Binding Energy in Nuclear Physics

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Abstract
Atomic Nucleus stability is discussed in terms of its Binding Energy (BE). New insights in understanding many Nucleus properties like limit of stability are obtained. BE generalized to the structured objects give some hints to find origin of the missing mass (dark matter). The very new CHAIN Model of the nucleus is confronted with the classical mass formula.. Some ideas for studies of Super Heavy Elements (SHE) related to the role of high spin states and their importance in binding energy calculation are presented.

Keywords: nucleus, nucleons, binding energy, nuclear models, SHE

1 Introduction

One of the most important characteristic of atomic nucleus is the Binding Energy. Apparently, the simple arithmetic's TOTAL = SUM OF PARTS is not fulfilled by nuclear mass. If we do the sum of masses of protons and neutrons which form the nucleus with $\mathbf{M_{sum}}$ and we compare with the experimental mass of the nucleus $\mathbf{M_{experiment}}$ given by the usual spectrographic measurements which resolution is 10^{-6} or better, we find for the ratio $\mathbf{(M_{sum} - M_{experiment})/M}$ approximately 10^{-3}, a big value which can't be due to experimental errors. This difference clearly indicates that the nucleons in the nucleus are strongly binded. We can understand these phenomena in the physical terms of interactions. The parts (nucleons) can be together through the interactions. Two metallic balls with the same kind of electrical charge can be put together with big effort because they repulse each other, they interact, and when they are touching they form a TOTAL. In the nucleus the amount of interactions between the nucleons gives us hints about of the internal dynamics.

It is believed that the atomic nucleus is like a bag, where the nucleons interact through their nuclear field creating the nuclear mean field in this bag, which keeps the nucleons in the nucleus. The mean field concept appeared earlier in nuclear physics in 1935 when Yukawa [1] gave his model of nuclear forces mediated by a field quantum «the pion». Each nucleon is surrounded by a field and the local field in a point of the nuclear bag is the arithmetic sum of the existing nucleons fields. Few years ago appeared the experimental evidence that this addition (of the fields) is not exactly fulfilled when the European Muon Collaboration studying the quark structure of the nucleons, discovered the EMC effect [2]. It was found that the momentum distribution of the quarks in a nucleon is related on its surrounding in a nucleus. For a nucleon embedded in a nucleus the number of quarks with small momenta increase and the number of quarks with large one decrease when the number of nucleons increase. Recently, according to experimental results from the electron accelerator experiments, Arrington et al. [16] believed that the quark structure inside protons and neutrons changes based on the local nuclear environment. To understand this effect we think that the binding energy of the nucleus play a very important role, even when the bombarding energies of muons is few thousand times larger than the 8.5 MeV, the binding energy of a nucleon in the target 56Fe.

In [3] we propose a new field model (shape) of the nucleus. We replace the bag by a ring and the simplest dynamics in the bag field is replaced by the nucleons dynamics inside of the ring and the dynamics of the ring itself (outside the ring). Volume energy inside the ring gives us a big part of binding energy, conversely the outside one (surface, Van der Waals kind), is the very important one for nucleus evolution. These ideas will be sketched in chapter **5**.

2 Binding Energy Characteristics

2.1 The Atom is a brick of the UNIVERSE

The Atom has a structure: a small nucleus, supposed to be a sphere, with the radius r_n of the order of 10^{-15} m and the electrons orbiting at large distances (relative) $R_{orbit} = 10^{-10}$ m; the ratio $R_{orbit}/ r_n = 10^5$. Conversely, from the mass point of view the nucleus is rather heavy; the hydrogen nucleus (the proton) is 1836 (ratio between m_p and m_e). times heavier than the electron The proton has an electrical charge q_p, the electron has the same amount of charge but of opposite sine q_e , $q_p = -q_e$. In the nucleus there are also neutrons, a kind of protons without charge, neutral particles. All these particles have a spin $s_p = s_e = s_n = (1/2\pi)h$ (h is Planck constant).

The Atom is a quantified and bound system, it means that the electrons are in the quantum states and to extract one of them we have **to pay** an amount of energy **(few eV)** named binding energy of an electron B_e in the Atom.

The Nucleus is also a quantified and binded system; it means that the nucleons (protons or neutrons) are in the quantum states and to extract one of them we have **to pay** an amount of energy **(few MeV)** named binding energy B_p, B_n for protons and neutrons. We remark that B_p or B_n are of the same order of magnitude, and the ratio B_p/ B_e is approximately 10^5 like the ratio $R_{orbit}./ r_n$ .Let say that when the object is smaller the binding energy (inside the object) is bigger.

The neutron was discovered in 1932 by Chadwick [4] in Manchester, 12 years after Rutherford suggestion of neutral particle in the Nucleus. In the same year Heisenberg suggested two models of the nucleus **a bag** [19] (became later Shell Model developed by Mayer Jansen [21] which explain the shells, bunch of nucleons: 2, 8, 20, 50, 82, 126) with the radius of less 10^{-14}m and **Fermi Gas** (the nucleus is interpreted like as a gas of free particles like Thomas and Fermi model for electrons) [20]. These original models are transformed and improved in our days.

Hideki Yukawa, a Japanese theoretician, proposed in 1935 [1], a model for nuclear forces. He claimed that the nucleons in the nucleus interact with all other nucleons through the nuclear mean field in the nuclear bag. This nuclear mean field is the result of the simplest sum of individual nucleons field, like in painting, where superposing the same color over and over we obtain a strong color. He proposed for the quanta of the field a "massive particle", **the pion** with the rest the mass $M_\pi \sim 200$MeV. This was a smart suggestion and was done in analogy with the Electromagnetic (EM) field theory studied by Maxwell in 1868 and with the field quanta suggested by Einstein in 1906. The photon does not have rest mass.

Another model for the nucleus was developed by von Weizsäcker, Bethe and Bacher and used by Niels Bohr in 1939 [18], the famous LDM (in the Liquid Drop Model, the nucleus is regarded as a liquid drop, with nucleons playing the role of molecules).

The difference between EM Field and Nuclear Field was the mass of the quanta exchanged. Yukawa roughly estimates the mass of the pion from the Nucleus diameter 6_fm. The wave length (with de Broglie's equation of wave length) of this particle of the mass m and moving in the nucleus, between two nucleons, with speed closer to the light speed is: $\lambda=h/(mc)$; then: $m=h/(\lambda c)$; $mc^2 = h c/\lambda = 1250$MeVfm/6fm = 208MeV which is a enormous mass, one fifth of the rest mass of the proton. This particle with short life time (recent measurements give the life time 2.6 10^{-8}s and $m_{\pi+} = m_{\pi-} =$ 139,58MeV) was discovered in 1947 in cosmic rays by Powell [5]. In the nucleus the field pions are virtual pions. They are created and absorbed in the processes of interaction of the nucleons.

2.2 Mass of the Nucleus

The difference between the calculated mass of the nucleus and experimental measured mass (already mentioned in the first chapter) is named also mass defect and noted with Δ.

$$\Delta = M_{sum} - M_{experiment} \qquad (1)$$

where:

$$\mathbf{M_{sum}} = Z^*m_p + N^*m_n \qquad (2)$$

with Z and N numbers of protons and neutrons in the nucleus, m_p, m_n being their experimental masses respectively for proton and neutron.

The notion of mass defect appears earlier after the mass measurements of some 200 known isotopes, realized by Aston from 1919 to 1922. The Rutherford suggestion since 1920 that the nucleus has neutral particles (N) inside and that the total number of particles in the nucleus A=Z+N particles (A number of mass) gives the possibility to evaluate the mass defect and to draw the classical graph **Δ** function of A. It appears clearly that the most bounded nuclei are the Fe.

By putting together nucleons (protons and neutrons) some mass «disappear», a part of the total mass is «evaporated». More mass disappear, more bounded is the nucleus. In the Mendeleyev table the binding energy per nucleon is growing up from deuteron to iron and after that is slowly decreasing to Uranium and further.

By using Einstein relation between mass and energy with the formula: $E=mc^2$ a we can interpret this mass defect in the terms of energy «the binding energy». Vice versa, the binding energy implies a relativistic mass m_B, which in the nucleus is negative and lower the total mass of the nucleus. We identify this relativist mass with the mass defect. This concept reflects the nucleons dynamics in the nuclei. Most usual is the formula:

$$B = \mathbf{\Delta}\, \mathbf{c^2/A} \tag{3}$$

We find tables (Wapstra 2003 [6]) with $B_n(Z,N)$, $B_p(Z,N)$ neutron, proton escape energy, $B_{2n}(Z,N)$ two neutron escape energy, $B_{2p}(Z,N)$ two protons escape energy or Q energy realized in a nuclear reaction.

In the nuclear reactions with heavy ions we can calculate the excitation energy through the Q of reaction. The most important for the DITR (Deep Inelastic Transfer Reaction) is the Q_{gg} (for the nuclei in the ground state):

$$Q_{gg} = (M_1 + M_2 - M_3 - M_4)\, c^2 \tag{4}$$

where M_1, M_2,are the masses of colliding nuclei (A_1, Z_1) and (A_2, Z_2) and M_3 , M_4 are the masses of outgoing nuclei (A_3, Z_3) and (A_4, Z_4), and supposing that the bombarding energies are big enough to overcome the coulomb repulsive barrier but not so high to avoid nucleons creation E<15*A MeV.

In the multinuclear transfer reaction it was experimentally found that the production cross section of the light nuclei (from H to Ar) in the reactions with heavy ions (^{11}B, ^{12}C, ^{16}O, ^{20}Ne, ^{40}Ar, etc.) satisfied the so named **Q_{gg} systematics** [14], the logarithmic plot of the experimental differential cross section function of (Q_{gg} -δ). The almost linear dependence of differential cross sections for the isotopes given element suggested the formula:

$$d\sigma_{exp}/d\Omega \sim \exp(((Q_{gg} - \delta) / T) \tag{5}$$

where Q_{gg} is given in (4), δ is the total energy necessary to break the pair before the transfer of the nucleons from projectile in the target (sum over transferred protons and neutrons, $\delta = \delta_p + \delta_n$). T is the temperature parameter related to the excitation energy in the reaction at the contact point of the colliding nuclei. Usually Q_{gg} is of tenth of MeV, δ few MeV and T is around 2 MeV for bombarding energies 5-10MeV/A. The transfer of the nucleons is supposed to take place sequentially.

In [7] we prolonged the analysis of experimental data for the heavy ion transfer reaction: Ar + Ag, Ar + Au ,Ar + Th, Ar + C. Instead of the proportional relation (5) we introduce [7] the function $K(Z_3)$ and (5) becomes an equality:

$$d\sigma_{exp}/d\Omega = K\,(Z_3)\, \exp((Q_{gg} - \delta) / T) \tag{6}$$

Having the experimental cross sections, T (the slope of the lines in the Q_{gg} systematics) and experimental masses, we can infer the K (Z_3). We remarked in [8] that the logarithmic plot of K (Z_3) on Z_3 (for the reactions Ar + Ni ,Ar +Ag, and Ar + Au) is a straight line, with different slopes b (parameter a is the value of K for $Z_3 = 0$ (neutrons)), characteristics of each reaction and covering almost 22 order of magnitude (see fig.1). Then, we find the relation

:

$$K_{exp}(Z_3) = k' \exp(a + b Z_3) \quad (7)$$

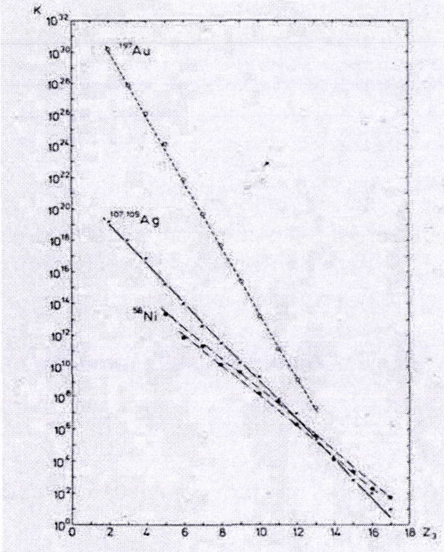

Fig. 1 K(Z) dependence for three mentioned reactions (from [8])

More information on binding energy can be obtained from the graph of the isospin dependence of the experimental measurements differential cross sections for multinucleon transfer.

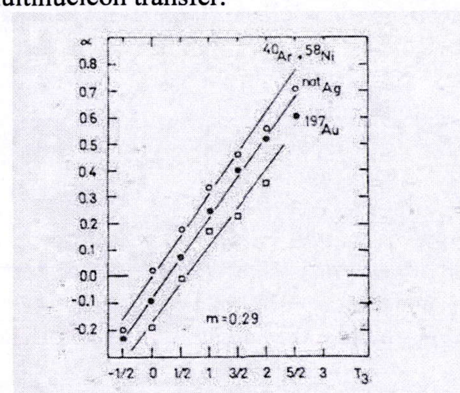

Fig. 2 The dependence of the slopes of the $d\sigma_{exp}/d\Omega(Z)$ for fixed $t_3$on the t_3 (from [8])

In [8] was obtained that:

$$d\sigma_{exp}/d\Omega = k'' \exp[(m\, t_3+q)\, Z_3] \qquad (8)$$

For different reactions was obtained the same value of $m = 0.3$, where the q the values for this dependence at origin ($t_3 = 0$). The t_3 is the isospin of the light detected particle

$$t_3 = (t_n N_3 + t_p Z_3)/2 \text{ with } (t_n = \tfrac{1}{2},\, t_p = -\tfrac{1}{2}). \qquad (9)$$

And putting together (6), (7) and (8) we find:

$$(m\, t_3+q)\, Z_3 = a' + b\, Z_3 + (Q_{gg} - \delta)\, /\, T \qquad (10)$$

In (10) the constant a' is determined by the k' and k", and gives the possibility to calculate the absolute differential cross sections for isotopes productions. Because the parameter T ("temperature") increases with the increasing of bombarding energy of the projectile, a' is also energy dependent and of nature of projectile and target.

This isospin dependence of the differential cross section in DITR indicates clearly through the relation (10) that the isospin of the nucleus should be taken into account in the mass formula, in the binding energy.

By studying the mentioned reactions at different bombarding energies it was found that the constant p is not changed.

This permits us to conclude that the binding energy of an atomic nucleus is the same even for high excitation energies.

3 Binding Energy Calculations

To calculate, the binding energy B(A,Z) of the nucleus (A,Z) where A and Z are the nucleons and protons number respectively, we take into account experimental observations: a rough constant density of nucleons in the nucleus, and a relatively sharp surface decrease of that density. The Total Binding Energy of the nucleus grows proportionally with the number of nucleons A. The binding energy per particle B(A,Z)/A stays in average constant for nuclei heavier than carbon:

$$B(A,Z)/A \approx -8{,}5 \text{ (MeV/nucleon)} \qquad (11)$$

The binding energy can be attributed to the saturation of the nuclear force: one nucleon interacts only with a limited number of nucleons (short range nuclear forces and the combined effect of the Pauli and uncertainty principles). The saturation property, also explains qualitatively the features found experimentally and mentioned higher. Consequently the nuclear radius is

$$R = r_0\, A^{1/3} \qquad (12)$$

where $r_0 = 1,2$ fm.

The best-known formula to reproduce the behavior of experimental values of B as function of A and Z (N=A–Z) is the semi-empirical formula of Bethe and Weizsäker [9]:

$$B(A,Z) = a_V A + a_S A^{2/3} + a_C Z^2/A^{1/3} + a_I (N-Z)^2/A + P(A) \qquad (13)$$

where:

$$a_V = -15.68 \text{ MeV}, a_S = 18.56 \text{ MeV}, a_C = 0.717 \text{ MeV}, a_I = 28.1 \text{ Me} \qquad (14)$$

and the pairing energy P(A) = (11.2 $A^{-1/2}$ for even-even; 0 for even-odd; and -11.2 $A^{-1/2}$ for odd-odd nuclei [13] formula issued from beta decay analysis) or like Peter Schuck [15] adopted: P(A) = (34 $A^{-3/4}$ for even-even; 0 for even-odd; and -34$A^{-3/4}$ for odd-odd nuclei formula deduced from level density analysis [17]). The two formulas for P(A) give sensitive equal values (less 50keV difference, rather small compared to 0.5MeV < P(A) < 2.5MeV over all A) for the isotopes with masses between 70 < A <100.

The parameters given in (14) are rather good, but they are rather old example. In our days, many other values are on the market. There are many formulas like (13) with tens and more parameters up to thousand. Almost every year, the experimental mass table is updated (some of old one are given with better precision). Even with hundreds parameters the far from stability line of nuclear mass are difficult to be predicted.

The first term in (12) is named **volume term**; the second one is **surface** (or tension) **term**; the third is **Coulomb term** (due to the protons repulsion), the fourth term is **symmetry energy term** (in the Fermi gas model) and the P is due to the so-called **pairing** effect (two nucleons with opposite spin form a pair).

The surface term is due to a surface force which is analog to the tension force at the surface of a liquid where the molecules are only partially attracted by the others.

Another interpretation of this term is from Gauss-Ostrogradsky integral equality: for part of nuclear forces function, the volume integral is equal to the integral taken on surface which closed this volume.

When B(A,Z) value become positive we reach the line of stability, or boundary line. Near the stability line the life time of the isotopes are rather short. The experimental limits are of order of nanoseconds for example the ^{26}O the $T_{1/2}$ <40ns.

4 Binding Energy and Dark Matter

The binding energy can be observed for different structured objects: from tiny objects like nucleus to the large scale one, galaxy or their assembly. For the quarks in a nucleon "the binding energy" is unknown and probably is almost infinite at our scale energy (TeV). It is impossible to extract a free quark from the nucleon through a simple reaction.

For usual ordinary structures: the nucleus, the atom, the crystal, the ratio D of the binding energy to the object mass ($D = \Delta M/M$, where ΔM is mass defect, or binding energy) decrease rapidly. The masses of the considered structured objects: nucleus, atom, crystal, increase and known experimentally bound energies (for nucleus, atom or crystals) decrease. The D value for a nucleus is approximately 10^{-2}, for the atom and the crystal D decrease rapidly from 10^{-8} to 10^{-11} (apparently negligible). The quotient D tells us about the missing mass. If we take into accounts the dimension L of the considered objects the nucleus, the atom, the crystal, the product DL seems to be a constant $C = 10^{-17}$ m (quark dimension!).

$$D\,L = C \tag{15}$$

We underline that the values mentioned are an average over the binding energies and the dimensions of the objects.

The Solar System, the Galaxy, the Black Holes (BH), etc. are bounded structures. The attractive gravitation keeps together the parts of the bound system and a total binding energy can be defined (sum of energies necessary to separate the parts at infinity). For some systems the gravific mass is not enough to explain the stability of the system and more mass has to act to stabilize the system (the dark matter). If we suppose that the interaction is slightly different from the Newton gravitation, (like MOND theory [24]) the binding energy will be different and can stabilize the system. For the moment we can't tell more about this "slightly different", but we can evaluate that through the new relation DL (funded empirical). We can't evaluate the mentioned ratio D because we don't know the average bound energy (if it exist). Through a simple arithmetic using the new average constant C we can find this average binding energy and consequently the missing mass for the celestial structures. Roughly the value of this missing mass is of the same order of magnitude like the dark matter.

Nottale [25] by inventing and using his scale general relativity (from nucleons o Universe) arrived at similar conclusions about dark matter.

Our thoughts through the extended binding energy, total different of Nottale ideas, come with the same conclusions, may be the dark matter does not exist.

5 Binding Energy in the CHAIN Model

Taking into account experimental evidence that the binding energy is roughly proportional to A (the number of the nucleons in the nucleus), we conclude that a nucleon in a nucleus does not interact with all A-1 nucleons, (otherwise B will be proportional with A*(A-1)). Like we mentioned before a nucleon is directly related with two neighbor nucleons. We conjectured [3] that the nucleons are in a kind of ring and we named this model CHAIN Model of the nucleus. One nucleon of the ring is in direct contact (two internal valence quark "are related" with two external virtual quarks of the pions, like in chemistry of the polymers and "the motif" of this chain is: virtual quark-valence quark–valence quark-virtual quark) only with two others. All the nucleons of the nucleus form a closed chain, a kind of DNA (life molecule, which is closed chain for some virus). The model was also inspired by the Toda Chain of coupled (through an exponential interaction) oscillators [23] and is completed with the exchange terms (Van der Waals one) which appeared in the self-crossed points of the chain. This model explains easiely all the decay modes of the nucleus.

If we suppose that the distance between two nucleons in a ring is "d" then, we obtain the volume energy of the ring, with A nucleons, is proportional with:

$$-\pi\, d^3\, A/4 \qquad (16)$$

By equating (156with the -15.68 A volume term from (13), we obtain d = 2.71 fm which is in very good agreement with the estimations from the mean field model. The most important part of nucleons dynamics come from the inside the ring evolution, which give us a big part of binding energy. The outside one (surface, Van der Waals kind) contributes to the overall shape change.

Our nuclear ring is distorted to minimize its total energy (volume), like DNA molecule (DNA is of almost 2 m long and few nanometers in diameter). The DNA volume in a cell nucleus is of few tens cubic nanometers. The coulomb energy of the compact ring is twice greater than that of the normal developed ring.

Our CHAIN Model will give new insights about the charge (protons) distribution density in the ring and for the decay properties of the nucleus. The new experimental data from Argonne [16] supports our model.

6 Binding Energy in SHE

SHE are the nuclei far above trans-uranium region (Z>100). The discovery of the shape fission isomers in 1962 by Polikanov in Dubna[10] and the new Strutinsky [11] idea in 1967 to calculate the shell model corrections for the liquid drop model calculations, lead nuclear theory calculations to a new concept, the multi hump fission barrier and new estimations of closed shell numbers, Z=114 instead of Z=126 and easier to reach through the fusion reaction with heavy projectile like ^{48}Ca or heavier one.

By breaking spherical symmetry a new minimum in the potential energy was found for higher deformations (in general twice bigger than deformation of the ground state). In these second minima the nucleus is quasi stable. These new calculations give hope to reach through the experiment the Super Heavy Elements

These new calculations suggested an island of stability of SHE around Z=114, N=184 [22]. Through the technique of the alpha chains (or fission products of decaying compound nucleus), it is possible to identify the nucleus formed by fusion of two heavy ions. In the last twenty years few laboratories in the world, like Dubna, Berkeley, GSI are announced measurements for few dozens of new isotopes. One of them, the $^{268}{}_{105}$Db (Dubnium) has a rather long life time of 30h.

In 2009 we [16] suggested to do calculations for SHE by taking into account the new quantum number $G = I + t_z$ Ripka idea [12]. By combining the spin and the isospin we believed we could obtain correct (more realistic) values for the binding energy of the SHE an to push for new experiments with high spin isomers. The spin-isospin relation furnishes a new kind of stability, for the SHE. Experimentally we can use for this purpose ions (targets) like $^{178}Hf^{2m}$.very high spin isomer with $I=16^{+}$, and with 31years life time.

We hope to reach SHE with high spin $I = 16^{+}$ for example, by bombarding $^{178}Hf^{2m}$ target with natural ^{178}Hf , the high spin will stabilize the SHE and compensate lake of neutrons (contributors to the stability), like G quantum number indicate.

Our calculations for the binding energy taking into account the G quantum number are in progress.

7 Conclusion

The physics of Nucleus through the binding energy notion was recalling. The binding energy seems to be the key of many phenomena in nuclear physics, like multi nucleons transfer reaction, of Super Heavy Elements or in cosmological problems (dark matter). We find that even at high excitation energy (hundred MeV) the Nucleus "remember" the tiny "binding energy" (approximate 8MeV). The quark CHAIN Model of the nucleus can explain the EMC effect. From the DITR cross section analysis was inferred that the "binding energy" is dependent on isospin and is independent of excitation energy. We introduced (in this article) a new constant $C = 10^{-17}$m to understand the dark matter. The G quantum number relating the spin (for example of the high spin isomer) and the isospin, is suggested for the calculations of levels energies to use in the binding energy evaluation, give a new perspective to obtain experimentally the far SHE.

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