

# Entanglement and Algorithmic Topology

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## Abstract

Algorithmic topology is the spanning of an algorithm on a topological structure. The common calculus with paper and pen shows that all the recursive functions can be spanned on Euclidean planes. It is known that two topological structures are identical if and only if cut-pasting operations don't need to transform one in the other. Dubois' third stage (identification of incursive algorithm last row and column respectively with its first row and column) gives to incursive algorithms a spanning only on a torus that can be transformed in Euclidean plane only by cut-pasting operations. Thus incursive algorithms couldn't reduce to recursive algorithms and Church's hypothesis couldn't be true. Now, observe the affinity between topologic cut-pasting operations, Dubois' third stage and quantum entanglement. This last one can be considered either two "entanglements" in incursive algorithms or a cut-pasting operation on Euclidean plane on which such an algorithm is spanned to transform such a plane in torus. Is quantum entanglement simply the inadequacy of algorithms that can be spanned only on Euclidean plains to represent quantum mechanics? The same question could have value for some complex biological systems.

**Keywords:** algorithmic topology, incursive algorithm, entanglement, quantum mechanics, biological systems.

## 1 Algorithmic Topology

We follow Turing's classical approach to computation.<sup>1</sup> Let a finite symbol set  $\{\emptyset, S_1, \dots, S_n\}$  be an *alphabet* where to print  $\emptyset$  in a place means to cancel the symbol present in such place. Let a finite sequence of  $S_i \in \{S_1, \dots, S_n\}$  be an *expression*. Let a finite symbol set  $\{q_1, \dots, q_m, q_{off}\}$  be an *internal state set* where internal state  $q_{off}$  means "stop". Let an expression in form  $q_r S_{i_1} \dots (S_{i_h}) \dots S_{i_k}$  be a *transitory achievement*. Let  $S_{i_1} \dots S_{i_h} \dots S_{i_k}$  be the *immediate conclusion* of  $q_r S_{i_1} \dots (S_{i_h}) \dots S_{i_k}$ .

Consider transitory achievements as space points. We can define these four generic applied vector typologies:

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<sup>1</sup> See Turing, 1950.

$$a^1V: q_r S_{i_1} \dots S_{i_u}(S_{i_h}) S_{i_v} \dots S_{i_k} \rightarrow q_s S_{i_1} \dots S_{i_u}(S_{i_w}) S_{i_v} \dots S_{i_k}$$

$$a^2V: q_r S_{i_1} \dots S_{i_u}(S_{i_h}) S_{i_v} \dots S_{i_k} \rightarrow q_s S_{i_1} \dots (S_{i_u}) S_{i_h} S_{i_v} \dots S_{i_k}$$

$$a^3V: q_r S_{i_1} \dots S_{i_u}(S_{i_h}) S_{i_v} \dots S_{i_k} \rightarrow q_s S_{i_1} \dots S_{i_u} S_{i_h}(S_{i_v}) \dots S_{i_k}$$

$$a^4V: q_r S_{i_1} \dots S_{i_u}(S_{i_h}(\in \alpha?)) S_{i_v} \dots S_{i_k} \rightarrow q_s S_{i_1} \dots S_{i_u}(S_{i_h}(\in \alpha)) S_{i_v} \dots S_{i_k}, q_r S_{i_1} \dots S_{i_u}(S_{i_h}(\notin \alpha)) S_{i_v} \dots S_{i_k}$$

Applied vector typologies  $a^1V$ ,  $a^2V$ ,  $a^3V$ , permit us to define respective non-applied vector typologies:

$${}^1V: q_r S_{i_h} S_{i_w} q_s$$

$${}^2V: q_r S_{i_h} L q_s$$

$${}^3V: q_r S_{i_h} R q_s$$

$${}^4V: q_r^\alpha S_{i_h} q_s q_t$$

these ones are a complete set of generic fundamental steps of Turing's computation.

Consider now a vector set that contains an alone applied vector  $a^iV_0$  and such a vector ordered sequence  ${}^iV_1, \dots, {}^iV_n$  that  ${}^iV_j$  can immediately be applied after  ${}^{i-1}V_{j-1}$  and  ${}^iV_n$  ends with  $q_{off}$ . Not only  $a^iV_0, {}^iV_1, \dots, {}^iV_n$  is a generic form of any Turing's algorithm but also it is an oriented segment of broken line if we consider transitory achievements as space points.

The described broken line segment is enough generic to span in an Euclidean space. We can conclude:

**Proposition 1.1:** *Any Turing's algorithm can be spanned in an Euclidean space.*

Standard results of computation theory permit us to put the following corollaries:

**Corollary 1.1.1:** *Any recursive function can be spanned in an Euclidean space.*

**Corollary 1.1.2:** *If Church's hypothesis (any computable function is recursive and vice versa) is true then any computable function can be spanned in an Euclidean space.*

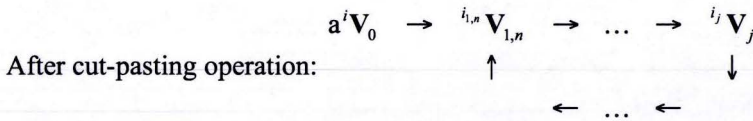
The last corollary shows that Church's hypothesis is very strong. We should deduce that if we span an algorithm on a torus or on a hypersphere or on a Klein's bottle or on another topological structure that is not equivalent to an Euclidean space then the same algorithm could be spanned in an Euclidean space. As two topological structures are different only if cut-pasting operations are necessary to transform one in the other we can conclude that an algorithm should be invariant to cut-pasting operation in the space where they are spanned. We can put:

**Corollary 1.1.3:** *If Church's hypothesis is true then any given algorithm does not change if cut-pasting operation are done in the space where the given algorithm is spanned.*

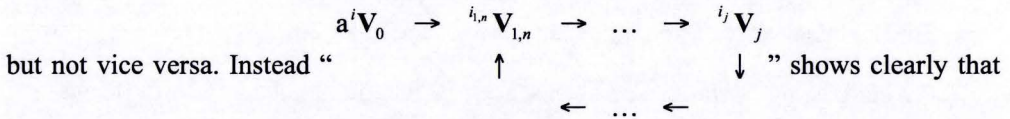


Corollary 1.1.3 has a conclusion very difficult to accept. Consider an algorithm  $a^iV_0, {}^iV_1, \dots, {}^iV_j, \dots, {}^iV_n$  that is spanned on an Euclidean space. To realize our purposes rewrite  $a^iV_0, {}^iV_1, \dots, {}^iV_j, \dots, {}^iV_n$  in form  $a^iV_0 \rightarrow {}^iV_1 \rightarrow \dots \rightarrow {}^iV_j \rightarrow \dots \rightarrow {}^iV_n$ . Now do a cut-pasting operation that provokes the overlapping of places of  ${}^iV_1$  and of  ${}^iV_n$  on the space where  $a^iV_0 \rightarrow {}^iV_1 \rightarrow \dots \rightarrow {}^iV_j \rightarrow \dots \rightarrow {}^iV_n$  is spanned. We obtain:

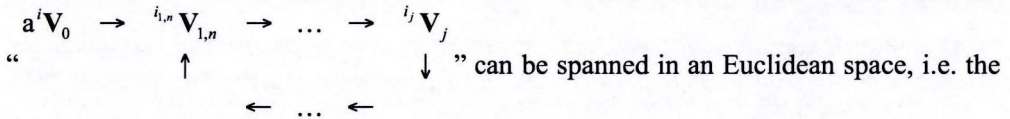
Before of cut-pasting operation:  $a^iV_0 \rightarrow {}^iV_1 \rightarrow \dots \rightarrow {}^iV_j \rightarrow \dots \rightarrow {}^iV_n$



Observe that our cut-pasting operation implies a mathematical “entanglement” between  ${}^iV_1$  and  ${}^iV_n$ . An important consequence is algorithm temporal order. “ $a^iV_0 \rightarrow {}^iV_1 \rightarrow \dots \rightarrow {}^iV_j \rightarrow \dots \rightarrow {}^iV_n$ ” defines clearly that  ${}^iV_n$  is future and dependent as regard  ${}^iV_1$



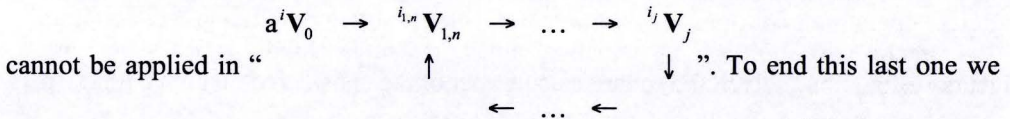
${}^iV_1$  anticipates and depends from successive (future)  ${}^iV_n$ . The creation of a mathematical entanglement between  ${}^iV_1$  and  ${}^iV_n$  by a cut-pasting operation on span space has given anticipatory properties to algorithm  $a^iV_0 \rightarrow {}^iV_1 \rightarrow \dots \rightarrow {}^iV_j \rightarrow \dots \rightarrow {}^iV_n$ . But, for Church’s hypothesis, no relevant changes are happens because also



same topological structure of  $a^iV_0 \rightarrow {}^iV_1 \rightarrow \dots \rightarrow {}^iV_j \rightarrow \dots \rightarrow {}^iV_n$ . We can conclude:

**Corollary 1.1.4:** *If Church’s hypothesis is true then there are not anticipatory mathematics that are not reducible to non-anticipatory mathematics.*

Observe also that  $a^iV_0 \rightarrow {}^iV_1 \rightarrow \dots \rightarrow {}^iV_j \rightarrow \dots \rightarrow {}^iV_n$  stops by  $q_{off}$  in  ${}^iV_n$  that



can adopt the fourth stage of Dubois’ incursive algorithms only:<sup>2</sup> the considered algorithm stops when  ${}^iV_n$  is equal to  ${}^iV_1$ , i.e. when  ${}^iV_1$  assumes the same value twice. However, we can have that  ${}^iV_1$  assumes the same value twice consecutively or periodically. The second case our cut-pasting operation on the span space transforms an algorithm with an alone achievement in an algorithm with more achievement contemporary. We can write:

<sup>2</sup> See Dubois and Resconi, 1992, p. 13.

**Corollary 1.1.5:** *If Church's hypothesis an algorithm with more achievements is reducible to an algorithm with an alone achievement. Thus Church's hypothesis should be rejected.*

To redefine a new concept of computable function we propose this procedure to build all the computable function as a work hypothesis:

- 1) Put standard zero-function.
- 2) Put standard projection-function.
- 3) Put standard successor-function.
- 4) Put standard substitution to obtain new functions from other ones.
- 5) Put standard recursion to to obtain new functions from other ones.
- 6) Put Euclidean space as default span space of the functions.
- 7) Put cut-pasting operations on span space of a function to obtain new functions

Observe that the steps 6) and 7) of previous procedure are operation of change of topological structure where the involved function is spanned. Thus we cannot study more an algorithm without considering the topological structure on which it is spanned. Classical computation theory becomes insufficient and a new study matter needs:

**Algorithmic topology:** *it considers any algorithm always spanned on a topological structure (the Euclidean space as default) and it studies as algorithms change when their span topological structures change.*

We can consider algorithmic topology a development of computational topology.<sup>3</sup>

## 2 Mathematical Entanglement

Let *mathematical entanglement* be the overlapping of two steps in a given algorithm after an operation of cut-pasting on the span topologic structure of the given algorithm.

A practical example of mathematical entanglement is Dubois' third stage<sup>4</sup> in an incursive function.<sup>5</sup> Given the square calculus net of the considered incursive function,

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<sup>3</sup> See Grappone, 2009.

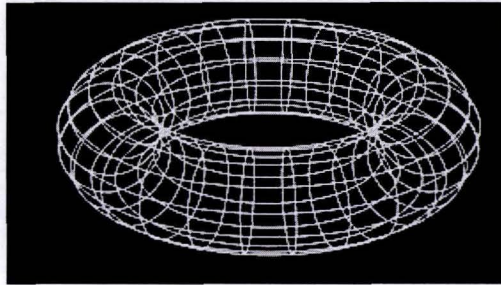
<sup>4</sup> See Dubois and Resconi, 1992, p. 12,

<sup>5</sup> In 1997, Daniel M. Dubois defined incursion and hyperincursion as follows: "The computation is incursive, for inclusive recursion, in the sense that an automaton is computed at future time  $t+1$  as a function of its neighboring automata at the present and/or past time steps but also at future time  $t+1$ . The hyperincursion is an incursion when several values can be generated for each time step. External incursive inputs cannot be transformed to recursion. This is really a practical example of the final cause of Aristotle. Internal incursive inputs defined at the future time can be transformed to recursive inputs by self-reference defining then a self-referential system. A particular case of self-reference with the fractal machine shows a non-deterministic hyperincursive field. The concepts of incursion and hyperincursion can be related to the theory of hyper-sets where a set includes itself. Secondly, the incursion is applied to generate fractals with different scaling symmetries. This is used to generate the same fractal at different scales like the box counting method for computing a fractal dimension. The simulation of fractals with an initial condition given by pictures is shown to be a process similar to a hologram. Interference of the pictures with some symmetry gives rise to complex



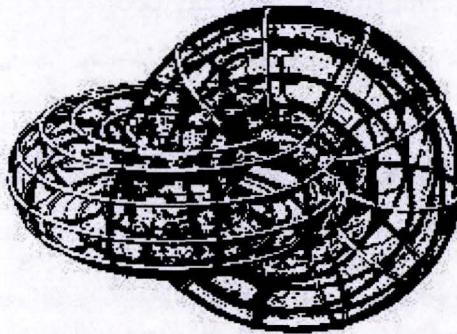
$k$	$x_{0,1}$	$x_{0,2}$	...	$x_{0,n-1}$	$x_{0,n}$
$x_{1,0}$	$x_{1,1}$	$x_{1,2}$	...	$x_{1,n-1}$	$x_{1,n}$
$x_{2,0}$	$x_{2,1}$	$x_{2,2}$	...	$x_{2,n-1}$	$x_{2,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$x_{n-1,0}$	$x_{n-1,1}$	$x_{n-1,2}$	...	$x_{n-1,n-1}$	$x_{n-1,n}$
$x_{n,0}$	$x_{n,1}$	$x_{n,2}$	...	$x_{n,n-1}$	$x_{n,n}$

the first row is identified with the last row and the first column is identified with the last column, i.e. we put an entanglement either between first and last row or between first and last column. Algorithmic topology shows easily that this double entanglement transforms a part of an Euclidean plain in a torus by a double cut-pasting operation.



**Fig. 1.** *Span space of the incursive functions.*

Another practical example of mathematical entanglement are hyper-incursive functions.<sup>5</sup> They can be viewed as systems of incursive functions with many-torus span topological structure that in some cases becomes an hyper-spherical structure.



**Fig. 2.** *Span space of some hyper-incursive functions.*

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*patterns. This method is also used to generate fractal interlacing. Thirdly, it is shown that fractals can also be generated from digital diffusion and wave equations, that is to say from the modulo N of their finite difference equations with integer coefficients."*

Observe that our work hypothesis implies that mathematical entanglement occurs in algorithms if and only if they cannot be spanned in an Euclidean space and it suppose cut-pasting operations always on the span space.

Observe also that an algorithm that is spanned in an Euclidean plain has a standard time sequence. Cut-pasting operations that provoke mathematical entanglements increase calculus anticipation too by creations of loops that are similar to Dubois' third stage and they can provoke achievement indetermination because such loops can be stopped only by Dubois' fourth stage (the repetition twice of the same result ends the algorithm). This stopping form can give the periodic repetition of an achievement sequence instead of an alone achievement. Thus the result is indetermined among the element of achievement sequence. Also, hyper-incursive function can give an achievent undefined among more values and with a frequence distribution of these values too.

We can conclude by emphasizing the strict relation that there is among cut-pasting operations on algorithm span space, mathematical entanglement, algorithm anticipation, indetermination of some achievements among more values with or without a frequence distribution.

### 3 Quantum Mechanics and Algorithmic Topology

Quantum mechanics differs from classical mechanics principally for the following phenomena too: quantum entanglement and indetermination of some achievement among more values with a frequence distribution.

Algorithmic topology can represent either entanglement or achievement indetermination with frequence distribution by oportune cut-pasting operation in algorithm span space. In other terms, classical mechanics can be transformed in quantum mechanics by cut-pasting operations on span spaces of its algorithms: quantum mechanics should be "strange" as regard classical mechanics only because the span spaces of their algorithms are not Euclidean spaces.

Algorithmic topology approach permits us to reveal another aspect of quantum mechanics: *its anticipatory nature*.<sup>6</sup> We have seen in the previous paragraph in fact as the entanglement and achievements indetermination are strictly linked with the anticipatory algorithms in Dubois' sense. To give some examples on quantum mechanics formulation in terms of hyper-incursive function could be useful.

Consider Gödel's function:<sup>7</sup>  $\beta(x_1, x_2, x_3) = \text{re}(1 + (x_3 + 1) \cdot x_2, x_1)$  where  $\text{re}(x_1, x_2)$  is the remainder of  $x_2$  divided by  $x_1$ . It has a very interesting property: *for every natural number sequence  $k_0, k_1, \dots, k_n$ , there are such two natural numbers  $b, c$  that*

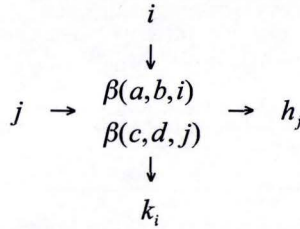
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<sup>6</sup> See Dubois, 2008.

<sup>7</sup> See Mendelson, 1964, Propositions 3.21, 3.22

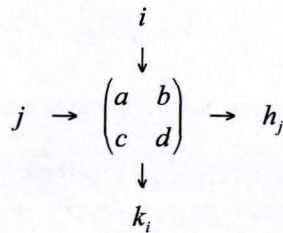


$\beta(b,c,i) = k_i$ . Thus, if we have any net node with two inputs and two outputs in an incursive algorithm then we can always represent it by Gödel's function in this way:



**Fig. 3.** Representation of any node in incursive net by Gödel's function

We can simplify the previous representation of node immediately in this way:



**Fig. 4.** Simplified representation of any node in incursive net by Gödel's function

In this way, any incursive net can be represented by a third order tensor, e.g.:

$$\left( \begin{array}{cc} \begin{pmatrix} a_{11} & b_{11} \\ c_{11} & d_{11} \end{pmatrix} & \begin{pmatrix} a_{12} & b_{12} \\ c_{12} & d_{12} \end{pmatrix} & \dots & \begin{pmatrix} a_{1n} & b_{1n} \\ c_{1n} & d_{1n} \end{pmatrix} \\ \begin{pmatrix} a_{21} & b_{21} \\ c_{21} & d_{21} \end{pmatrix} & \begin{pmatrix} a_{22} & b_{22} \\ c_{22} & d_{22} \end{pmatrix} & \dots & \begin{pmatrix} a_{2n} & b_{2n} \\ c_{2n} & d_{2n} \end{pmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{pmatrix} a_{n1} & b_{n1} \\ c_{n1} & d_{n1} \end{pmatrix} & \begin{pmatrix} a_{n2} & b_{n2} \\ c_{n2} & d_{n2} \end{pmatrix} & \dots & \begin{pmatrix} a_{nn} & b_{nn} \\ c_{nn} & d_{nn} \end{pmatrix} \end{array} \right)$$

**Fig. 5.** Representation of any incursive net

Thus we can always build the third order tensor that represents a bigness that is indeterminated among some values that appears all ones with the same frequency.

To represent a bigness that is indeterminated among some values that have got distinct frequencies we has to use hyper-incursive functions.

Let  $p_0, p_1, \dots, p_m$  be parameters. Thus we can represent a hyper-incursive function by a third order tensorial function:

$$\left( \begin{array}{cc} \left( \begin{array}{cc} a_{11}(p_0, p_1, \dots, p_m) & b_{11}(p_0, p_1, \dots, p_m) \\ c_{11}(p_0, p_1, \dots, p_m) & d_{11}(p_0, p_1, \dots, p_m) \end{array} \right) & \left( \begin{array}{cc} a_{12}(p_0, p_1, \dots, p_m) & b_{12}(p_0, p_1, \dots, p_m) \\ c_{12}(p_0, p_1, \dots, p_m) & d_{12}(p_0, p_1, \dots, p_m) \end{array} \right) & \dots & \left( \begin{array}{cc} a_{1n}(p_0, p_1, \dots, p_m) & b_{1n}(p_0, p_1, \dots, p_m) \\ c_{1n}(p_0, p_1, \dots, p_m) & d_{1n}(p_0, p_1, \dots, p_m) \end{array} \right) \\ \left( \begin{array}{cc} a_{21}(p_0, p_1, \dots, p_m) & b_{21}(p_0, p_1, \dots, p_m) \\ c_{21}(p_0, p_1, \dots, p_m) & d_{21}(p_0, p_1, \dots, p_m) \end{array} \right) & \left( \begin{array}{cc} a_{22}(p_0, p_1, \dots, p_m) & b_{22}(p_0, p_1, \dots, p_m) \\ c_{22}(p_0, p_1, \dots, p_m) & d_{22}(p_0, p_1, \dots, p_m) \end{array} \right) & \dots & \left( \begin{array}{cc} a_{2n}(p_0, p_1, \dots, p_m) & b_{2n}(p_0, p_1, \dots, p_m) \\ c_{2n}(p_0, p_1, \dots, p_m) & d_{2n}(p_0, p_1, \dots, p_m) \end{array} \right) \\ \vdots & \vdots & \ddots & \vdots \\ \left( \begin{array}{cc} a_{n1}(p_0, p_1, \dots, p_m) & b_{n1}(p_0, p_1, \dots, p_m) \\ c_{n1}(p_0, p_1, \dots, p_m) & d_{n1}(p_0, p_1, \dots, p_m) \end{array} \right) & \left( \begin{array}{cc} a_{n2}(p_0, p_1, \dots, p_m) & b_{n2}(p_0, p_1, \dots, p_m) \\ c_{n2}(p_0, p_1, \dots, p_m) & d_{n2}(p_0, p_1, \dots, p_m) \end{array} \right) & \dots & \left( \begin{array}{cc} a_{nm}(p_0, p_1, \dots, p_m) & b_{nm}(p_0, p_1, \dots, p_m) \\ c_{nm}(p_0, p_1, \dots, p_m) & d_{nm}(p_0, p_1, \dots, p_m) \end{array} \right) \end{array} \right)$$

**Fig. 6.** Representation of any hyper-incursive net

Every value assegnation to  $p_0, p_1, \dots, p_m$  defines an incursive net and a set of achievements. As the same achievement can appear for distinct value assegnations to  $p_0, p_1, \dots, p_m$  an oportune set of such assignations can give a set of achievements with the wanted frequency. In other terms, any undetermined quantum bigness that has a given frequency distribution on a value set can be represented by a hyper-incursive function.

Precisely, let  $\mathbf{I}$  be an oportune starting input vector, let  $\mathbf{P}$  be an oportune parameter matrix and let  $\Omega$  be an oportune hyper-incursive net. Thus  $\Omega(\mathbf{I}, \mathbf{P})$  can represent an indetermined quantum bigness with a frequency distribution on a value set.

As incursive and hyper-incursive functions permit us to represent directly either quantum entanglement or quantum indetermination and so the whole quantum mechanics we can conclude that quantum mechanics is anticipatory in Dubois' sense and can be entirely obtained by cut-pasting operations on Euclidean algorithm span space.

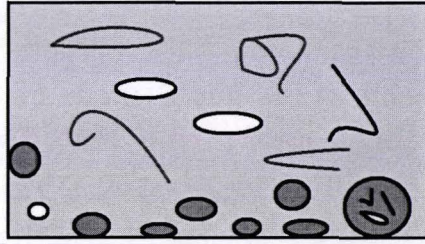
In other terms, the spanning of ordinary algorithms on oportune non-Euclidean topological structure eliminates every strangenes of quantum mechanics.

### 3 Histochemistry: Biological Anticipation of Algorithmic Topology

Histochemistry is the branch of histology that deals with the identification of chemical components in cells and tissues. It has introduced the importance of localization for biochemical reation that often change meaning with their place.

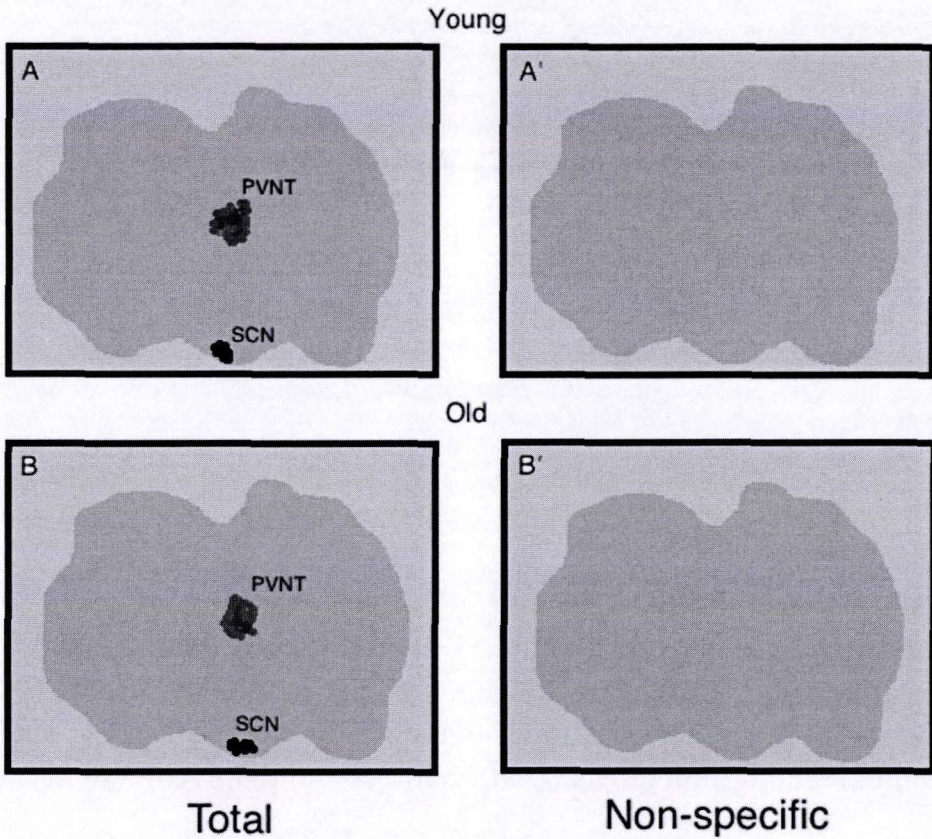


Some examples can clarify:



**Fig. 6.** *Acetylcholinesterase activity with localization*

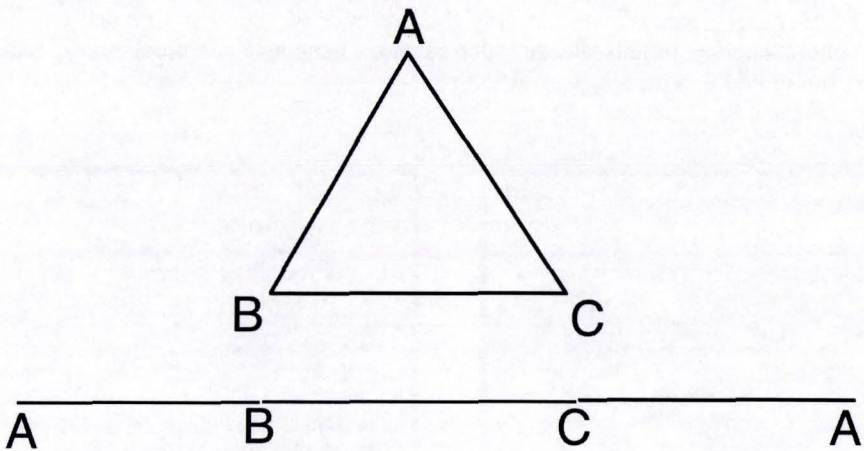
The previous figure shows not only the presence of acetylcholine decomposition by acetylcholinesterase but also *where* such reaction happens. The biochemical reaction has to be integrated in a topological structure.



**Fig. 7.** *Iodomelatonin-binding with localization*

#### 4 Matte Blanco Tridim Structure: Psychological Anticipation of Algorithmic Topology

Matte Blanco<sup>8</sup> affirms that the structure tridim is the representation of objects of the unconscious that, for him, have a greater number of dimensions in the Mad aware thought that Blanco considers three-dimensional. The condensations and the displacements of meaning that Freud described for first are examples of structures tridim.



*Fig. 7. Representation of a two-dimension object in one dimension*

Observe as the representation of a two-dimension object in one dimension provokes an entanglement between the two extreme points of the line segment. Freud describes such psychical entanglements as condensations and displacements.

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<sup>8</sup> See Matte Blanco,



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