Local and Nonlocal

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Abstract
While nonlocality has been considered by some to be a problem for quantum physics, it is, in fact, an essential component of understanding how *local* interactions actually operate. Locality and nonlocality are fundamental components of a dual system in which each determines the behaviour of the other. The exact characteristics of the different local interactions (weak, strong and electric) can be shown to be completely determined by the nonlocal vacuum structures with which they are associated. At the same time, gravity provides a nonlocal dual to the combined interactions, which has inertia as its local manifestation. A completely integrated description of local interactions and nonlocal vacuum structures is proposed, based on nilpotent quantum mechanics and its unique algebraic structure.
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1 Introduction

A significant problem in quantum physics is in accounting for the nonlocality or instantaneity of quantum correlations, while recognising that all direct physical measurement or observation must be local, or confined to information transmitted at the speed of light or less. Nonlocality, in various forms, has now been demonstrated by the experiments of Aspect[1] and many others, but there persists a view in certain quarters that it is somehow a problematic aspect of quantum physics, one that defies 'common sense' and all previous expectations. This is tied up with the strong feeling that the fact 'information' cannot be transmitted faster than *c* is one of the fundamental principles of physics. In fact, the fundamental duality of nature (which is evidenced in the universal rewrite system proposed as emerging from zero totality[2-4]) requires nonlocality as much as locality. And they are not separate occurrences: local interactions ultimately stem from nonlocal processes. This is evident in the nilpotent version of quantum mechanics, where it is possible to show that both the local and nonlocal aspects of the strong, weak and electric interactions contribute to their special characteristics. In addition, 'information', in the computational sense of a code symbol equivalent to 1 or 0, *can* be transmitted faster than *c* through the spin correlations of entangled states. This is in partial contradiction to the use of the term in physics, where 'information' is usually restricted to the results of bosonic transfer. Really, however, we should define two types of information transfer, local (through bosonic transfer) and nonlocal (through quantum correlation), and both provide 'information' in the computational sense. The holistic nature of physics means that the local cannot, in fact, be separated from the nonlocal. Even the terms can be misleading, because 'local' refers to the entire universe as much as nonlocal does. The nilpotent version of quantum mechanics shows that there is no such thing as an isolated system, and so a complete local description, which originates in the individual particle, will still require knowledge of the contents and disposition of the whole universe. In principle, the difference between local and nonlocal is not in the phenomena they describe, but in the method of description, essentially whether we use an iterative or recursive computational paradigm. It is for this reason that, as proposed by Dubois[5], we should expect to see anticipatory features in many physical systems, especially in ones driven by gravity, which is the most holistic of all the physical interactions. Local interactions are determined by the collective nonlocal effect of the rest of the universe, and, in this sense, could not take place without anticipation. Gravity, however, as we shall argue, has a special anticipatory significance among the four known interactions in providing the most directly observable aspect of nonlocality.

2 Nilpotent Quantum Mechanics

One of the advantages of using nilpotent quantum mechanics[4] is that it gives us a very precise definition of the boundary between the local and the nonlocal. It is important here that the version of quantum mechanics used is relativistic, for without the Lorentzian connection between space and time, and energy and momentum, we cannot really define the local, and, without a proper account of the local, we cannot specify what we mean by nonlocal. The simplest way to derive nilpotent quantum mechanics is to begin with Einstein's energy-momentum conservation equation (with the usual convention that $c = 1$):

$$E^2 - p^2 - m^2 = 0. \qquad (1)$$

We now need to define an algebra to factorize this equation, and, as in previous publications, we use a combined algebra made from the tensor product of four quaternion units and a set of four multivariate vector units to which these are commutative. The four quaternion units, $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$, 1, follow multiplication rules which are the same as those of Pauli matrices:

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1$$
$$\boldsymbol{ij} = -\boldsymbol{ji} = \boldsymbol{k}$$
$$\boldsymbol{jk} = -\boldsymbol{kj} = \boldsymbol{i}$$
$$\boldsymbol{ki} = -\boldsymbol{ik} = \boldsymbol{j}.$$

The multivariate vector units, $\mathbf{i}, \mathbf{j}, \mathbf{k}, i$, are effectively those of complexified quaternions $(i\boldsymbol{i}) = \mathbf{i}$, $(i\boldsymbol{j}) = \mathbf{j}$, $(i\boldsymbol{k}) = \mathbf{k}$, $(i1) = i$, and follow the multiplication rules:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -i\mathbf{ijk} = 1$$
$$\mathbf{ij} = -\mathbf{ji} = i\mathbf{k}$$
$$\mathbf{jk} = -\mathbf{kj} = i\mathbf{i}$$
$$\mathbf{ki} = -\mathbf{ik} = i\mathbf{j}\,.$$

In multivariate vector algebra, which is also described as the Clifford algebra of 3D space, two vectors $\mathbf{a}$ and $\mathbf{b}$ have a full product $\mathbf{ab} = \mathbf{a.b} + i\mathbf{a} \times \mathbf{b}$. Terms like $i\mathbf{i}$, $i\mathbf{j}$, $i\mathbf{k}$ are pseudovectors (e.g. area, angular momentum) and i is a pseudoscalar (e.g. volume). The eight base units have a similar structure to Penrose's twistors, with four real or norm 1 components and four imaginary or norm –1 components. There is a significant difference, however, in that the connection between the units of space and time is a quantum rather than classically relativistic one.

In the combined algebra, each of the four units units, $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$, 1, is commutative to each of $\mathbf{i}, \mathbf{j}, \mathbf{k}, i$. This algebra, which is isomorphic to that of the gamma matrices of the Dirac equation, requires 64 units, which are + and – versions of:

$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$	$i\mathbf{i}$	$i\mathbf{j}$	$i\mathbf{k}$	i	1
$\boldsymbol{i}$	$\boldsymbol{j}$	$\boldsymbol{k}$	$i\boldsymbol{i}$	$\mathrm{i}\boldsymbol{i}$	$i\boldsymbol{k}$		
$\mathbf{i}\boldsymbol{i}$	$\mathbf{i}\boldsymbol{j}$	$\mathbf{i}\boldsymbol{k}$	$i\mathbf{i}\boldsymbol{i}$	$\mathrm{i}\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{k}$		
$\mathbf{j}\boldsymbol{i}$	$\mathbf{j}\boldsymbol{j}$	$\mathbf{j}\boldsymbol{k}$	$i\mathbf{j}\boldsymbol{i}$	$\mathrm{i}\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{k}$		
$\mathbf{k}\boldsymbol{i}$	$\mathbf{k}\boldsymbol{j}$	$\mathbf{k}\boldsymbol{k}$	$i\mathbf{k}\boldsymbol{i}$	$\mathrm{i}\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{k}$		

However, the 64 can be generated from a subset of 5, which can be matched to the 5 gamma matrices in a number of ways, for example:

$$\gamma^0 = i\boldsymbol{k} \qquad \gamma^1 = \mathbf{i}\boldsymbol{i} \qquad \gamma^2 = \mathbf{j}\boldsymbol{i} \qquad \gamma^3 = \mathbf{k}\boldsymbol{i} \qquad \gamma^5 = i\boldsymbol{j}$$

The algebra now allows us to factorize (1) in the form:

$$(i\boldsymbol{k}E + i\mathbf{i}p_x + i\mathbf{j}p_y + i\mathbf{k}p_z + \boldsymbol{j}m)\,(i\boldsymbol{k}E + i\mathbf{i}p_x + i\mathbf{j}p_y + i\mathbf{k}p_z + \boldsymbol{j}m) =$$
$$(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)\,(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m) = 0.$$

If we now apply a canonical quantization procedure to the first bracket in each of these squared expressions, to replace the terms E and $\mathbf{p}$ by the operators $E \to i\partial/\partial t$, $\mathbf{p} \to -i\nabla$ (this time equating $\hbar$ to 1), and assume that the operators act on the phase factor for a free fermion, $e^{-i(Et - \mathbf{p.r})}$, we obtain the nilpotent Dirac equation for a free fermion:

$$\left(\mp \boldsymbol{k}\frac{\partial}{\partial t} \mp i\boldsymbol{i}\nabla + \boldsymbol{j}m\right)\left(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m\right)e^{-i(Et - \mathbf{p.r})} = 0\,. \tag{2}$$

One of the most convenient aspects of using a multivariate vector for the $\mathbf{p}$ or ∇ term is that, as Hestenes showed,[6] it then automatically includes spin (essentially through the extra cross product term in the full product). So, in equation (2), $\mathbf{p}$ is interchangeable with $\boldsymbol{\sigma}.\mathbf{p}$ and ∇ with $\boldsymbol{\sigma}.\nabla$. However, if we should revert to using ordinary vectors at any time, we would have to include an explicit spin or angular momentum term. As in standard relativistic quantum mechanics, four simultaneous solutions are required for the wavefunction – 2 for fermion / antifermion × 2 for spin up / spin down. The conventional formalism uses a 4 × 4 matrix differential operator and a column vector wavefunction, but the nilpotent Dirac equation uses a row vector operator and a column vector wavefunction, each of which may be represented in abbreviated form by $(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)$. In the nilpotent formalism, the four solutions can be represented, typically, as

$(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$	fermion spin up
$(i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m)$	fermion spin down
$(-i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$	antifermion spin down
$(-i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m)$	antifermion spin up

The observed particle state is the first in the column, while the others are the accompanying vacuum states, or states into which the observed particle could transform by respective P, T and C transformations:

$\boldsymbol{P}$	$\boldsymbol{i}\,(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{i}$	$\to$	$(i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m)$
T	$\boldsymbol{k}\,(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k}$	$\to$	$(-i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$
C	$-\boldsymbol{j}\,(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{j}$	$\to$	$(-i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m)$

Replacing fermion state spin up as the observed with any of the others would simultaneously transform all four states by *P*, *T* or *C*. A full description of the nilpotent wavefunction should always include all four terms, but it will often be convenient to specify just the first term, and with the others assumed to be automatic consequences.
The dual status of the energy term $\pm \boldsymbol{ik}E$ originates in the dual nature of the pseudoscalar term $\pm i$, which is purely mathematical in origin. As is evident from (2), the time coordinate (in $\partial / \partial t$) follows the same relative sign, so 'negative energy' particles go backwards in time. In the single observable direction of time allowed by the second law of thermodynamics, all material objects are observed to have positive energy. Negative energy, as we have argued elsewhere, represents vacuum rather than the local quantized state, and the apparent disparity between matter and antimatter in the universe is really a result of the fact that, as in the four-component Dirac wavefunction, one set of states exists in observable real space, and the other in an unobservable 'vacuum space' or 'antispace', as required by zero totality. The dual nature of $\pm i$ is also a factor in creating existence of two states of helicity. Once we have decided on a sign convention for **p**, the spin state of the particle (or, more conventionally, the helicity or handedness **σ.p**) is determined by the ratio of the signs of *E* and **p**. So $\boldsymbol{i}\mathbf{p} / \boldsymbol{ik}E$ has the same helicity as $(-\boldsymbol{i}\mathbf{p}) / (-\boldsymbol{ik}E)$, but the opposite helicity to $\boldsymbol{i}\mathbf{p} / (-\boldsymbol{ik}E)$.
Nilpotent quantum mechanics produces all the standard results of conventional relativistic quantum mechanics, which can easily be obtained by replacing equation (2) with

$$-i\gamma^5\left(\gamma^0\frac{\partial}{\partial t}+\gamma^1\frac{\partial}{\partial x}+\gamma^2\frac{\partial}{\partial y}+\gamma^3\frac{\partial}{\partial z}+im\right)\psi=0$$

Classic results include spin ½, one-handed helicity for weakly interacting states, and the *zitterbewegung* which emerges as an automatic switching process between the four states in the wavefunction, and which is interpreted as a mass-generating switching between the fermion and its antifermion vacuum partner, and the two helicity states, which are already mixed in real fermions.[4] However, it also produces many new ones. Among the most important are the descriptions of three different boson-type states, which are combinations of the fermion state which any of the *P*, *T* or *C* transformed ones, the result being a scalar wavefunction.

$(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\mp \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	spin 1 boson
$(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\mp \boldsymbol{ik}E \mp \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	spin 0 boson
$(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\pm \boldsymbol{ik}E \mp \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	fermion-fermion combination

One of the most significant aspects of this formalization is that a spin 1 boson can be massless, but a spin 0 boson cannot, as then $(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p})\ (\mp \boldsymbol{ik}E \mp \boldsymbol{i}\mathbf{p})$ would immediately zero: hence Goldstone bosons must become Higgs bosons in the Higgs mechanism.

3 Vacuum and the Separation of Local and Nonlocal

The key aspect of nilpotent quantum mechanics, in fact, is not the equation (which is seldom used directly) but the fact that an operator of the form $(ik E + \mathbf{i}\mathbf{p} + jm)$ automatically generates a phase term on which it operates to produce a nilpotent amplitude of the form $(ikE + \mathbf{i}\mathbf{p} + jm)$, that is, one that squares to zero. In fact, it is not necessary to assume that the fermion is free, and we could also incorporate field terms or covariant derivatives into the operator, with, for example, $E \to i\partial / \partial t + e\phi + \ldots$, and $\mathbf{p} \to -i\nabla + e\mathbf{A} + \ldots$ (though often, of course, the $\mathbf{p}$ terms can be transformed by choice of frame). In this case, we could still represent the operator as $(ikE + \mathbf{i}\mathbf{p} + jm)$, but the phase term would no longer be $e^{-i(Et - \mathbf{p}.\mathbf{r})}$. It would be whatever is needed to create an amplitude of the general form $(ikE + \mathbf{i}\mathbf{p} + jm)$, which squares to zero, with the eigenvalues E and $\mathbf{p}$ representing the more complicated expressions that will result from the presence of the field terms.

The nilpotent structure immediately gives us a *formal* way of separating the local from the nonlocal. The bracketed term representing the fermion creation operator or wavefunction determines how conservation of energy applies to that fermion, as squaring the wavefunction and equating to zero gives us back the energy-momentum equation, and, of course, it is local, as the required Lorentzian structure is intrinsic. However, the addition and multiplication of nilpotent wavefunctions construct the nonlocal processes of superposition and combination, and these processes do not require a Lorentzian structure. In effect, anything inside the fermion bracket is local and anything outside it is nonlocal.

Nilpotent quantum mechanics is predicated on the idea that the total structure of the universe is exactly zero. Once we have defined the structure of the wavefunction as a nilpotent we can identify the meaning of Pauli exclusion, which is a fundamentally nonlocal phenomenon. If we imagine creating a fermion wavefunction of the form $\psi_f = (ikE + \mathbf{i}\mathbf{p} + jm)$ from absolutely nothing, then we must simultaneously create the dual term, $\psi_v = -(ikE + \mathbf{i}\mathbf{p} + jm)$, which negates it both in superposition and combination. So we have

$$\psi_f + \psi_v = (ikE + \mathbf{i}\mathbf{p} + jm) - (ikE + \mathbf{i}\mathbf{p} + jm) = 0$$

$$\psi_f \psi_v = -(ikE + \mathbf{i}\mathbf{p} + jm)(ikE + \mathbf{i}\mathbf{p} + jm) = 0$$

We have identified the dual term as the vacuum appropriate to that fermion state, in principle, the rest of the universe needed to maintain a fermion in that particular state.[4,7] Pauli exclusion then tells us that no two fermions can have the same quantum numbers because the combination state would be zero. It also implies that no two fermions can share the same vacuum. Now, vacuum is intrinsically nonlocal. Because the fermion is localized, then the rest of the universe is necessarily nonlocalized. If the fermion is a point, as experiments suggests that it may be, then the rest of the universe is defined as everything outside that point. So the nonlocal connection which makes Pauli exclusion possible can be thought to occur through the vacua for each fermion.

The nilpotent structure clearly demands a holistic approach to physics. When we write down an operator or amplitude in the form $(\pm ikE \pm \mathbf{i}\mathbf{p} + jm)$, the brackets may suggest that we have created a closed system, but in fact the E and $\mathbf{p}$ terms may contain an unlimited number of potentials. We have created a system but it is open. Closure or energy conservation is only maintained over the entire universe, and requires the second law of thermodynamics as well as the first. So, though the bracket may define locality, locality does not imply a closed system. The creation of the fermion state is the creation of a local region in phase space, to which everything else becomes nonlocal; the creation of the two regions is simultaneous. Any subsequent change inside the bracket (a rewriting of the structures of E and $\mathbf{p}$) also affects everything else outside it, and vice versa.

In the nilpotent formalism, the vacuum can be structured, directly reflecting that of matter. If we take $(\pm ikE \pm i\mathbf{p} + jm)$ and post-multiply it by the idempotent $k(\pm ikE \pm i\mathbf{p} + jm)$ any number of times, the only effect is to introduce a scalar multiple, which can be normalized away.

$$(\pm ikE \pm i\mathbf{p} + jm)\ k(\pm ikE \pm i\mathbf{p} + jm)\ k(\pm ikE \pm i\mathbf{p} + jm) \ldots \rightarrow (\pm ikE \pm i\mathbf{p} + jm)$$

The same applies with post-multiplication by $j(\pm ikE \pm i\mathbf{p} + jm)$ or $i(\pm ikE \pm i\mathbf{p} + jm)$ (except for an extra unit vector, in the last case, which disappears on every second multiplication). The three idempotent terms have the mathematical characteristics of vacuum operators. They also correspond to the respective transformations of alternate brackets by T, C and P, or to the production of the spin 1, spin 0 and fermion-fermion bosonic states. The character of the original fermionic state remains unchanged, while vacuum versions of the three bosonic states are created. In principle, the quaternionic operators k, j, i, like T, C and P, which are associated respectively with E, m and $\mathbf{p}$, split the continuous vacuum into discrete units, and we can to some extent regard these units as the respective 'charges' or sources of the weak, electric and strong interactions, acting to create the vacuum necessary for these forces to act.

4 The Coulomb Interaction

Locality and nonlocality are, in every respect, dual processes. It is impossible to imagine one without the other. Though we think of interactions in a local sense, they also have equivalent nonlocal aspects. These are expressed in the symmetry groups for the interactions, and these can be derived from the nilpotent structure. The translation to the local effects comes about when we use the same structure to determine the potentials to be added to the operators E and $\mathbf{p}$ in the fermionic nilpotent. (We can consider decoherence to be an example of the reverse transformation, using local potentials, for example, to remove the nonlocal quantum coherence.) The simplest of all interactions is the Coulomb interaction, which follows the $U(1)$ symmetry group and is part of the structure of each of the electric, strong and weak forces. In the case of the electric force it is the entire interaction; for the strong force it is the one gluon exchange; and for the weak force it provides the hypercharge and the B^0 gauge field, which creates the combination with the electric force in the Weinberg-Salam electroweak theory.

The $U(1)$ symmetry group comes from the characterization of a fermion as a point source with spherical symmetry. Effectively, it is the statement that spherical symmetry is preserved and demonstrated by rotation whatever the length of the radius vector used. It is a purely scalar symmetry defined only by the magnitude of the charge, or source of the interaction. It is equivalent, in effect, to defining a coupling constant for the interaction, which maintains its value independent of the distance from the source. The nonlocal effect which is responsible for this symmetry can be identified as the one which is least specific and most general for all fermions: Pauli exclusion.

Now, a free fermion has no interactions, by definition. An interacting fermion is one that has been brought within the range of influence (or field) of another or others. If fermions are point particles and their influence is spherically symmetric, then it will be convenient to express the influence of one point-source on another by changing the coordinates of the 'receiving' particle from Cartesian to polar, with the point-particle source at the centre of the coordinate system also defined as the centre of physical influence. To use polar coordinates, it is convenient to revert to ordinary, rather than multivariate vectors. We can then use a version of Dirac's standard prescription[8] for converting the differential operator to polar coordinates, with the explicit inclusion of a fermionic spin / angular momentum term:

$$\boldsymbol{\sigma}.\nabla = \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \pm i\frac{j+½}{r} .$$

This means that the nilpotent differential operator now takes the form:

$$(i\boldsymbol{k}E - \boldsymbol{i}i\nabla + \boldsymbol{ij}m) \rightarrow \left(\boldsymbol{k}E + \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j+½}{r}\right) + i\boldsymbol{j}m\right) .$$

The fundamental condition necessary to assign this operator to a fermion state is that it maintains Pauli exclusion and leads to a nilpotent solution when applied to a phase factor. This leads to the local manifestation of the $U(1)$ symmetry. It can be seen, simply by inspection, that it will be impossible to obtain a nilpotent solution (i.e. a nilpotent amplitude) with any phase factor unless the operator $i\boldsymbol{k}E$ includes potential energy term varying with $1 / r$ to cancel out the effect of that in the $\boldsymbol{i}i$ part of the operator. The definition of one point-fermion as being within the range of another (or, in alternative terms, within the nonlocal *vacuum* of another) requires the existence of a potential energy term with the characteristic features of a Coulomb interaction, leading to an inverse linear potential and an inverse square force. All four interactions have a component of this kind, and all three parts of the nilpotent structure contribute, as all are required to fulfil the nilpotent condition. The symmetry, of course, is most closely associated with the electric interaction, because this force has no other component, just as the $\boldsymbol{j}m$ term has no other aspect than scalar magnitude. The fact that one term in the nilpotent operator has no structure other than magnitude ensures that it must be possible to have an interaction with no symmetry other than $U(1)$.

The addition of the inverse linear potential to $i\boldsymbol{k}E$ now changes the nonlocal $U(1)$ (nilpotent or Pauli exclusion) condition to a local one, since the potential is added *inside* the bracket. The solution is well known and is very quickly effected using the nilpotent condition. It is significant because it provides a model for the more complicated strong and weak interactions, which have not otherwise succumbed to analytical calculation. With the added potential, the nilpotent operator will be of the form

$$\left(\pm \mathbf{k}\left(E - \frac{A}{r} \right) \pm \mathbf{i}\left(-a + \frac{\gamma}{r} + \frac{\nu}{r} + \ldots \frac{1}{r} \pm i \frac{j + \frac{1}{2}}{r} \right) + \mathbf{ij}m \right) . \qquad (3)$$

To establish that it is truly nilpotent, we have to find the phase factor to which the operator must apply to result in a nilpotent amplitude. The phase factor is, of course, the well known

$$\phi = e^{-ar_{\nu}{}^{\gamma}} \sum_{\nu=0} a_{\nu} r^{\nu} .$$

To find the amplitude, we simply apply the differential operator (3), and, to prove that it is nilpotent, we immediately square and equate to zero:

$$4\left(E - \frac{A}{r} \right)^2 = -2\left(-a + \frac{\gamma}{r} + \frac{\nu}{r} + \ldots \frac{1}{r} + i \frac{j + \frac{1}{2}}{r} \right)^2 - 2\left(-a + \frac{\gamma}{r} + \frac{\nu}{r} + \ldots \frac{1}{r} - i \frac{j + \frac{1}{2}}{r} \right)^2 + 4m^2 .$$

The positive and negative $i(j + \frac{1}{2})$ terms notably cancel out over the four solutions.
The solution is completed in only a few more steps. Equating constant terms leads to

$$a = \sqrt{m^2 - E^2}$$

Equating terms in $1/r^2$, following the standard procedure, with $\nu = 0$, gives:

$$\left(\frac{A}{r} \right)^2 = -\left(\frac{\gamma + 1}{r} \right)^2 + \left(\frac{j + \frac{1}{2}}{r} \right)^2 .$$

Assuming the power series terminates at n', again following standard procedure, and equating coefficients of $1/r$ for $\nu = n'$,

$$2EA = -2\sqrt{m^2 - E^2}\,(\gamma + 1 + n'),$$

and

$$\frac{E}{m} = \frac{1}{\sqrt{1 + \frac{A^2}{(\gamma + 1 + n')^2}}} = \frac{1}{\sqrt{1 + \frac{A^2}{\left(\sqrt{(j + \frac{1}{2})^2 - A^2} + n' \right)^2}}} .$$

With $A = Ze^2$, this becomes the well-known 'hydrogen atom' solution for a one-electron nuclear atom or ion with hyperfine structure.

The main purpose of deriving this well-known result is to show that the local Coulomb interaction, and its very particular consequences, can be shown to be the result of a nonlocal symmetry ultimately stemming from the fact that, at the quantum level, local sources are point-like with unique vacua, that are distinguished by application of the nilpotent condition. It is completely consistent with the idea, established since the time of Immanuel Kant, that inverse-square forces result purely from spherical symmetry. A similar reasoning can be applied to both the strong and the weak interactions, where standard solutions are not yet established.

5 The Strong Interaction

The strong interaction begins in a combination state, which reflects the *vector* nature of the **p** term in the nilpotent wavefunction. Effectively, the vector aspect requires a source term and corresponding vacuum with three components. A combination is outside the bracket and so is nonlocal. Nilpotency, of course, prevents us writing a combination of the form

$$(ikE + i\mathbf{p} + jm)\ (ikE + i\mathbf{p} + jm)\ (ikE + i\mathbf{p} + jm)$$

which would automatically zero. However, if we separate out three components of **p** in the form

$$(ikE + i\mathbf{i}p_x + jm)\ (ikE + i\mathbf{j}p_y + jm)\ (ikE + i\mathbf{k}p_z + jm)$$

then we have a nonzero combination. Spin is defined in only one direction at a time, so, at any given instant, the wavefunction will reduce to something like $(ikE + i\mathbf{i}p_x + jm)$ $(ikE + jm)$ $(ikE + jm)$, which, after normalization, becomes simply $(ikE + i\mathbf{i}p_x + jm)$, or to $(ikE + jm)$ $(ikE + i\mathbf{j}p_y + jm)$ $(ikE + jm)$, which becomes $(ikE - i\mathbf{j}p_y + jm)$, or to $(ikE + jm)$ $(ikE + jm)$ $(ikE + i\mathbf{k}p_z + jm)$, which becomes $(ikE + i\mathbf{k}p_z + jm)$.

The sign change in the second term is significant. Since we need to maintain the symmetry between the three directions of momentum, there will be six possible outcomes, using both + and – values of momentum terms, that is, a superposition of six combination states:

$$\begin{array}{lcl}
(ikE + i\mathbf{i}p_x + jm)\ (ikE + jm)\ (ikE + jm) & \rightarrow & (ikE + i\mathbf{i}p_x + jm) \\
(ikE - i\mathbf{i}p_x + jm)\ (ikE + jm)\ (ikE + jm) & \rightarrow & (ikE - i\mathbf{i}p_x + jm) \\
(ikE + jm)\ (ikE + i\mathbf{j}p_y + jm)\ (ikE + jm) & \rightarrow & (ikE - i\mathbf{j}p_y + jm) \\
(ikE + jm)\ (ikE - i\mathbf{j}p_y + jm)\ (ikE + jm) & \rightarrow & (ikE + i\mathbf{j}p_y + jm) \\
(ikE + jm)\ (ikE + jm)\ (ikE + i\mathbf{k}p_z + jm) & \rightarrow & (ikE + i\mathbf{k}p_z + jm) \\
(ikE + jm)\ (ikE + jm)\ (ikE - i\mathbf{k}p_z + jm) & \rightarrow & (ikE - i\mathbf{k}p_z + jm)
\end{array}$$

The six 'phases' represented here must all be valid at the same time, as required by the rotation symmetry of space or of gauge invariance. The symmetry is recognisably that of the group $SU(3)$, with the same structure as the three symmetric and three antisymmetric 'colour' combinations in the standard representation:

$$\psi \sim (\mathrm{BGR} - \mathrm{BRG} + \mathrm{GRB} - \mathrm{RGB} + \mathrm{RBG} - \mathrm{GBR}).$$

The simultaneous existence of all phases means that *individual* quarks, and such identifying characteristics as electric charges, are not identifiable by their spatial positions (unlike the proton and electron constituting a hydrogen atom), thus explaining, for example, why the neutron has no electric dipole moment. Just as $U(1)$ establishes that spherical symmetry of a point source requires the rotation to be performed independently of the length of the radius vector, so $SU(3)$ requires the rotation to be independent of the coordinate system used. In terms of Noether's theorem, while $U(1)$ conserves the magnitude of angular momentum, $SU(3)$ conserves the direction.

The $SU(3)$ symmetry represents a nonlocal interaction between the parts of the wavefunction or 'quarks' incorporating p_x, p_y, and p_z. This interaction is independent of the physical separation of the components (and they must be spatially or temporally separated to have different local and nonlocal manifestations). The local effect of this is a transfer of **p** between the three brackets of each wavefunction. As it is local, with **p** changing within each bracket, this is not instantaneous, unlike the actual combinations and superpositions, and so there will be a nonzero local force or rate of change of momentum. However, because it does not depend on separation, this force will be constant. The local manifestation of a constant force is a potential that varies linearly with distance. As the $U(1)$ condition for spherical symmetry / Pauli exclusion / nilpotence is also a requirement, then the minimal potential to describe this local strong interaction is a combination of terms that are linear and inverse linear with distance. It may be significant that the linear potential of the strong interaction is the only one that is optional to the fermion state, the nilpotency not being dependent directly on the vector nature of **p**.

Now it is significant that, because we have both positive and negative **p** terms for the same E term, the six phases incorporate a (maximal) superposition of left- and right-handed components, thus explaining the zero observed chirality in the interaction. This means that the mass term m is necessarily nonzero, though the unbroken gauge invariance which results from no momentum direction being preferred, means that, in quantum field terms, the boson mediators managing the transitions between the six component states must be massless, as well as spin 1. The so-called mass gap problem has been a significant one in particle physics for a number of decades, and is part of one of the mathematical problems for which the Clay Institute has offered prizes. It would seem that nilpotency suggests that the problem is now within our grasp. Significantly, also, the solution would appear to be that of the Higgs mechanism, which supplies mass to all the fermions. Many commentators have argued that the bulk of the mass of a proton comes from the boson or gluon exchange and not from the Higgs mechanism. Our analysis would suggest, however, that they are in fact the same process.

The $SU(3)$ symmetry requires the existence of eight generators which can be identified with six bosons of the form:

$(\pm ikE \mp \boldsymbol{i}\,\mathbf{i}p_x)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{j}p_y)$ $\quad$ $(\pm ikE \mp \boldsymbol{i}\,\mathbf{j}p_y)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{i}p_x)$

$(\pm ikE \mp \boldsymbol{i}\,\mathbf{j}p_y)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{k}p_z)$ $\quad$ $(\pm ikE \mp \boldsymbol{i}\,\mathbf{k}p_z)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{j}p_y)$

$(\pm ikE \mp \boldsymbol{i}\,\mathbf{i}p_z)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{i}p_x)$ $\quad$ $(\pm ikE \mp \boldsymbol{i}\,\mathbf{i}p_x)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{i}p_z)$

and two combinations of the three bosons of the form:

$(\pm ikE \mp \boldsymbol{i}\,\mathbf{i}p_x)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{i}p_x)$ $\quad$ $(\pm ikE \mp \boldsymbol{i}\,\mathbf{j}p_y)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{j}p_y)$

$(\pm ikE \mp \boldsymbol{i}\,\mathbf{k}p_z)\,(\mp ikE \mp \boldsymbol{i}\,\mathbf{k}p_z)$

These structures are identical to an equivalent set in which both brackets undergo a complete sign reversal,

$$(\mp ikE \pm \boldsymbol{i}\,\mathbf{i}p_x)\,(\pm ikE \pm \boldsymbol{i}\,\mathbf{i}p_y) \quad \text{or} \quad (\pm ikE \pm \boldsymbol{i}\,\mathbf{i}p_y)\,(\mp ikE \pm \boldsymbol{i}\,\mathbf{i}p_x), \quad \text{etc.,}$$

and, with normalization, to ones equivalent to a massless fermion acting on a 'strong' vacuum,

$$(\pm ikE \pm \boldsymbol{i}\,\mathbf{i}p_x)(\pm ikE \mp \boldsymbol{i}\,\mathbf{i}p_y) = (\pm ikE \pm \boldsymbol{i}\,\mathbf{i}p_x)\,\boldsymbol{i}\,(\pm ikE \pm \boldsymbol{i}\,\mathbf{i}p_y)\,\boldsymbol{i}.$$

A 'colour' transitions can be seen as either an exchange of the components of **p** between the individual quarks, or a relative switching of quark positions. The colours, therefore, either move with the respective p_x, p_y, p_z components, or switch with them. The effect of either process is the same, and a sign reversal in **p**, as with $\boldsymbol{i}\,(\pm ikE \pm \boldsymbol{i}\,\mathbf{i}p_x)\,\boldsymbol{i}$, is an additional necessary result.

To establish that the local interaction produced by the combination of direct and inverse linear potential results in a force with the characteristics of the strong interaction, we need to extend the Coulomb solution of for this new nilpotent operator. In principle, it will be the same whether we are referring to a quark-antiquark combination, with a temporal cycle of the components p_x, p_y, and p_z, and a separation of quark and antiquark, or to a combination of three quarks, with a separation of each from the centre of the system. In either case, we will produce an operator of the form:

$$\left(\pm \boldsymbol{k}\left(E + \frac{A}{r} + Br\right) \mp \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j+½}{r}\right) + \boldsymbol{i}\boldsymbol{j}m\right) \tag{4}$$

Once again, we need to identify the phase factor to which this operator applies, but a relatively simple extension of the one for the Coulomb potential suggests that its form is:

$$\phi = \exp\left(-ar - br^2\right) r^{\gamma} \sum_{\nu=0} a_{\nu} r^{\nu} .$$

Applying operator (4), and the nilpotent condition, we obtain

$$E^2 + 2AB + \frac{A^2}{r^2} + B^2 r^2 + \frac{2AE}{r} + 2BEr = m^2$$

$$-\left(a^2 + \frac{(\gamma+\nu+\ldots+1)^2}{r^2} - \frac{(j+½)^2}{r^2} + 4b^2r^2 + 4abr - 4b(\gamma+\nu+\ldots+1) - \frac{2a}{r}(\gamma+\nu+\ldots+1)\right)$$

with the positive and negative $i(j + ½)$ terms again cancelling out over the four solutions. Then, once again assuming a termination in the power series (as we did with the Coulomb solution), we can equate:

coefficients of r^2 to give $\quad B^2 = -4b^2$
coefficients of r to give $\quad 2BE = -4ab$
coefficients of $1/r$ to give $\quad 2AE = 2a(\gamma+\nu+1)$

The immediate consequences of these equations are:

$$b = \pm\frac{iB}{2}$$

$$a = \mp iE$$

$$\gamma + \nu + 1 = \mp iA .$$

The ground state case (where $\nu = 0$) then requires a phase factor of the form:

$$\phi = \exp\left(\pm iEr \mp iBr^2/2\right) r^{\mp iqA-1} .$$

We can identify the imaginary exponential terms in ϕ as representing asymptotic freedom; the exp ($+,^{-}$ iEr) is typical for a free fermion. The complex $r^{\gamma-1}$ term can be restructured as a component phase, $\chi(r) = \exp(\pm\, iqA \ln(r))$, which varies more slowly with r than the rest of ϕ. We can therefore express ϕ in the form:

$$\phi = \frac{\exp(kr + \chi(r))}{r},$$

where

$$k = \pm iE \mp iBr/2.$$

At high energies, and small r, the first term dominates, which approximates to a free fermion solution or asymptotic freedom. At low energies, however, when r is large, the second term, which incorporates the confining potential Br, dominates, leading to infrared slavery. The Coulomb term, which emerges purely from spherical symmetry, is the component which defines the strong interaction phase, $\chi(r)$, and this can be related to the directional status of **p** in the state vector. Once again, a nonlocal symmetry, related to the nilpotent structure, determines the known characteristics of a local interaction.

6 The Weak Interaction

We have so far discussed nonlocal combination and superposition states that lead to the local Coulomb and strong interactions. There is, however, in all fermion states an additional superposition resulting from the spinor structure of the Dirac wavefunction, and the *zitterbewegung* that connects the superposed states. The fermion wavefunction has two fermion and two antifermion states, each representing two directions of spin or helicity. While the co-existence of two spin states can be thought of as, in some sense, 'real', and accounted for by the presence of mass, the co-existence of two energy states can only be meaningfully understood in the context of the simultaneous existence of fermion and vacuum. A fermionic source cannot be separated from its vacuum partner. So a fermion or antifermion cannot be created or annihilated, even with an antifermionic or fermionic partner, unless its vacuum is simultaneously annihilated or created.

The nonlocal connection between fermion and antifermion, or fermion and vacuum, which ultimately emerges from the dual nature of the pseudoscalar aspect of the term ikE, can be seen as creating a kind of virtual dipole, which will necessarily have local consequences. In the case of the *zitterbewegung*, this involves a switching between fermion and antifermion states which is an aspect of the *weak* interaction, in which one state is annihilated and another created by a boson exchange, the only interaction in which such a process occurs. So, even though the transitions between the two energy states, ikE and $-ikE$, may be virtual, the *zitterbewegung* would seem to require the production of an intermediate bosonic state at a vertex where one fermionic state is annihilated and another is created to replace by it. In fact, all weak interactions operate according to the same mechanism, and the mediator is a spin 1 boson $(\pm \boldsymbol{i}\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ $\boldsymbol{k}(\pm \boldsymbol{i}\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\boldsymbol{k}$, equivalent to the state produced by a fermion acting on a 'weak' vacuum. Fermions are the only particles which act as weak sources, and the fact that they are a kind of dipole under the weak interaction suggests that all weak sources act, in effect, as weak dipoles, and the immediate manifestation of this dipolar nature occurs with the fermion's ½-integral spin. Though the superposition of fermion and antifermion states is nonlocal, we can expect it to have local consequences for a fermion nilpotent. Locally, a dipole manifests itself through a squaring of the distance dependence of the force. So inverse square forces become inverse fourth power dipoles and linear forces become squared distance dipoles. There is also the possibility that the existence of multiple dipoles will produce multipole forces.

It would seem that the weak interaction, like the electric and strong interactions, is built into the structure of the nilpotent operator, and its nature is determined by that of the pseudoscalar *iE* operator, whose sign uniquely determines the helicity of a weakly interacting particle, or more specifically its weakly interacting component. So, the question then becomes: how do we accommodate a local interaction which emerges from this nonlocal superposition? The answer requires us to investigate the nilpotent solutions that would emerge from an operator incorporating a Coulomb potential together with any other potential which is a function of distance from a point source with spherical symmetry, apart from the linear potential which is characteristic of the strong interaction. So, let us assume a nilpotent operator with a form such as

$$\left(\mathbf{k}\left(E-\frac{A}{r}-Cr^{n}\right)+\mathbf{i}\left(\frac{\partial}{\partial r}+\frac{1}{r}\pm i\frac{j+½}{r}\right)+\mathbf{ij}m\right).$$

where n is an integer greater than 1 or less than –1. As before, we will need to find a phase factor which will make the amplitude nilpotent. Again, we will seek to extend the Coulomb solution, using the information that polynomial potential terms which are multiples of r^n require the incorporation into the exponential of terms which are multiples of r^{n+1}. So, we may suppose that the phase factor is of the form:

$$\phi=\exp\left(-ar-br^{n+1}\right)r^{\gamma}\sum_{\nu=0}a_{\nu}r^{\nu}.$$

As in the previous examples, we apply the operator and square to zero, with a termination in the series, to obtain

$$4\left(E-\frac{A}{r}-Cr^{n}\right)^{2}=-2\left(-a+(n+1)br^{n}+\frac{\gamma}{r}+\frac{\nu}{r}+\frac{1}{r}+i\frac{j+½}{r}\right)^{2}$$
$$-2\left(-a+(n+1)br^{n}+\frac{\gamma}{r}+\frac{\nu}{r}+\frac{1}{r}-i\frac{j+½}{r}\right)^{2}$$

Equating constant terms, we find:

$$a=\sqrt{m^{2}-E^{2}}$$

Equating terms in r^{2n}, with $\nu = 0$, gives:

$$C^{2}=-(n+1)^{2}b^{2}.$$

Equating coefficients of r, where $\nu = 0$:

$$AC=-(n+1)\,b\,(1+\gamma),$$
$$(1+\gamma)=\pm\, iA.$$

Equating coefficients of $1 / r^2$ and coefficients of $1 / r$, for a power series terminating in $\nu = n'$, leads to

$$A^2 = -(1 + \gamma + n')^2 + (j + ½)^2$$

and

$$-EA = a\,(1 + \gamma + n').$$

Combining these results produces:

$$\left(\frac{m^2 - E^2}{E^2}\right)(1 + \gamma + n')^2 = -(1 + \gamma + n')^2 + (j + ½)^2$$

$$E = -\frac{m}{j + ½}(\pm iA + n').$$

The last equation has the form of a harmonic oscillator, with evenly spaced energy levels deriving from integral values of n'. If we now make an additional assumption that A, which is the phase term required for spherical symmetry, has its origin in the random directionality of fermionic spin, we might assign to it a half-unit value ($\pm ½\, i$), or ($\pm ½\, i\hbar c$ if we explicitly include the constants), and obtain the complete formula for the fermionic simple harmonic oscillator:

$$E = -\frac{m}{j + ½}(½ + n').$$

The constant A has the dimensions of charge (q) squared, or interaction energy × range, and if numerically equal to $\pm ½\, \hbar c$ has exactly the value required by the uncertainty principle, if the range of an interaction mediated by the Z boson is calculated at $\hbar / 2M_Zc = 2.166 \times 10^{-18}$ m, as observed. In this interpretation, we can describe the *zitterbewegung* as a dipolar switching between fermion and vacuum antifermion states, with a weak dipole moment $(\hbar c / 2)^{3/2} / M_Zc^2$, of magnitude $8.965 \times 10^{-18}\, e$ m, and a weak magnetic moment, of order 4.64×10^{-5} × the magnetic moment of the electron. The possible appearance of an imaginary factor i in A has an interesting relation to the fact that a complex potential or vacuum is required for CP violation in the pure weak interaction. The dual state $\pm \boldsymbol{ik}E$ which produces the dipolar switching is also manifestly an $SU(2)$ symmetry, and this can be related to the fact that the spherical symmetry of the point source proceeds from the fact that it is independent of the handedness of the rotation, which, in terms of Noether's theorem, becomes the conservation of the handedness of angular momentum.

There are, however, two different $SU(2)$ symmetries involved, though they are related. The $SU(2)$ of spin describes two helicity states, left- and right-handed. However, there is another $SU(2)$ symmetry, weak isospin, which describes a fermionic weak interaction as being independent of whether or not an electric charge is present and generating its own contribution to mass. In effect, weak isospin tells us that the weak interaction, though often occurring simultaneously and in combination with the electric interaction, has an independent origin. In the case of weak isospin, the $SU(2)$ is a switching between different ratios of left- and right-handedness, and so of mass, determined by the presence or absence of the electric charge in one of the two states. So a superposition of, say, $\alpha_1(\boldsymbol{ik}E_1 + \mathbf{i}\mathbf{p}_1 + \boldsymbol{j}m_1) + \alpha_2(\boldsymbol{ik}E_1 - \mathbf{i}\mathbf{p}_1 + \boldsymbol{j}m_1)$ might become $\beta_1(\boldsymbol{ik}E_2 + \mathbf{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$ +

$\beta_2(ikE_2 - \mathbf{ip}_2 + jm_2)$, with both spin 1 and spin 0 vertices, the additional spin 0 vertex (the one which changes the mass) being equivalent to a fermion $(\pm ikE \pm \mathbf{ip} + jm)$ acting on an 'electric' vacuum of the form $-j(\pm ikE \pm \mathbf{ip} + jm)\,j$. In effect, a combination of the electric vacuum operator (j) and the weak one (k) produces a partial transition in the sign of $\mathbf{p}$. Significantly, while all the forces contribute to the production of the scalar mass term, the electric force is the only which contributes to nothing else.

Overall, the calculation shows that an additional potential of the form Cr^n, where n is an integer greater than 1 or less than –1, that is any potential emerging from a system in which there is complexity, aggregation, or a multiplicity of sources, leads to a harmonic oscillator solution for the nilpotent operator, irrespective of the value of n, and, in fact, any polynomial sum of potentials of this form would produce the same result. A dipolar weak sources would require a minimum extra term of the form Cr^{-3}, and the harmonic oscillator solution would provide the precise characteristics that we would expect such a weak source would produce. It is significant that, apart from the Coulomb and strong interaction solutions, this is the only one which a fermion interaction specified in relation to a spherically symmetric point source can produce. We have, in effect, shown that there are only three local physical manifestations which result from the nonlocal combinations and superpositions associated with a nilpotent fermion, and these are the ones associated with the electric (or other pure Coulomb), strong and weak interactions. They are also aspects which can be derived from the *algebraic structure* of the three terms, jm, $\mathbf{ip}$, and ikE, which make up the nilpotent operator or wavefunction.

7 Gravity and the Nature of Mass

Gravity presents a different problem from the other three forces. Here, we have the $U(1)$ symmetry present in all the other forces, but at least two facts distinguish it from them. One is the fact that gravity has not yet been successfully quantized. The other is that the value of the cosmological constant predicted from a quantum-style argument is 10^{123} times higher than experimental data, based on 'dark energy', would suggest. If, as Seth Lloyd suggests on the basis of the holographic principle, there are 10^{123} possible bits of data in the observable universe,[9,10] then the prediction is as wrong as it could conceivably be, suggesting that we should look for an answer in a totally different direction. In fact, it suggests, that, rather than applying to a single quantum state (local), it actually applies to the whole universe (nonlocal). Other indications which point in this direction are the gravity-gauge theory correspondence now being seriously considered by string theorists, which suggests that gravity is somehow a totality of all the other gauge forces, a fact which has always seemed obvious in the context of the nilpotent structure, where the vacua of the weak, strong and electric forces seem to be derived by partitioning of the general gravitational vacuum. In addition, the Higgs field, the zero-point energy, the cosmic microwave background radiation, and even ordinary fields point to a continuity in mass (i.e. mass-energy, as the source of gravity) which would explain the fact that, unlike charges, it is unable to change sign. The Higgs field might well suggest that the amount of mass-energy is the same at every point in space, and that changes occur only in its method of manifestation because of the presence of charges of various kinds.

Everything points to the fact that gravity is really a nonlocal property, a manifestation of the total vacuum, $-(\pm ikE \pm \mathbf{ip} + jm)$, even in the negative potential energy with which it is associated, and is not a local force, as is often maintained, though it has, of course, a local manifestation. In this case, gravity would be the carrier of all the information separately available from the other three forces, and, in fact, of all their local manifestations, but it would be their inverse rather than their summation, exactly as reflected in its negative energy, and we could even regard the attractive nature of the gravitational interaction between like particles as a *result* of its being a vacuum, rather than a local force, and therefore requiring negative energy. It would seem from this analysis that the local manifestation associated with gravity is the positive energy inertial local reaction, which is what we really observe and equate to the gravity of localized clumps of matter, and which, for a fermion we could represent by the nilpotent structure $(\pm ikE \pm \mathbf{ip} + jm)$ itself. This force, however, is fictitious and repulsive, which means that it could be quantized, as a repulsive force requires a spin 1 gauge boson, rather than the spin 2 particle which creates the renormalization problems associated with quantum gravity.

We have shown previously that, if gravity is instantaneous and nonlocal, but physical *observation* is local, involving time-delayed luminal or subluminal interaction, we can no longer use the Lorentzian space-time of a local coordinate system in the description of gravity.[11] To do so effectively creates a noninertial frame for the gravitational system, with the resulting appearance of fictitious inertial forces. Relativistic and 'gravomagnetic' effects would appear but would be properties of the local coordinate system and not due to the gravitating source. One such effect would be an acceleration-dependent inductive force analogous to that appearing in electromagnetic theory:

$$F = \frac{G}{c^2 r} m_1 m_2 \sin\theta \frac{dv}{dt} \tag{5}$$

Sciama[12] proposed using a real force of this kind to explain inertia. He assumed that the inertia of a body of mass $m = m_1$ could be derived from the action of the total mass $m_u = m_2$ within a Lorentzian event horizon of radius r_u, making the force in equation (5) equal to Kma, with K a constant. However, in our understanding, the continuous mass-field or vacuum which provides gravitational nonlocality must also define the standard by for a unit inertial mass for the entire universe, in the same way as the almost constant gravitational field **g** allows us to define a unit mass at the Earth's surface. In this case, the mass m_u will define a radial inertial field of constant magnitude from the centre of a given local coordinate system to the event horizon defined by r_u, at the same time as the gravitational field (Gm_u / r_u^2), independently of the local coordinate system, defines a unit of gravitational mass within the same radius. Allowing isotropy to remove the angular dependence, we obtain:

$$\frac{Gm_u}{c^2 r} \frac{dv}{dt} = \frac{Gm_u}{r_u^2}$$

The calculation produces both an acceleration

$$a = \frac{dv}{dt} = v\frac{dv}{dr} = \frac{c^2 r}{r_u^2}$$

and, by integration with respect to *r*, a velocity related to the Hubble constant, H_0:

$$v = \frac{cr}{r_u} = H_0 r$$

In terms of this constant, the acceleration now becomes:

$$a = \frac{v^2}{r_u} = H_0^2 r$$

For a nonlocal gravity, this acceleration would be a fictitious one, the result of using a local Lorentzian coordinate system to model the instantaneous interaction. It is a fundamental physical process, analogous to a centrifugal force, which determines the cosmology (however we interpret this) rather than being a result of it. Combining it with the gravitational force due to total mass *m* at any distance, allows us to calculate an equivalent vacuum density, ρ, from

$$F = \frac{Gm}{r^2} - H_0^2 r = \left(\frac{4}{3}\pi G\rho - H_0^2\right) r$$

In terms of a Poisson-Laplace equation, this becomes:

$$\nabla^2\phi = 4\pi G\left(\rho - \frac{3H_0^2}{4\pi G}\right) = 4\pi G\left(\rho + \frac{3P}{c^2}\right) = 4\pi G(\rho - 3\rho_{vac}),$$

The vacuum density here,

$$\rho_{vac} = \frac{H_0^2}{4\pi G},$$

then becomes equivalent to a 'dark' energy density or negative pressure

$$-P = \frac{H_0^2 c^2}{4\pi G},$$

which can also be expressed through the cosmological constant

$$\lambda = 8\pi G\rho_{vac} = 2H_0^2$$

Defining the critical density for a 'flat' universe as

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

we obtain

$$\frac{\rho_{vac}}{\rho_{crit}} = \frac{2}{3}.$$

As stated previously, this value is within the limits of current observations, but was obtained twenty years before the earliest measurements from supernovas.[4,11,13-15]
Of course, if inertia is local, it must be quantizable, even if nonlocal gravity is not. Here, we may make use of a discrete gravity theory by de Souza and Silveira,[4,16] based on a

concept of *extended causality*, which is a development of the special relativistic 4-vector space-time, including the proper time, which in our notation would be represented by a nilpotent ($\boldsymbol{ikt} + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$), analogous to ($\boldsymbol{ikE} + \boldsymbol{i}\mathbf{p} + \boldsymbol{jm}$). In this theory, a single object (either a particle or field) at two points in Minkowski space-time (s) must satisfy the causality constraint $\Delta\tau^2 + \Delta s^2$. In our interpretation this becomes $\Delta(\boldsymbol{ikt} + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau)^2 = 0$. 'Extended causality' applies when we shift τ and x by infinitesimal steps $d\tau$ and dx. Applying a massless scalar gives us a discrete field equation, and a field source represented by a scalar charge to generate a 'graviton'-like object and a metric for a discrete gravitational field. In our terms, the field is a repulsive inertial one, fully quantized via the Dirac nilpotent, and the 'graviton'-like object is correspondingly identified as a spin 1 boson or photon-like pseudo-boson. The field can only be quantized in this way because it is inertial, with positive energy, not gravitational, with negative energy, as negative energy, in our understanding, represents nonlocal vacuum rather than the local quantized state. In effect, we have reversed the argument used for the other forces, working from vacuum to the quantized local state.

8 Conclusion

The fundamental basis of the work in this paper has been a long-held conviction that physics ultimately requires just four parameters, space, charge, time, mass, which are symmetrical elements of a noncyclic group of order 4, and which are respectively characterized by multivariate vector, quaternion, pseudoscalar and scalar algebras, or, in Cliffordian terms by vector, bivector, trivector and scalar algebras. The combination of the four algebras (with units **i**, **j**, **k**; ***i***, ***j***, ***k***, *i*; 1) gives us the 64-part algebra of the fundamental physical unit, the fermionic state, with a minimal set of five generators, typically ***ik***; **i*i***; **j*i***; **k*i***; ***ij***. The process of creating these generators, however, by 'compactification' of the units, requires the creation of new physical quantities: quantized energy (E), quantized momentum (**p**) and rest mass (m). At the same time the symmetry of one of the two 3-dimensional quantities (space or charge) must be broken. In practice, it is charge, and the weak, strong and electric components of charge take on the respective characteristics of pseudoscalar, vector and scalar algebra.[4]

In effect, the modification of charge shows the nonlocal or vacuum side of the compactification process, while the compactification to energy, momentum and rest mass shows the local. The algebraic characteristics acquired are manifested nonlocally through the vacua associated with the energy, momentum, and rest mass components of the nilpotent wavefunction. The different algebraic characteristics of the three components then ultimately determine the nature of the local interactions which result from the local symmetries. We have shown that it is possible to go all the way from the nonlocal vacua to the recognizable physical effects which are characteristic of the different forces.

At the same time, gravity presents a kind of inverse effect to that of all the other forces, as we can see from the negative energy involved. While we can do the calculations for weak, strong and electric forces from the perspective of the local structure of the fermion state, and work iteratively to determine the effect of the rest of the universe, with gravity we have to do the calculation universally or recursively to work back to the inertial effect on the quantized fermionic state.

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