Quantum Causal Analysis

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Abstract
We suggest a new method of quantum causal analysis. The causality is defined without invoking the time relation. It clarifies Cramer principle of weak causality, which admits time reversal in the entangled states and directly leads to the anticipation. The quantitative quantum measure of causality is the pseudoscalar velocity of irreversible information flow called the course of time. The smaller course of time modulus, the stronger causal connection. The causal parameters for the examples of three-qubit entangled states have been computed. The results have been compared with the degrees of mixedness and entanglement of the states. In the simplest cases the formal measure of quantum causality does not contradict its intuitive understanding. But even in slightly more complicated situations intuition may be a failure.
Keywords: causality, entanglement, time, anticipation

1 Introduction

Causality is one of the universal physical principles. It plays the twofold role. On the one hand, in the theoretical problems, this principle allows selecting of the physically realizable solutions among a plethora of the mathematically admissible ones. On the other hand, establishing of causal-effect connections in analysis of the complicated systems is the first step to constructing of a phenomena model.

In references to the causality principle, usually one does not bear in mind anything except retardation of the effect relative to the cause. However what is the "cause" and "effect" remains indefinite. In the theoretical problems it may lead to the confusions. In the complicated phenomena investigation the serious mistakes are possible.

The necessity of formal taking into account of really existing causal connections was felt by many researchers. Moreover deeper insight into causality problem had been led to the interesting theoretical and experimental consequences concerning anticipation [26]. In answer to this challenge the formal method of classical causal analysis was suggested [9]. This method had been successfully applied to the various theoretical and experimental problems of classical electrodynamics, magnetohydrodynamics and geophysics, e.g. [8, 10]. Later it was also applied to the experiments on macroscopic entanglement [11, 13-25]. But the classical approach to a quantum phenomenon is rather limited. Recently the quantum causal analysis has been suggested [12].

Quantum causality, unlike classical one, can exist only in the mixed states, i.e. in a system which is or was open. Another interesting property of the mixed entangled states is that they obey the *weak* causality [5] (for which reverse time is allowed).

In Ref. [12] the series of examples of two-qubit causeless (symmetric) and causal (asymmetric) states has been considered. In this paper we consider more complicated three-qubit states. In fact this consideration demonstrates the qualitative peculiarities of the many-qubit states as compared to two-qubit ones. In Sec. 2 we review the idea of quantum causal analysis. The main new results are contained in Sec. 3, were we consider the examples of causal analysis application to the three-qubit states. Conclusion is in Sec. 4.

2 Quantum Causal Analysis

The main idea of causal analysis is formalization of usual intuitive understanding of asymmetry of a cause and effect, owing to which we can usually distinguish them without measuring the retardation (although the finite retardation is, of course, a necessary condition of causal connection).

For any classical variables (subsystems) A and B the independence function i can be introduced:

$$i_{B|A} = \frac{S(B\,|\,A)}{S(B)},\; i_{A|B} = \frac{S(A\,|\,B)}{S(A)},\; 0 \le i \le 1, \tag{1}$$

where S are conditional and marginal Shannon entropies. As it has been shown in Ref [12] in classical causal analysis there are two measures of causality. One of them is the causality function γ:

$$\gamma = \frac{i_{B|A}}{i_{A|B}},\; 0 \le \gamma \le \infty, \tag{2}$$

Another on is the course of time:

$$c_2 = k\frac{(1-i_{A|B})(1-i_{B|A})}{i_{A|B} - i_{B|A}}, \tag{3}$$

where $k = \Delta r / \delta t$; we suppose that the subsystems A and B are separated by some finite effective distance Δr; and δt is duration of an elementary signal. The c_2 has a meaning of the velocity of cause-effect information transition. The direction of irreversible information flow is determined by the sign of c_2.

It can define that A *is the cause and* B *is the effect if* $\gamma(A,B) < 1$ *or, equivalently,* $c_2(A,B) > 0$. And inversely: B *is cause and* A *is effect if* $\gamma(A,B) > 1$ *or, equivalently,* $c_2(A,B) < 0$. The case $\gamma = 1$ or, equivalently, $c_2(A,B) \to \pm\infty$ corresponds to adiabatic (causeless) relationship A and B.

For the quantum variables von Neumann entropies S in Eq (1) are used. For the entangled states the conditional entropies can be negative [2, 3]. Therefore $-1 \le i \le 1$. In particular, for the pure bipartite states from Schmidt decomposition it is follows $i_{B|A} = i_{A|B} = -1$. Next $-\infty \le \gamma \le \infty$ and the value of γ becomes meaningless when it is negative. Moreover even positive γ failures when the both i are negative, while c_2 as shown in Ref. [12] remains to be always consistent. That is the course of time c_2 is the appropriate measure of quantum causality. Thus we have:

Definition: *The cause* A *and the effect* B *are the subsystems for which* $c_2(A,B)>0$.

Then, introducing the demand of the effect retardation τ, we can formulate the axiom of strong causality, embracing local and nonlocal correlations, as follows:

$$c_2>0\Rightarrow\tau>0,\ c_2<0\Rightarrow\tau<0,\ |c_2|\to\infty\Rightarrow\tau\to 0. \quad (4)$$

Notice, that nonlocal correlations are often treated as instantaneous and causeless ones. Our approach includes such treatment, but only as a particular case.

The axiom (4) is the principle namely of strong causality. Cramer was the first to distinguish the principles of strong and weak causality [5]. The strong causality corresponds to the usual condition of retardation of the effect relative to the cause. Without this axiom we have the weak causality. The weak causality corresponds only to nonlocal correlations and implies a possibility of information transmission in reverse time, but only related with unknown states (hence "the telegraph to the past" is impossible). Note, that in the examples of the Sec. 3 we shall nowhere use the axiom (4). Reverse time is allowed.

Interpretation of entanglement of a quantum system as the resource serving for information transfer through it, justified in Ref. [27], gives c_2 the additional physical meaning. Specifically in Ref. [27] it has been proven that negative conditional entropy is "an amount of information which can be transmitted through <the subsystems> 1 and 2 from a system interacting with 1 to another system interacting with 2. The transmission medium is quantum entanglement between 1 and 2". Causality characterized by c_2 value reflects the asymmetry of this process (the greater causality is expressed by the less $|c_2|$).

Zyczkowski and Horodecki's [31] were the first to put forward the hypothesis on asymmetry in the transfer of quantum information with respect to its direction. It has been just that we have studied in this work. As compared to the first article on quantum causal analysis [12], where only the simplest two-qubit states were considered, in this work we can demonstrate the specific causal properties of the compound parties of the quantum systems.

But though defined by Eq. (3) the course of time c_2 with accuracy to the coefficient k is of great interest by itself, it is desirable to show the way of its full determination for the natural processes. For this there is no remain δt to be duration of "an elementary signal", that is pertinent only for a technical channel. Since δt in any case plays a role of some elementary time, it is natural to suppose it, according to Ref. [1], to be time of brachistochrone evolution. In the case of time independent Hamiltonian this time is easily expressed explicitly:

$$\delta t=\frac{\hbar\theta}{2\omega}, \quad (5)$$

where 2ω is the difference between the largest and smallest eigenvalues of the Hamiltonian and θ is the length of geodesic (according to Fabini-Study metric) connecting the initial and final states. If they are orthogonal, $\theta=\pi$. In realistic Hamiltonian ω depends on distance Δr and k becomes definite. It is readily shown [9] that for the simplest Coulomb interaction $k=e^2/\hbar$, that corresponds to Kozyrev order estimation of c_2 obtained from the semiclassical reasoning [26].

To keep the examples described bellow from becoming too involved; we shall restrict ourselves by calculations of c_2 with accuracy to $k=1$, because, as it has been demonstrated in Ref. [12], the precise estimation of c_2 with regard to variable k calculated through the eigenvalues of Hamiltonian does not lead to a qualitative change in c_2 behavior.

3 Examples

By entropic symmetry stemmed from Schmidt decomposition causality is absent in any bipartite pure state. Therefore the mixedness is a necessary condition of quantum causality. Following the line of Ref. [12] we consider the dissipation as a way of partial decoherence which leads to the mixedness. The dissipation is reduced to the following qubit transformation [7, 30]:

$$\begin{aligned}
&|0\rangle\langle 0| \to |0\rangle\langle 0|,\\
&|1\rangle\langle 1| \to (1-p)|1\rangle\langle 1| + p|0\rangle\langle 0|,\\
&|1\rangle\langle 0| \to \sqrt{1-p}\,|1\rangle\langle 0|,\\
&|0\rangle\langle 1| \to \sqrt{1-p}\,|0\rangle\langle 1|,
\end{aligned} \tag{6}$$

where $0\le p\le 1$ is decoherence degree.

We consider the three-qubit systems. The first qubit we call the subsystem A, the second and third – the subsystems B and C respectively. Another source of mixedness of a two-particle subsystem is its interaction with the rest of the system, which is the third particle. We consider the bipartite states, where one party may consists of two particles, that is some more complicated in comparison to Ref. [12], where only one-particle partition has been analyzed. Thus the rather simple three-qubit examples will help to understand the peculiarities of many-particle causality in comparison to two-particle ones.

Except the measure of causality c_2 and independence functions i, every example we shall provide with the negativity N as a measure of entanglement and the entropy of full system $S(ABC)$ (or two-particle subsystems $S(AC)$ etc.) as a measure of mixedness.

3.1 Coffman-Kundu-Wootters State

Coffman, Kundu and Wootters [4] have discovered the (CKW) state:

$$|CKW\rangle = \frac{1}{\sqrt{2}}|100\rangle + \frac{1}{2}\left(|001\rangle + |010\rangle\right), \tag{7}$$

which is notable by maximal pairwise entanglement (measured by concurrence) of the subsystems AB and AC. The entanglement properties of this remarkable state have also been considered in Ref. [6]. In Ref. [12] it has been found that those subsystems are causal, a party A is a common cause for B and $C: c_2(A,B)=c_2(A,C)=5.30$ (the link $B-C$ is causeless: $|c_2(B,C)|=\infty$).

Let the particle C be dissipated as in above examples. Then the state is:

$$\rho_{CKW}^{dissC}=\frac{1}{4}|010\rangle\langle 010|+\frac{1}{2\sqrt{2}}(|010\rangle\langle 100|+|100\rangle\langle 010|)+$$
$$\frac{1}{2}|100\rangle\langle 100|+\frac{1}{4}(1-p)|001\rangle\langle 001|+\frac{1}{4}p|000\rangle\langle 000|+ \quad (8)$$
$$\sqrt{1-p}\left(\frac{1}{4}|001\rangle\langle 010|+\frac{1}{2\sqrt{2}}|001\rangle\langle 100|+\frac{1}{4}|010\rangle\langle 001|+\frac{1}{2\sqrt{2}}|100\rangle\langle 001|\right).$$

One may expect that as a result of dissipation of C $c_2(A,C)$ must be lowered, while $c_2(A,B)$ must remain constant; the finite causality must appear in all the other links. The results of all calculations are presented in Fig. 1 (except the link $A-B$, where all the parameters are constants: $c_2(A,B)=5.30$, $S(AB)=0.811$, $i_{B|A}=-0.233$, $i_{A|B}=0$, $N(A,B)=\frac{1}{4}$).

In Fig. 1a it is seen that indeed $c_2(A,C)$ lowers from 5.30 as p increases and tends to 0 at $p\to 1$. $c_2(A,C)$ is minimal among the others at any p. Indeed, just in the link $A-C$ directionality of causal connection owing to original asymmetry and owing to dissipation is the same and resulting causality turns out the strongest. In the link $B-C$ causality is only due to dissipation and, accordingly, it is weaker: $c_2(B,C)>c_2(A,C)$ at any p. Next, in Fig. 1a it is seen that causality in both the two-particle links $A-C$ and $B-C$ is stronger (c_2 is less) than in the three-particle links $AB-C$, $A-BC$ and $AC-B$. It is explained by the fact that mixedness of the formers is more – the both $S(AC)$ and $S(BC)$ are more than $S(ABC)$ (Fig. 1b). It should be stressed that the relationship of causality and mixedness is only a tendency, but not a rule, e.g. $c_2(B,C)$ and $S(BC)$ both decrease as p increases.

A nontrivial result is that dissipated particle C can belong not only to the party-effect (in the partitions $AB-C$ and $A-BC$), but to the party-cause too (in the partition $AC-B$). At full dissipation ($p=1$) the particle C "disappears" from its two particle party and as a result $c_2(AC,B)=c_2(A,BC)=c_2(A,B)=5.30$.

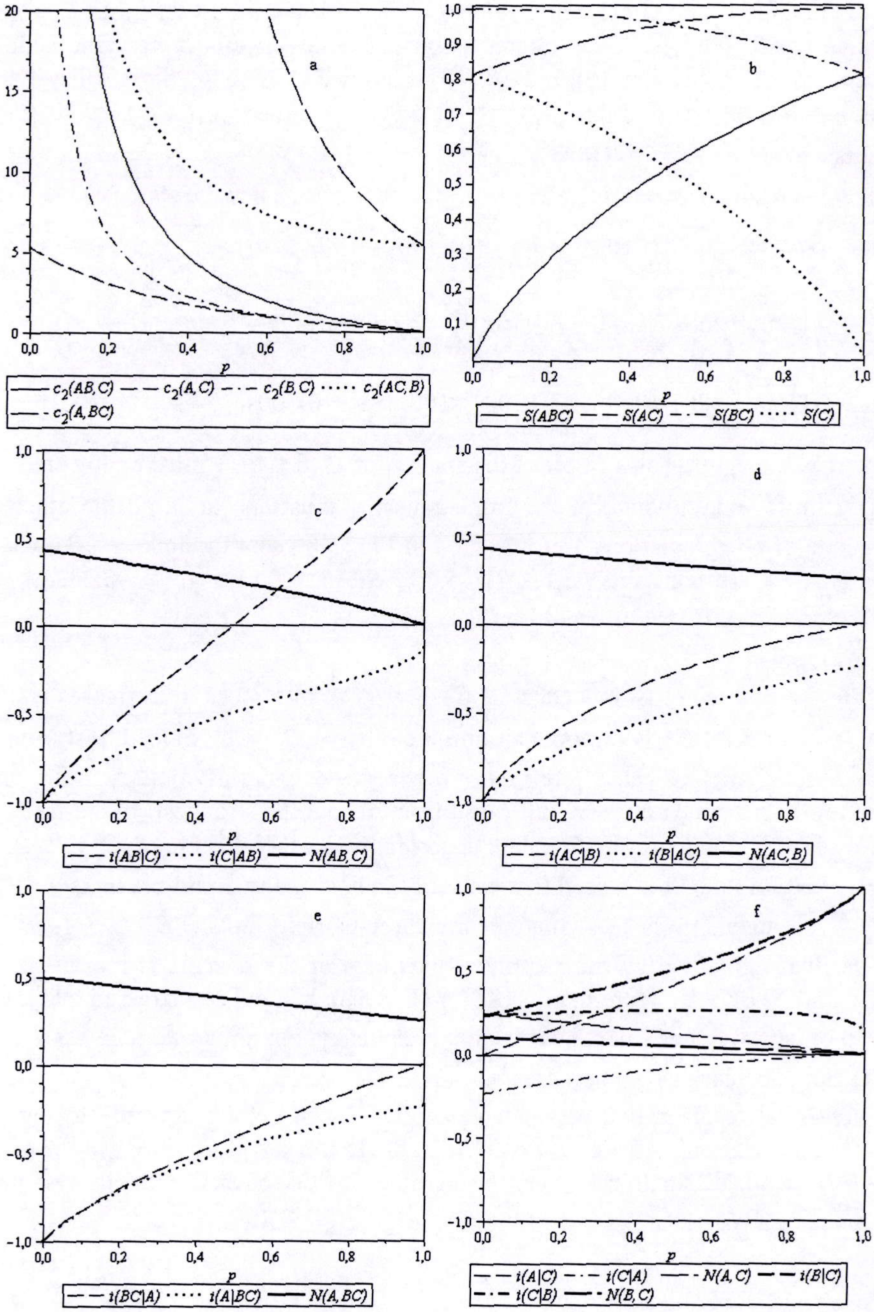

Figure 1. Dependence of c_2 (a), S (b), i and N (c, d, e, f) on degree of dissipation p of the particle C for the different partition of the state (8).

When Fig. 1a is compared with Figs 1c-e it is apparent that for most of the partitions the stronger entanglement the weaker causality: $N(A,C) < N(AB,C) < N(AC,B) < N(A,BC)$ correspond to $c_2(A,C) < c_2(AB,C) < c_2(AC,B) < c_2(A,BC)$. But the partitions $A-B$ and $B-C$ do not obey this relationship. That is the relationship of causality and entanglement is only a tendency, but not a rule too.

In Fig.1f it is seen that in the link $B-C$ at any $p<1$ the both i and N are positive. The subsystem BC is entangled in spite of the entropic classicness.

Now consider dissipation of the particle A:

$$\rho_{CKW}^{dissA} = \frac{1}{4}(|001\rangle\langle 001| + |001\rangle\langle 010| + |010\rangle\langle 001| + |010\rangle\langle 010|) + \frac{1-p}{2}|100\rangle\langle 100| + \frac{p}{2}|000\rangle\langle 000| + \frac{1}{2}\sqrt{\frac{1-p}{2}}(|001\rangle\langle 100| + |010\rangle\langle 100| + |100\rangle\langle 001| + |100\rangle\langle 010|) \quad (9)$$

One may expect that as a result of increasing dissipation of A, the original causal connection $A \to C$ will at the beginning attenuate until disappear at some p, after that direction of causality will reverse with further utmost amplification of the connection $C \to A$ as p will tend to 1. The finite causality must appear in all the other links, except $B-C$ because of its symmetry. The results of calculations are presented in Fig. 2, except the link $B-C$, where all the parameters are constants: $c_2(B,C) \to \pm\infty$, $S(BC)=1$, $i_{B|C} = i_{C|B} = 0.233$, $N(B,C) = \frac{\sqrt{2}-1}{4}$ (the particles B and C are entangled and classically correlated due to availability of the common cause). The partition $AB-C$ is equivalent to the presented one $AC-B$.

In Fig. 2a it is seen that indeed $c_2(A,C)$ changes its sign at $p=\frac{1}{2}$. But the variation of positive $c_2(A,C)$ (corresponding to directionality of the causal connection $A \to C$) proves to be not monotonous; it has the intuitively unexpected minimum equal to 5.08 at $p=0.103$. The monotonous increase of negative $c_2(A,BC)$ simply reflects amplification of causality along with increase of dissipation of the effect A. It is notable that $\min c_2(AC,B) = \min c_2(A,C)$. There is an interesting relation, which is valid not only in this example: $p(\min c_2(AC,B)) = 1 - p(|c_2(A,C)| = \infty) + p(\min c_2(A,C))$.

There is no a relationship of different c_2 with the degree of mixedness (Fig. 2b). There is only a relationship of $c_2(AC,B)$ and $c_2(A,BC)$ with the degree of entanglement (Figs. 2c and 2d): at $p<\frac{1}{2}$ $N(AC,B) < N(BC,A)$ corresponds to $|c_2(AC,B)| < |c_2(A,BC)|$; at $p>\frac{1}{2}$ $N(AC,B) > N(BC,A)$ corresponds to $|c_2(AC,B)| > |c_2(A,BC)|$.

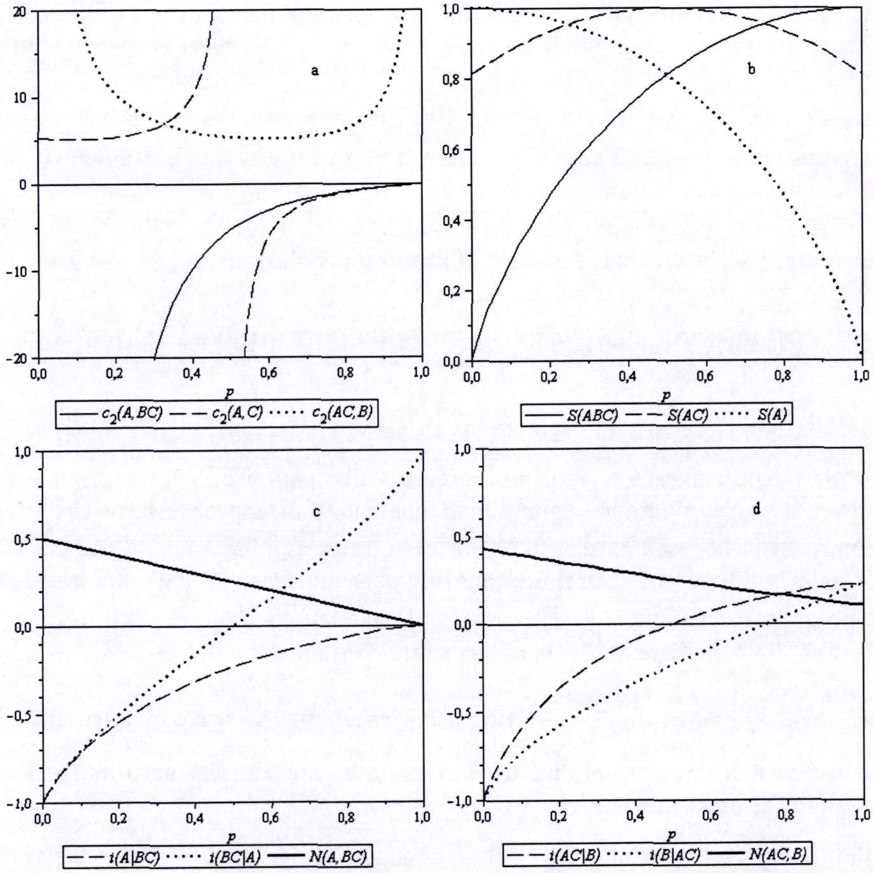

Figure 2. Dependence of c_2 (a), S (b), i and N (c, d) on degree of dissipation p of the particle A for the different partition of the state (9).

In Fig. 2d it is seen that on the interval $\frac{3}{4} < p < 1$ the partition $AC - B$ is classically correlated (the both i are positive), but entangled. The same is observed in the subsystem AC (Fig.3), but on the wider interval $\frac{1}{4} < p < 1$.

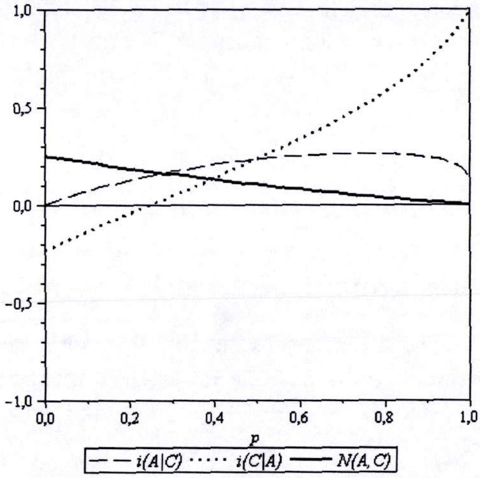

Figure3. Dependence of i and N on degree of dissipation p of the particle A for reduced $\rho(AC)$ of the state (9).

A comparison between the cases $AB-C^{diss}$ and $BC-A^{diss}$ shows:

(i) $|c_2(AB,C^{diss})|\to 0$ at $p\to 1$ quicker than $|c_2(A^{diss},BC)|$. It reflects the influence of the original (at $p=0$) causality in the link $A-C$ (where A is the cause and C is the effect).

(ii) $S(A^{diss}BC)$ increases faster than $S(ABC^{diss})$ as p increases. It means that the dissipation of the original cause enhances mixedness stronger than of the effect. Therefore opening of the system through the cause (information source) is more dramatic than through the effect (information runoff).

(iii) $i_{A^{diss}|BC}\to 0$ at $p\to 1$ quicker than $i_{C^{diss}|AB}$. Therefore dissipation of the original cause quicker destroys quantum correlation than of the effect.

(iv) At $0<p<\frac{1}{2}$ $N(AB,C^{diss})<N(BC,A^{diss})$, but at $\frac{1}{2}<p<1$ $N(AB,C^{diss})>N(BC,A^{diss})$. Therefore dissipation of the original cause destroys entanglement to a greater extent than of the effect.

3.1 WRr-State

In Refs. [28, 29] the different three-particle states related by the symmetry transformations, the particular cases of which are GHZ and W-states, have been investigated. In particular the duplet has been obtained:

$$|WRr\rangle=\frac{1}{\sqrt{6}}(|001>+|010>-2|100>). \qquad (10)$$

In Ref. [12] it has been found that, similar to CKW state, the subsystems AB and AC are causal, a party A is a common cause for B and C, but quantitatively the causality is expressed stronger: $c_2(A,B)=c_2(A,C)=3.43$ (the link $B-C$ is also causeless: $|c_2(B,C)|=\infty$).

Again at the beginning let the particle C is dissipated. Then the state is:

$$\rho_{WRr}^{dissC}=\frac{1}{6}|010\rangle\langle 010|-\frac{1}{3}(|010\rangle\langle 100|+|100\rangle\langle 010|-2|100\rangle\langle 100|)+\frac{1-p}{6}|001\rangle\langle 001|+ \\ \frac{1}{6}p|000\rangle\langle 000|+\frac{\sqrt{1-p}}{6}(|001\rangle\langle 010|-2|001\rangle\langle 100|+|010\rangle\langle 001|-2|100\rangle\langle 001|) \quad (11)$$

The results of calculations are presented in Fig. 4 (except the link $A-B$, where all the parameters are constants: $c_2(A,B)=3.43$, $S(AB)=0.650$, $i_{B|A}=-0.413$, $i_{A|B}=0$, $N(A,B)=\frac{\sqrt{17}-1}{12}$).

It easy to see, that the state (11) qualitatively is similar to (8). The quantitative distinctions are stemmed from stronger causality in (11).

Then consider dissipation of the particle A:

$$\rho_{WRr}^{dissA}=\frac{1}{6}(|001\rangle\langle 001|+|001\rangle\langle 010|+|010\rangle\langle 001|+|010\rangle\langle 010|)+\frac{2}{3}(1-p)|100\rangle\langle 100|+ \\ \frac{2}{3}p|000\rangle\langle 000|-\frac{1}{3}\sqrt{1-p}(|001\rangle\langle 100|+|010\rangle\langle 100|+|100\rangle\langle 001|+|100\rangle\langle 010|) \quad (12)$$

The results of calculations are presented in Fig. 5 except the link $B-C$, where all the parameters are constants: $c_2(B,C)\to\pm\infty$, $S(BC)=0.918$, $i_{B|C}=i_{C|B}=0.412$, $N(B,C)=\frac{\sqrt{5}-2}{6}$, that is again particles B and C are entangled and classically correlated due to availability of the common cause. The partition $AB-C$ is equivalent to $AC-B$.

In contrast to dissipation of C, dissipation of A leads to a number of qualitative distinctions in Fig. 5 as compared to Fig. 2. Contrary to all the above cases, the entropy of dissipated particle $S(A)$ does not decrease monotonously, but has a maximum at $p=\frac{1}{4}$, while mixedness of the whole system $S(ABC)$ does not increase monotonously, but has a maximum at $p=\frac{3}{4}$ (Fig. 5b). Therewith $S(AC)$ is the same (with a maximum at $p=\frac{1}{2}$).

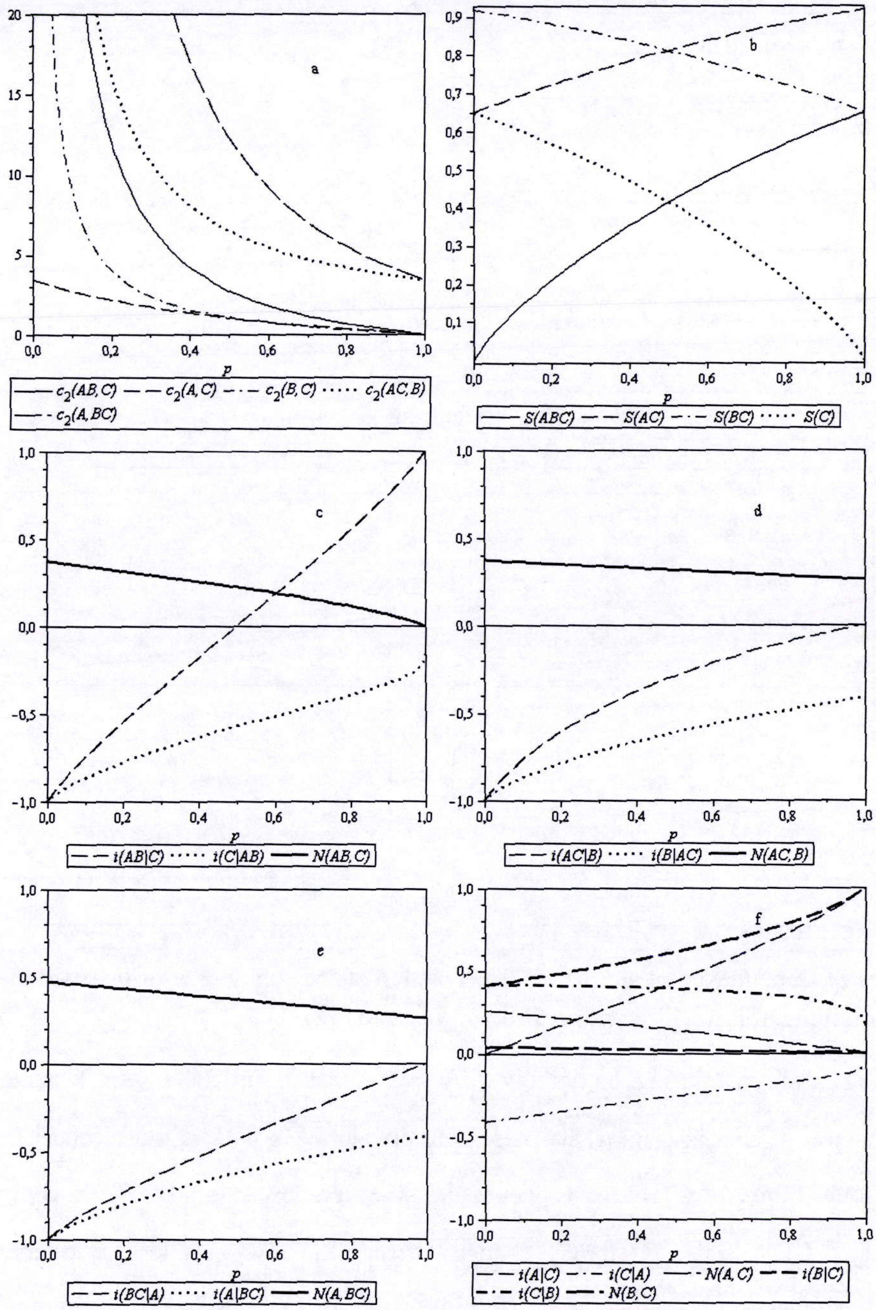

Figure 4. Dependence of c_2 (a), S (b), i and N (c, d) on degree of dissipation p of the particle C for the different partition of the state (11).

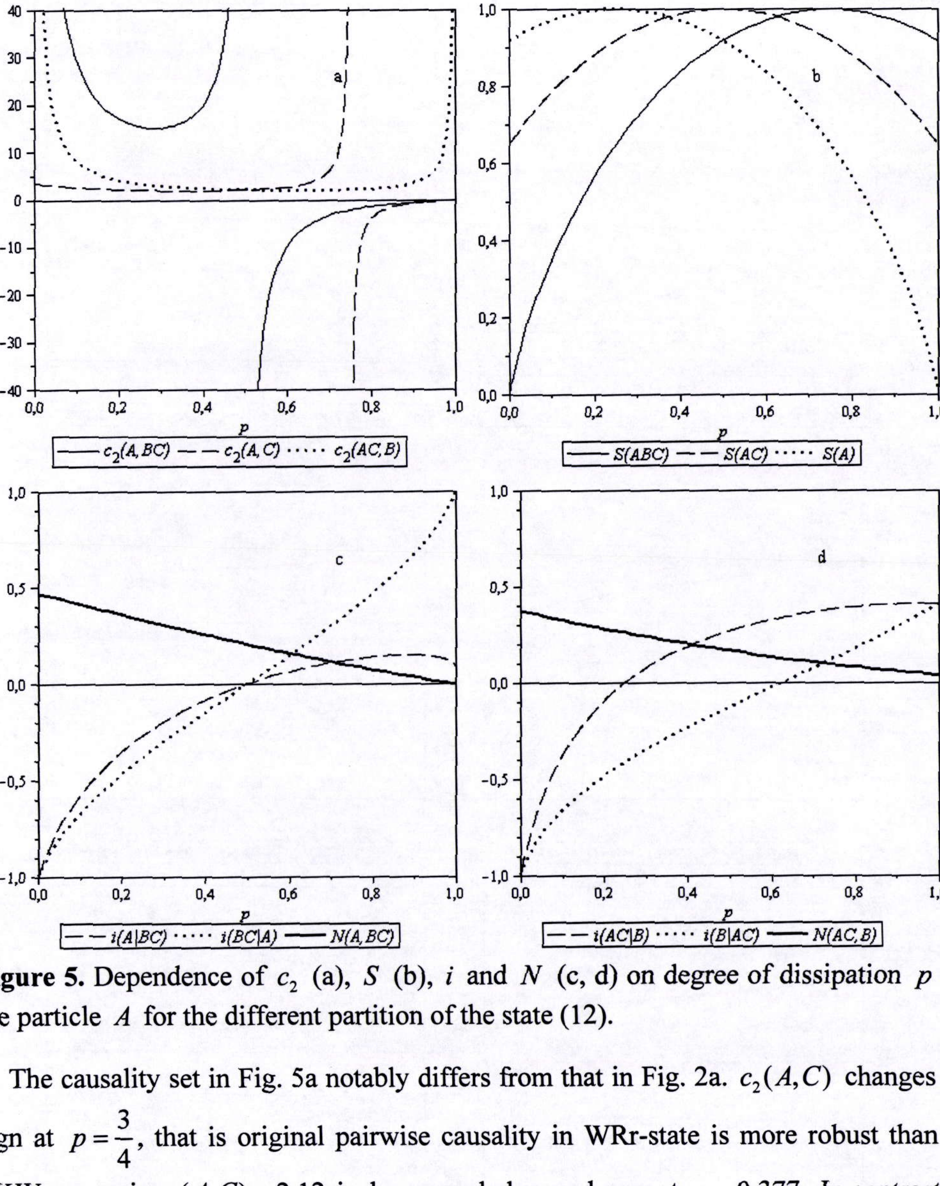

Figure 5. Dependence of c_2 (a), S (b), i and N (c, d) on degree of dissipation p of the particle A for the different partition of the state (12).

The causality set in Fig. 5a notably differs from that in Fig. 2a. $c_2(A,C)$ changes its sign at $p=\frac{3}{4}$, that is original pairwise causality in WRr-state is more robust than in CKW one. $\min c_2(A,C)=2.12$ is deeper and observed now at $p=0.377$. In contrast to dissipated CKW state, $c_2(A,BC)$ changes its sign at $p=\frac{1}{2}$. At less p direction of causal connection is $A\rightarrow BC$, at higher p it is $A\leftarrow BC$. The minimum of $c_2(A,BC)=15.2$ corresponding to $A\rightarrow BC$ is observed at $p=0.288$. The curve of $c_2(AC,B)$ in Fig. 5a is similar to that in Fig. 2a, although $\min c_2(AC,B)=1.97$ at

$p = 0.627$ is not equal to $\min c_2(A,C)$ but their position also obey the relation: $p(\min c_2(AC,B)) = 1 - p(|c_2(A,C)| = \infty) + p(\min c_2(A,C))$.

The same relationship of $c_2(AC,B)$ and $c_2(A,BC)$ with the negativity is observed in Figs. 5c and 5d): at $p < \frac{3}{4}$ $N(AC,B) < N(BC,A)$ corresponds to $|c_2(AC,B)| < |c_2(A,BC)|$; at $p > \frac{3}{4}$ $N(AC,B) > N(BC,A)$ corresponds to $|c_2(AC,B)| > |c_2(A,BC)|$. Note, that the inversion points: $p = \frac{3}{4}$ in this case, and $p = \frac{1}{2}$ in the case of CKW state dissipation, exactly coincide with break points $c_2(A,C) \to \pm\infty$.

In Fig. 5c it is seen that on the interval $\frac{1}{2} < p < 1$ the partition $A - BC$ is classically correlated (the both i are positive, unlike Fig. 2c), but entangled. The same is observed in Fig 5d at $p > 0.625$ for the partition $AC - B$. The subsystem AC (Fig. 6) is classically correlated, but entangled, but on the wider interval $0.375 < p < 1$.

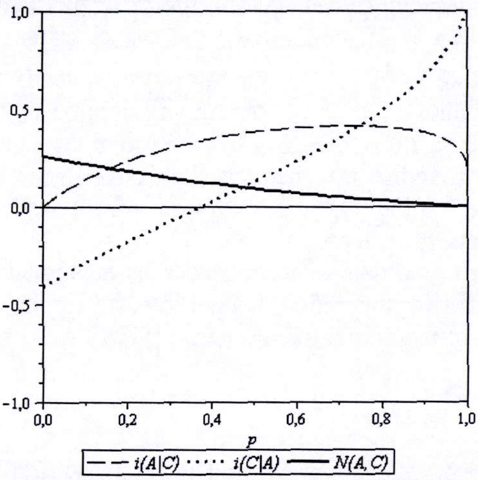

Figure 6. Dependence of i and N on degree of dissipation p of the particle A for reduced $\rho(AC)$ of the state (12).

The conclusions (i) – (iv) concerning dissipated CKW state remains true for WRr one (with less $p = 0.390$ in the quantitative justification of (iv)) and obviously are general.

4 Conclusion

The classical causal analysis, which already has rather rich history of applications, has given two equivalent measures of causality. One of them, the course of time c_2 proved to be adequate for its quantum extension. The direction of causal connection is defined as the direction of irreversible information flow which is determined by the sign of c_2; the quantitative measure of this connection is value of c_2: the absence of causality corresponds to $|c_2| \to \infty$, accordingly, the degree of causal connection is inversely related to $|c_2|$. This formal definition of causality is valid at any time direction. In turn the causality in reverse time implies the direct anticipation.

The possibilities of causal analysis have been demonstrated by a couple of examples of the three-qubit states. The causality parameters for both the examples have been computed and compared with the degree of mixedness and entanglement. In the simplest cases the results of formal causal analysis correspond to the intuitively expected ones, but even at small complication of a quantum system the intuition fails. Thus its employment leads to the nontrivial conclusions about quantum information propagation.

In contrast to the classical case, a finite causality can exist only in the open systems, because a necessary condition of quantum causality is a finite mixedness. Correspondingly consideration of various causal links in the different states has shown that often (although not always) the greater mixedness the less c_2. The mixedness of the asymmetric subsystems inside even closed quantum system leads to their original causal connection. In the originally causal systems the dissipation leads to nontrivial redistribution of the causal connections. Opening of the system through the cause (information source) leads to more mixedness of the state than through the effect (information runoff). Dissipation of the original cause destroys quantum correlations and entanglement to a greater extent than of the effect.

In addition we have found that some states can be entangled but classically correlated. This fact is important for theoretical insight into the results of experiments on macroscopic entanglement of the dissipative systems [13-27].

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