) following two to three reinforcement learning trials. For lower pattern storage densities, or sets of patterns which embody some degree of similarity in structure (i.e. patterns that are not random), the response recall error may be significantly less than that estimated by (25).

1.5 Higher Order Systems

Conventional ANS models are restricted in terms of the numbers of stimulus-response associations that may be learned, due primarily to a functionally linear basis of operation. Within the holographic/ quantum neural process, limitations on the storage density are overcome through a preprocessing operation involving the generation of higher order products from the stimulus basis set $[S]$. The response recall error relationship presented in equation (25) remains valid for the situation where the stimulus field is expanded to a higher dimensionality using higher order combinatorics. These higher order products must however form unique sets. Within this context, "unique sets" are defined as the set of all possible combinations derived from the input basis, providing that no two product sets contain the same group of basis elements. An input field expanded to 4th order products, for example, may be generated by the following product solution for a higher order term k:

$$s_j = \prod_{k=1}^{4} \lambda_{f(j.k)} e^{i\theta_{f(j.k)}} \tag{33}$$

where $f(j.k)$ is some arbitrary function which selects the input element as a function of k and the index to the cortical cell synapse j.

The above product operation performs a cumulative phase rotation over the stimulus basis elements, and evaluates a net confidence level for the result. The evaluation of confidence levels for these higher order terms maintains a congruency with probability theory. In addition, individual $\lambda e^{i\theta}$terms used in the above product (i.e., over $k = 1$ to 4) may be conjugated to increase the number of unique terms attainable for a specific product order. Such combinatorics are illustrated within the basic learning and response recall operation producing the following sigma-pi forms:

Basic learning:

$$x_j = \sum_{t=1}^{N} \gamma_t e^{i\phi_t} \prod_{k=1}^{K} \lambda_{f(j,k)} e^{-i\theta_{f(j,k)}} \tag{34}$$

Response recall:

$$r = \Omega_S \sum_{j=1}^{M} x_j \prod_{k=1}^{K} \lambda^*_{f(k,j)} e^{i\theta^*_{f(k,j)}} \tag{35}$$

Reiterating, the number of complex elements within the above product terms (K) defines the order of the product. The response recall error characteristic for any set of complex product terms of order >= 1 follows the relationship defined in (25) provided that the product set forms unique combinatorics. The limit imposed by this uniqueness criterion establishes an upper limit for the total number of higher order terms that may be generated for a given basis set and product order. The number of unique higher order terms can be extremely large given a basis stimulus set of modest size.

As an example, Table 1 lists the number of unique product sets given stimulus signals of varying sizes. Consider the situation involving a stimulus signal containing 20 degrees of freedom (20 elements). Expanding this basis set up to 8th order products yields greater than 1×10^6 product combinatorics, allowing a proportional number of distinct and separate (Monte-Carlo based) stimulus-response patterns to be learned. One million associative patterns stored within a single cell, and confined within a state space bound by 20 degrees of freedom, defines a system that is not limited in any respect by linear non-separability concerns, permitting single cortical cells to learn highly complex (non-linear) stimulus-response environments.

Table 1: Number of Combinatorics as Function of the Product Order

Stimulus Size	**Order of Product Expansion**				
	1	**3**	**5**	**7**	**10**
1	1	1	1	1	1
5	5	35	126	330	1001
10	10	220	2,002	11,440	92,378
15	15	680	1,628	116,280	1,961,256
20	20	1,540	42,504	657,800	20,030,010

The HNeT Application Development System allows one to control the statistical nature of higher order product generation. Two options regarding the composition of complex conjugates are provided, and the user may specify the order of terms generated. In addition to this, one has the ability to specify the *spin number* for higher order products, evaluated as follows:

spin number = non-conjugated basis elements - conjugated basis elements

Applying these features, one may adjust the mapping density within the cell to suit requirements of the application. On the other hand, one requires a more complete knowledge of these features, as arbitrary specification of the higher order product expansion may not result in the desired generalization or interpolation. Neural plasticity provides a method by which more appropriate (or optimal) higher order combinatorics are selected by the cell in an automated manner, producing in most cases a dramatic improvement in learning and generalization.

1.6 The Commutative Property of Cells

Holographic/quantum neural cells display a commutative aspect when connected into multi-cellular structures (i.e. cell assemblies). This refers to a characteristic whereby the storage capacity for stimulus-response patterns increase in proportion to the number of cortical memory elements stored within all cells connected together to form an assembly. Such cells when connected together, operate in a similar manner to single cortical cells possessing the sum total of all memory elements over the cell assembly. This concept may be illustrated by the following equality, considering $[X]_r$ as the cortical memory values stored within each cell (r) and $[S]_r$ the stimuli processed through each of those memory elements:

$$[X]' \cdot [S]' = \sum_{r}^{N} [X]_r \cdot [S]_r \tag{36}$$

The above primed sets represent the appended set of cortical memory elements and stimulus signals over a group of cells indexed by 1 to N. Such sets may be represented as a collection of the linear matrices used to evaluate each cell independently i.e.

$$[X]' = \{[X]_1, [X]_2, [X]_3, \ldots\ldots, [X]_N\} \tag{37}$$
$$[S]' = \{[S]_1, [S]_2, [S]_3, \ldots\ldots, [S]_N\}$$

This property is utilized in the HNeT system for the neo-cortical assembly, allocated using the **AllocStellateCell** and **AllocPyramidalCell** functions. This cell assembly is comprised of multiple stellate cells with output connections into one pyramidal cell. The stellate cell stores cortical memory, while the pyramidal cell does not. The pyramidal cell however, permits the outputs across the multiple stellate cells to be summed in a manner that the commutative property is observed. The cell assembly formed by the combination of stellate and pyramidal cells is illustrated below:

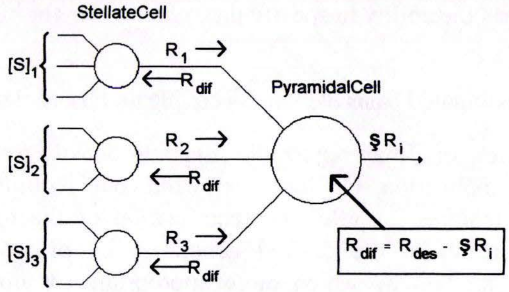

Fig. 6: Illustration of a Cell Assembly Based on the Neo-Cortical Model

This property is unique to the holographic/quantum neural model and permits one to construct assemblies that are effectively unrestricted in terms of their mapping (learning) densities. This aspect of operation also has some interesting implications for cell structures that resemble biological constructions. For instance in the human neo-cortex, cell assemblies similar to Figure 7 are observed in which several stellate cells relay signals via axonal processes into the synaptic spines of a pyramidal cell.

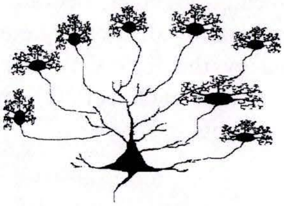

Fig 7: Illustration of a Biological Stellate\Pyramidal Cell Assembly

Current estimates regarding the synaptic populations of neuron cells indicate that stellates have typically from 10,000 to 50,000 synaptic connections along their dendrites. A pyramidal cell may receive a proportionately large number of axonal inputs from stellate cells. If one applies the holographic/quantum model to these structural aspects, there exists the capability for neo-cortical based assemblies to learn and recall several hundred million stimulus-response associations (memories). Practicably, these assembly structures have the ability to learn or *enfold* such quantities of associative patterns that could represent an organisms life span, applying single epoch training (i.e. real time learning).

The actual connective structures observed in biological systems are far more complex than the supervised feed-forward structures described previously. These biological structures possess a high degree of interaction among pyramidal chains, often displaying an extensive degree of collateralization along cell axons. It is suspected that recurrent (hyperincursive) structures exist in the dense inter-connectivity among actual biological assemblies, providing the basis for unsupervised cognitive based operation. Hyperincursive in this context refers to assembly structures which exhibit recurrent data flow via the environment, and illustrate a self-referential aspect. An example of hyperincursive feedback is provided by Hofstadter [4] through the simple experiment of aiming a camera at a monitor to which it is connected. Other examples of hyperincursion for generating fractals in automota are provided by Dubois [5]. The HNeT system allows the user to structure multiple pyramidal chains in a rather flexible manner that permits both collateralization and hyperincursive data flow for modeling and simulation of more elaborate cell assembly structures.

2 The Biological Model

Neural networks attempt to construct information processing systems based on known features within neurobiology. Little is currently understood regarding the mechanisms of information processing within neuron cells. Some aspects with respect to the morphology of various neurological cell types (i.e. their structure), inter-connectivity, as well as aspects concerning signal transmission may however be measured and observed. The following provides an structural overview of cell assembly construction in the cerebellum [6], with analogies to the holographic/quantum process.

The cerebellum is associated principally with motor coordination and is believed to perform a fundamental role in postural control and integration of movement. It's general structure consists of an external layer of gray matter in which the cell bodies and synaptic areas of the neuron cells are located. This cortical region surrounds a core of white matter which contains, again, efferent (outgoing) and afferent (incoming) signal tracts. The gray matter within the cerebellum may be divided into separate layers. These are: the outmost molecular layer, Purkinje layer and the granule layer. Several distinct cell types and cell assembly structures exist within the cortical region. The most predominant cell types are the Purkinje and granule cells. As opposed to the three layer structure of the neo-cortex, the cerebellum in based on a two layer structure (granule→Purkinje). The arrangement of these cells within the various cortical regions is illustrated in Figure 9, and the general features of cerebellar neuron cells are discussed below.

2.1 Purkinje Cell

This is the principal cell of the cerebellum. The Purkinje cell body is located in the Purkinje layer between the molecular layer and granule layers. Dendrites from this cell course through the full extent of the molecular layer to synapse with axons of primarily granule cells. The structure of the Purkinje's dendritic processes are highly elaborate, with extensive arborization and many dendritic spines located along the course of these processes. The Purkinje cell possesses up to 150,000 synaptic spines. Each Purkinje cell possesses one axon, which courses down through the white matter to deep cerebellar nuclei. These cerebellar nuclei are believed to operate in some part as relay stations, propagating signals to cell structures within the brain stem and thalamus. It is estimated that there are approximately 15 Million Purkinje cells in the human cerebellum.

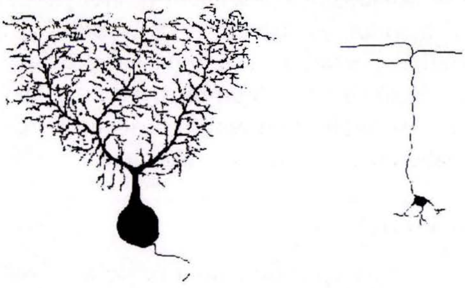

Fig 8: Structure of the Purkinje and Granule Cell

2.2 Granule Cell

Like the granule cells of the cerebral cortex, these cells are very small in comparison to the Purkinje cells but are far more numerous. It is estimated that approximately 2×10^9 granule cells exist within the cerebellum. Granule cells predominate in the lowest cortical layer of the cerebellum, named the granular layer. These cells have from three to five dendrites which establish contact with climbing fibers (afferent signals) relaying input signals from the brain stem and cerebral cortex. The axon from the granule cell courses vertically through the Purkinje layer and arborizes forming many collateral branches within the more superficial molecular layer. These collaterals run parallel to the cerebellar surface and establish contact with up to 500 Purkinje cells. The axonal collateral originating from a granule cell will come in contact with a Purkinje cell generally only once. A single Purkinje cell however, can receive up to 150,000 synaptic inputs from the granule layer.

2.3 General Structure of the Cerebellum

A simplified schematic of the cerebellum is presented in Figure 9. Afferent signals are received from the climbing fiber system relaying signals from the mid brain, thalamus and cerebral cortex. These climbing fibers establish synaptic contact primarily with granule cells. Each climbing fiber will synapse with between 20 and 40 granule cells. In response to these signals, the granule cells integrate these afferent inputs propagating a signal via axonal processes to Purkinje cells. Signals from the granule layer establish synaptic connections on the Purkinje cell's dendritic processes located within the molecular layer. The Purkinje cells themselves have axonal processes which form the only efferent (outgoing signal) from the cerebellum.

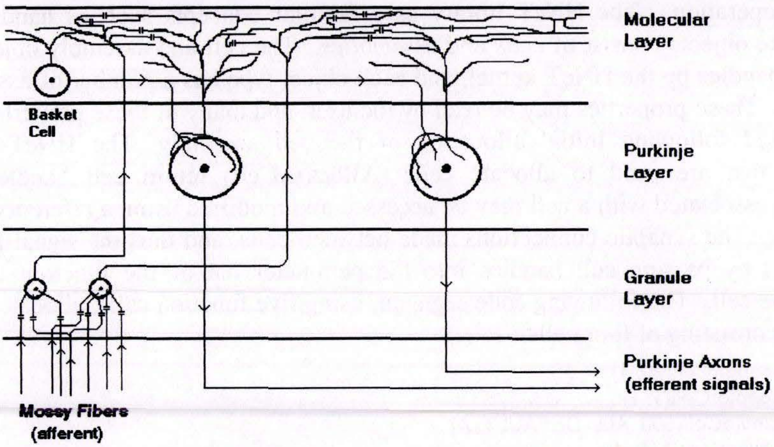

Fig 9: Neural Pathways within the Cerebellum

Structure at the granule layer is again highly indicative of the preprocessing required to perform a signal expansion through generation of higher order scalars. Granule cells display the appropriate placement and structure for this operation. Third to fifth order product terms as indicated from the dendritic structure of actual granule cells are quite sufficient to integrate and expand the incoming signals to provide densities in the millions of patterns, and a non-linear mapping capability. The cell assembly constructed from the granule and Purkinje cell types establish the cerebellar model within the HNeT Application Development System:

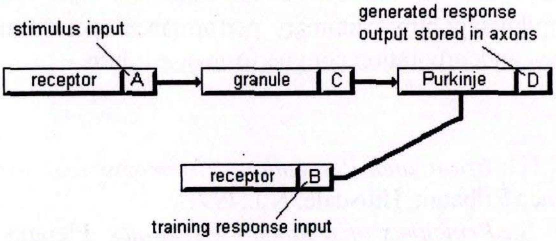

Fig. 10: The Cerebellar Model

One may use the cerebellar model to construct quite simple assemblies consisting of only four cells. Despite the simpler structure, cerebellar assemblies may also be configured for real time learning of large numbers of stimulus-response patterns (i.e. >> several thousands). The above cerebellar structure forms the basis for the HNeT SL (supervised learning) Platform.

3 The HNeT Library

The HNeT2000 Application Development System is based on a 32 bit Windows 95, 98, 2000, and NT dynamic link library (DLL). This DLL permits multi-process and multithreaded operation. The HNeT library uses familiar concepts such as handles and objects; the objects consist of *cells* and *assemblies*. The cell and assembly objects are assigned handles by the HNeT kernel, and each object type has a number of associated properties. These properties may be read by the user, and many of these properties may be modified following initial allocation of the cell assembly. The HNeT library functions that are used to allocate cells (**AllocxxxCell**) return cell handles. The properties associated with a cell may be accessed and modified using a reference to the cell handle. The synaptic connections made between cells, and thus the signal flow, is established by passing cell handles into the parameter list of the function used to allocate the cell. The following code segment, using five function calls, allocates a cell assembly consisting of four cells.

```
A = AllocReceptorCell(20, 1);
B = AllocReceptorCell(1, 1);
C = AllocGranuleCell(500, ALL, DEFAULT, A);
D = AllocPurkinjeCell(1, C, A);
AssemblyHandle = EndAssemblyCode();
```

The above code allocates the cell assembly illustrated above in Figure 10. The first receptor cell receives the stimulus signals from the host application, and is assigned handle A. The above code allocates a granule cell that receives its input signal from the receptor cell (by referencing handle A). The granule cell is used to performs an expansion of the stimulus signal stored by receptor cell A, using product combinatorics. Subsequently, the Purkinje cell reads its input from the granule cell (by referencing handle C). The Purkinje cell also reads a training response signal from the second receptor cell (referenced by handle B).

A summary list of HNeT DLL functions are provided at www.andcorporation.com/hnetapi.htm. Summary performance specifications for the HNeT process start at www.andcorporation.com/performance1 .htm

References

[1] Pribram, K.H., *Brain and Perception: Holonomy and Structure in Figural Processing*, Lawrence Erlbaum, Hillsdale, N.J.,1991.

[2] Ramamurti, S., *Principles of Quantum Mechanics*, Plenum Press, New York, NY, 1984.

[3] Hebb, D. O. *The Organization of Behavior*, Wiley, New York, NY, Introduction and Chapter 4, "The first stage of perception: growth of the assembly", pp. xi-xix, 60-78, 1949.

[4] Hofstadter, D.R., *Godel, Esher, Bach*, Basic Books, New York, NY, pp. 490-548. 1979.

[5] Dubois, D.M., "Hyperincursive Methods for Generating Fractals in Automota Related to Diffusion and Wave Equations", Int. J. General Systems, Gordon and Breach Science, Vol 27, pp. 141-180, 1998.

[6] Afifi, A.K., Bergman, R.A. *Basic Neuroscience*, Urban & Schwartzenberg, Baltimore, MA, Chapter 11, "Cerebellum", pp. 215-240, 1080.