A Delayed Gamma Process for Bridge Lifetime Assessment and Maintenance

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Abstract
The gamma process is a stochastic process with independent non-negative increments having gamma distributions. It is an infinite collection of probability distributions, correlated in a manner suitable for modeling gradual wear and degradation over time. Degradation generally consist of cumulative amounts of deterioration and the advantage of the gamma process is recognized by Jan M. van Noortwijk in the 1990's and applied in many structural studies. We introduce a novel stochastic process we call a *delayed gamma process*, designed for the estimation of infrastructure lifetime. An overview of the development of the model is given and the methods for estimation are reviewed.
Keywords : gamma process, structural deterioration, Reliability.

1 Introduction

It is difficult to estimate the deterioration rate of bridge elements. Each bridge is unique, subject to various physical and chemical stresses. There are a number of factors that affect the condition of a bridge, for example the traffic load. Road traffic is hard to estimate and is often approximated. The time and quality of construction, the environment, the management policies, all contribute differently to ageing in bridge elements. To be effective in estimating a bridge lifetime, one must develop mechanistic-based deterioration models to predict the micro-response of the structure. For the purpose of bridge management, macro level models are built. They are of a statistical nature, used to estimate the deterioration rates. Bridge Management Systems are established for the purpose of managing effectively large stocks of bridges. The most notable one is Pontis, a bridge management system developed at the request of the U.S. Federal Highway Administration. Pontis was developed via contracting to two consulting companies and in collaboration with six U.S. states. Pontis was designed based on previous work in maintenance. A pavement management system was successfully deployed for the State of Arizona to produce optimal maintenance policies for the 7,400-mile network of highways (Golabi et al, 1982). At the heart of the system is a Markov decision model. In a similar approach, a Markov chain was made to drive the Pontis bridge management system. The Markov model formulation is appealing because it provides a framework that

Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-930396-15-6

accounts for the uncertainty and the optimal policies can be obtained by solving simple programming problems. However, a number of criticisms have been made against the usefulness of the model (Madanat et al, 1995; Frangopol et al, 2001). Among the issues raised, the Markov chain has a restrictive stationarity assumption by which the time effect is not introduced effectively. There have been attempts at relaxing the assumptions of the Markov chain model, for example Ng and Moses (1998). A more recent approach, developed for the state of New York by Agrawal et al. (2010) uses the Weibull distribution for the calculation of deterioration rates of bridges. This approach represents a fundamental departure from the traditional Markov chain decision model. In a similar exercise, we seek a different approach from that of Pontis. We develop a novel model based on the gamma process. The advantage of the gamma process is recognized and applied in structural studies by van Noortwijk and co-authors (Pandey and van Noortwijk, 2004; van Noortwijk et al, 2005; Pandey at al, 2007). We recognize the applicability of the stochastic process and use it for the calculation of deterioration rates of bridge elements. In the next sections, we introduce the gamma process and discuss the estimation of its parameters. We then develop the new model as an extended version of the gamma process.

2 Modeling Deterioration Using the Gamma Process

Deterioration in structures is caused by corrosion penetration and similar inherent degradation processes. For example, a large sulphate attack, freeze-thaw action, alkali silica reaction, chloride ingress and carbonation lead to the deterioration of concrete structures (Gaal, 2004; van Beek et al.,2003). Time dependent stochastic processes are sequences of random variables that can be used to model these deteriorations. A particularly convenient stochastic process for modeling the cumulative aspect of deterioration of structures is the gamma process. The gamma process can be found in its modern application to structures in the late 1990's by van Noortwijk (1998) and van Noortwijk and Klatter (1999). In some structural material, empirical studies showed that the expected deterioration in some cases followed the power law at^b, where t is the time. This function of time is incorporated into the gamma process and used to model structural deterioration (van Noortwijk and Klatter, 1999). The gamma process is a sophisticated statistical model that is well suited for modeling the temporal deterioration of components of structures. It gives the stochastic flexibility that other models lack, while remaining a mathematically tractable model. van Noortwijk (2009) provides a comprehensive overview of the use of the gamma process in maintenance of structures.

In bridge management and following a practice that has been established in the maintenance of civil infrastructures, the state of an element of a bridge is judged upon visual inspection to be in one of n states. n can be 4, 5 or up to 9. Regular inspections are conducted and repair decisions are made based on the conditions of the elements on the structures. One of the possible condition states, say the first one, represents the 'as new' condition, no-deterioration state, while the other condition states mark increasing levels of deterioration. Among recorded inspection entries are q, the total quantity of the inspected bridge element, q_1 the quantity in condition state 1, q_2 the quantity in state 2, to q_n the quantity in condition state n. The inspected quantities for each element are measured either in square meters (m^2) if it is a surface, in meters (m) for some elements such as railing and joints, or some other units (Manamperi et al, 2009). To better study the deterioration of bridge elements, the data of the observed condition states are converted to continuous variables. The condition data $(q_1, \ldots, q_n)$ of an element are converted to a univariate measure C, using the notion of 'Condition Index'. A number of condition indices can be formulated. These indices are related to the California bridge health index (Shepard and Johnson, 1999), a ranking system that takes values in [0,100]. The California Department of Transportation was involved in the development and implementation of Pontis. The variable C can be modeled with a gamma process.

The gamma process was first applied by the Australian scientist Moran (1954) to model water flow into a dam. Abdel-Hameed (1975) was the first to propose the gamma process as a deterioration model. The gamma process is suitable for modeling gradual wear and degradation. Degradation generally consist of cumulative amounts of deterioration and the advantage of the gamma process is recognized and is applied in structural studies. In the context of structural deterioration, the gamma process is defined (van Noortwijk, 2009) as follows: Let $v(t)$ be a non-decreasing, right continuous, real-valued function for $t \geq 0$, with $v(0) = 0$. The gamma process with shape function $v(t) > 0$ and scale parameter $u > 0$ is a continuous-time stochastic process $\{X(t), t \geq 0\}$ with the properties; (1) $X(0) = 0$ with probability 1, (2) $X(\tau) - X(t) \sim G(v(\tau) - v(t), u)$ and (3) $X(t)$ has independent increments, where $G(x|v, u) = u^v x^{v-1} e^{-ux}/\Gamma(v)$ is the gamma probability density function defined for $x \in (0, \infty)$. The process can be parameterized. Letting $v(t) = \mu^2 t^q/\sigma^2$ and $u = \mu/\sigma^2$, the mean and variance of the deterioration $X(t)$ are $E(X(t)) = \mu t^q$ and $V(X(t)) = \sigma^2 t^q$. We consider the condition $C(t)$ of a structural element to be a function of the deterioration $X(t)$, where $X(t)$ is modeled by a gamma process. $C(t) = r_0 - X(t)$ where r_0 is the starting condition, often considered to be 100%. $r_0 - \mu t^q$ is the mean of the stochastic process representing the condition of the structural element under consideration. To illustrate, we apply the process to a concrete element on a bridge and observe the parameter estimates $\hat{q} = 2$, $\hat{\mu} = 0.37$, $\hat{\sigma} = 0.105$ (Figure 1). Two data points are used; condition 99.26% is observed at time 15.09 years and condition 98.15% at time 17.12 years with a full 100% condition last observed at time 13.5 years. Time zero is the start of the data recording. This example shows a typical condition path of some structural elements on a bridge. The rates of deterioration vary and the shape of the curve can also be convex if it is a case of less severe growth in degradation, accommodated by a q value less than 1 in the gamma process. The flexibility of the gamma process and its ability to capture deterioration

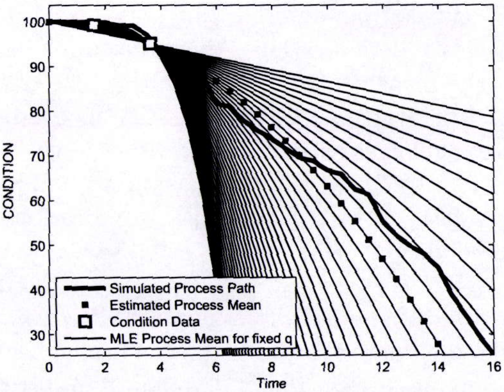


Fig. 1: Gamma process estimation: $\hat{q} = 2$ $\hat{\mu} = 0.37$ $\hat{\sigma} = 0.105$

(Aboura et al, 2009) take the modeling of bridge lifetimes to the stochastic process level.

3 Estimation of the Stochastic Process Parameters

There are three approaches in estimating the parameters of the gamma process; (1) Method of Moments (Cinlar et al, 1977), (2) Maximum Likelihood Estimation (Cinlar et al, 1977) and (3) Bayesian estimation (Dufresne et al, 1991). The first two methods give the same estimate for μ, but different values for σ. To estimate the parameters μ, σ and q, one uses deterioration data in the form of amounts $x_i, i = 1, ..., n$ observed during inspection times $0 = t_0 < t_1 < t_2 < \ldots < t_n$. The Likelihood function is constructed from the independent observations that are gamma distributed. Given a set of observations of the deterioration process $X(t)$, $\{x_i\}_{i=1}^n$ for times $\{t_i\}_{i=1}^n$, the likelihood function is a function of the three parameters μ, σ and q. The range of q can be made as large as desired, is descitized, taken one value at a time and the likelihood function is maximized over μ and σ. We choose that value of q that leads to an overall maximum likelihood function value. Cinlar et al. (1977) provide the maximum likelihood estimates for q fixed. The Maximum Likelihood estimate of μ is $\hat{\mu} = x_n / t_n^q$ and $\hat{\sigma}$, the Maximum Likelihood estimate of σ, is solution of

$$\sum_{i=1}^{n}(t_i^q - t_{i-1}^q)\{\Psi(\frac{\hat{\mu}^2}{\sigma^2}(t_i^q - t_{i-1}^q)) - \log\delta_i\} = t_n^q \log(\frac{x_n}{t_n^q \sigma^2}) \quad (1)$$

Although $\hat{\sigma}$ is not provided in a closed form solution, it is relatively easy to solve the 1-dimensional equation. The MLE algorithm is to set $q = q_1, \ldots, q_m$. For each q, compute $\hat{\mu}$ and $\hat{\sigma}$ and compute $\log(\mathcal{L}(\hat{\mu}, \hat{\sigma}, q))$. Then we set $\hat{q}$ such that

$\log(\mathcal{L}(\hat{\mu},\hat{\sigma},q))$ is maximized. The MLE parameters are $\hat{q}$ and corresponding $(\hat{\mu},\,\hat{\sigma})$. We experimented with the process using simulation and estimating the parameters with the maximum likelihood approach (Aboura et al, 2009).

3.1 Maximum Likelihood Estimates in the General Case

The data for each individual structure aren't enough to estimate the parameters μ, σ and q properly. Often, in a 15 years data, there will be about 7 to 8 inspection times for a structure, leading to 7 or 8 element condition values. When using a stochastic process as a deterioration model, one looks for renewal points and the deterioration data following those renewal points in time. At most, there could be 2 or 3 renewals in the 15 years, and quite commonly 1 or none. This translates into condition path of few data points. In other words, in the data $(\{x\}_{i=1}^{n}, \{t\}_{i=1}^{n})$, n would be 1, 2 or at most 3. This information can still provide an estimate, but the uncertainty about those estimates would be large. One way around this problem, and the advantage of using the gamma process, is to aggregate the data by dividing the bridges into similarity classes. Bridges can be grouped so that the element in question that exists on them behaves similarly on those bridges. Influencing factors, such as traffic load, age, region and environmental stress are used to conduct the stratification. Once the bridges are divided into classes, the corresponding data result in better assessment and prediction of the lifetimes. Due to the independent increments property of the gamma process and with the assumption of conditional independence between the element's data on the different structures within a class, the likelihood function can be written and maximized to estimate the parameters μ, σ and q. The likelihood function is extended from a single element to multiple elements by considering m independent elements, $j = 1, \ldots, m$, for which n_j inspections are performed resulting in n_j independent deterioration increments (Nicolai et al, 2007). The likelihood is

$$\prod_{j=1}^{m}\prod_{i=1}^{n_j} f_{X(t_{i,j})-X(t_{i-1,j})}(\delta_{i,j}) = \prod_{j=1}^{m}\prod_{i=1}^{n_j} G(\delta_{i,j}|\frac{\mu^2[t_{i,j}^{q}-t_{i-1,j}^{q}]}{\sigma^2}, \frac{\mu}{\sigma^2}) \tag{2}$$

where $x_{i,j}$ is the cumulative amount of deterioration at the i^{th} inspection time $t_{i,j}$ for the j^{th} component and $\delta_{ij} = x_{i,j} - x_{i-1,j}$ is the i^{th} deterioration increment for the j^{th} element. This formulation leads to the solution of the estimation problem when an element exists on many structures that are part of a structure class and when data exist on the deterioration paths of these elements. It also applies to the case where an element is brought back to the 'as good as new' condition upon maintenance work, and eventually starts to deteriorate again, creating a second path. That second path can be included in the likelihood of eq. 2. This is due to the independence of paths for the same element once a renewal occurs. The Maximum Likelihood estimates of μ is:

$$\hat{\mu} \;=\; \frac{\sum_{j=1}^{m}\sum_{i=1}^{n_j}\delta_{ij}}{\sum_{j=1}^{m}\sum_{i=1}^{n_j} w_{ij}} \tag{3}$$

and $\hat{\sigma}$ is solution of

$$\sum_{j=1}^{m}\sum_{i=1}^{n_j} w_{ij}\{\Psi(\frac{\hat{\mu}^2}{\sigma^2}w_{ij}) - \log\delta_{ij}\} = \sum_{j=1}^{m}\sum_{i=1}^{n_j} w_{ij}\log(\frac{\hat{\mu}}{\sigma^2}) \tag{4}$$

where $w_{ij} = t_{i,j}^q - t_{i-1,j}^q$. The MLE algorithm proceeds by fixing q, determining $(\hat{\mu}, \hat{\sigma})$, computing the likelihood function value for $(q, \hat{\mu}, \hat{\sigma})$. The q value that provides the maximum likelihood value is the estimates $\hat{q}$.

3.2 Method of Moments

According to Cinlar et al. (1977) the method of moments estimates are:

$$\hat{\mu} = \frac{x_n}{t_n^q} \tag{5}$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(\delta_i - \hat{\mu}w_i)^2}{t_n^q(1 - \frac{\sum_{i=1}^{n} w_i^2}{(\sum_{i=1}^{n} w_i)^2}}} \tag{6}$$

van Noortwijk and Pandey (2004) provide the derivation. We extend the derivation to the case of several elements, more precisely to the case of several deterioration paths. The Method of Moments estimates for the general case are:

$$\hat{\mu} = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n_j}\delta_{i,j}}{\sum_{j=1}^{m}\sum_{i=1}^{n_j} w_{ij}} \tag{7}$$

$$\hat{\sigma}^2 = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n_j}(\delta_{ij} - \bar{\delta}w_{ij})^2}{[\sum_{j=1}^{m}\sum_{i=1}^{n_j} w_{ij} - \frac{\sum_{j=1}^{m}\sum_{i=1}^{n_j} w_{ij}^2}{\sum_{j=1}^{m}\sum_{i=1}^{n_j} w_{ij}}]} \tag{8}$$

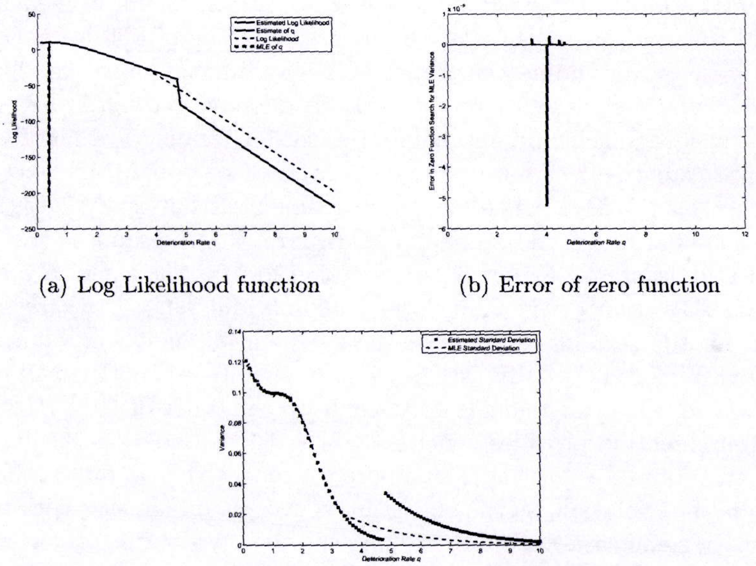
These estimates are easy to compute. They serve as initial estimates for μ and σ for fixed q, and as input in a search for the zero of the function of eq. 4.

3.3 Robust MLE Computations

In computing the Maximum Likelihood Estimates of q, μ and σ, one must find the zero of the function in eq. 4. For each q value, the MLE of μ is computed analytically and the MLE of σ requires the application of an algorithm for the search of the zero of a function. In addition to this last operation being iterative, the search algorithm may not converge, thus creating a robustness issue in computations. A possible approximation is the replacement of the MLE of σ by its Method of Moment estimate (MME). The MME and MLE estimates are the same for μ, but in general they differ for σ. We choose to approximate the MLE of σ by it MME. We conducted an analysis of the use of this approximation and found that in most cases it provides the same estimates for the three parameters at the optimality point in the parameter space. To do the analysis, recall that the final estimates are obtained, according to the MLE criteria, by finding the triplet $(\hat{q}, \hat{\mu}, \hat{\sigma})$ that maximizes the likelihood function $\mathcal{L}(\hat{\mu}, \hat{\sigma}, q)$, or equivalently the log-likelihood function $\log(\mathcal{L}(\hat{\mu}, \hat{\sigma}, q))$. We compute the exact log-likelihood and maximize it using a numerical method. Then we use the approximation where we replace the MLE of σ by its MME and compare the results. In many cases, when the data is smooth, that is the deterioration paths follow a similar trend, the MLE and MME for σ are the same at the point of optimality. In addition, the log-likelihood function for q is the same in both cases and the estimated variances are very close. The only difference is where the error in the search for the zero in the function of eq. 4 is a bit higher in values of q in which the MME replaced the MLE estimates for σ when the search algorithm didn't converge. Even so, the error remains very small, to the order of 10^{-10}. In all such cases, the approximation provides a perfect match to the true log-likelihood, and therefore to the optimal solution. This approach makes the overall application a very robust and fast solution, as the algorithm converges in all cases with an exact final solution. In some cases, using the robust solution, we do not obtain a perfect match in all points of the log-likelihood function. Looking at the error, we can see the values of q at which the substitution of the MME of σ is made for the MLE, and we observe consequently that the corresponding log-likelihood values are a bit off from the true values. However, at the point of optimality, where it matters, the match is perfect and therefore the solution is good (Figure 2). A similar case is encountered when the error in estimating the likelihood is more pronounced when the MLE of σ is replaced by its MME. However, again, the solution is perfect as the match is perfect at the optimality point. Many cases where studied, and the most typical noticeable departure from the log-likelihood still leads to a perfect match at the optimality point. A computational problem often occurs when q is large. The log-likelihood function does not compute at some points. The values are returned as ∞ by the computational routine at these values of q. While one can study the exact reason for this and try to remedy, the point is that often one does not consider very high values of q. When q is high, it means that there is possibly a very sharp drop in the condition, an event that may be considered catastrophic. By forcing q to be limited to the right, one does not loose much in estimation. If q optimal reaches the highest allowed value, it usually means that the user has to return to the data and study whether parts of it can be excluded from the analysis. Most likely some outliers may be explained by some special event and do not belong in the analysis where a 'normal' trend is sought.

4 The Delayed Gamma Process

Following the analysis of bridge inspection data and after extracting all relevant condition information, taking into account renewals in the maintenance process, it



(a) Log Likelihood function (b) Error of zero function

(c) Estimated variance

Fig. 2: Typical error in the log-likelihood approximation

was observed that all elements follow a time period where they stay in full condition then start deteriorating (Aboura and Samali, 2011). The full condition time period can be modeled as zero when the first inspection after a renewal reveals a positive deterioration amount. It is very likely that no full condition time is zero. However, since it is not possible to know the real start of deterioration, one considers a random variable that represents the last time the condition was observed to be 100%. Statistical investigations in both parts of the process, the full condition period and the deterioration period, were conducted. They resulted in two statistical models (Aboura and Samali, 2011). One is a probability distribution for the time in full condition and the other is a gamma process. The amalgamation of the two parts creates a new stochastic process. The novel stochastic process characteristics are derived in the remaining of the section to provide a lifetime assessment model for the element, bridge and network conditions.

4.1 Derivation of the New Model

Let $C(t)$ be the condition of an element at time t after a renewal, $0 \leq C(t) \leq 100$, $C(0) = 100$. Let T_d be the time to the start of deterioration, from a renewal (Figure 3). T_d is the first observable time before a deterioration is observed, looking backward in time. It is the last time the element was in full condition, and observed to be, before a deterioration is recorded. T_d is an observable whose data can be

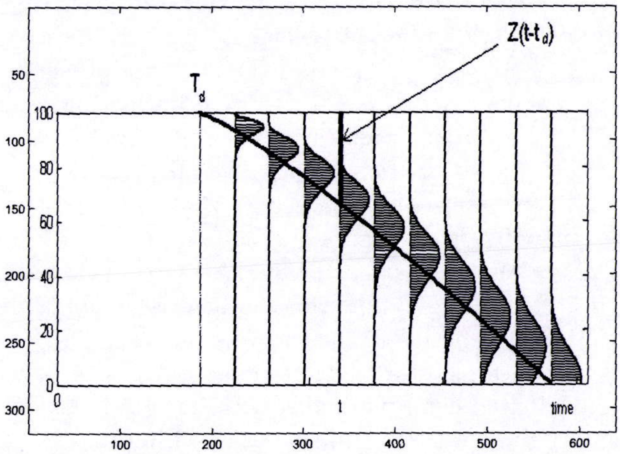
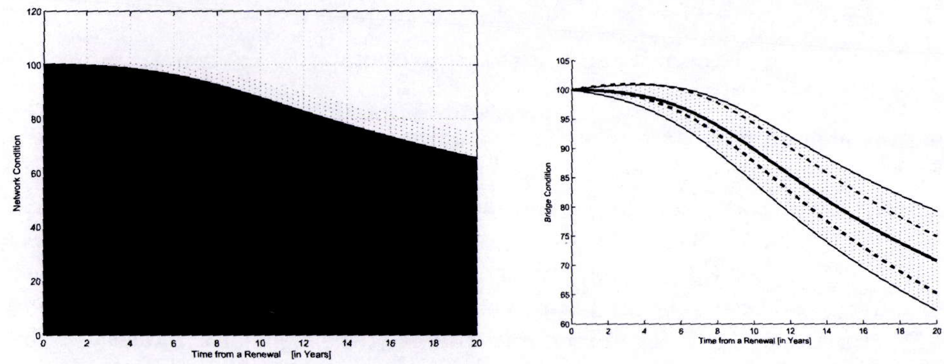


Fig. 3: Time to start of deterioration

used in an inferential procedure to estimate and predict the element condition after a renewal. A renewal is a point in time at which the element is brought back from a deteriorated state to a full condition. Time t is then reset to zero. T_d has a probability density function $f_{T_d}(t_d)$, $0 \leq t_d \leq \infty$, estimated from the data. A skewed distribution like the Weibull and the Gamma probability density functions are found to be good candidates. Let the mean and variance of T_d be $E(T_d) = m$ and $V(T_d) = s$, respectively. The deterioration part of the element is modeled with a gamma process $Z(.)$. The process starts at time T_d. In this case, we consider $Z(t - T_d)$ as a deterioration amount for $t \geq T_d$. The condition of the element is $C(t)|T_d, Z(.) = 100$ if $0 \leq t \leq T_d$ and $C(t)|T_d, Z(.) = 100 - Z(t - T_d)$ if $t \geq T_d$. Using this definition of the new stochastic process and the laws of probability, we derive the mean and variance of this new deterioration process. The expected value of $C(t)$, $\hat{C}(t) = E(C(t)|T_d, Z(.)) = 100 - \int_0^t \mu(t-t_d)^q f_{T_d}(t_d)dt_d$. The variance of $C(t)$ is $V(C(t)) = \int_o^t \{\sigma^2(t-t_d)^q + [\mu(t-t_d)^q]^2\}\, f_{T_d}(t_d)\, dt_d - [\int_0^t \mu(t-t_d)^q f_{T_d}(t_d)dt_d]^2$. Aboura and Samali (2011) provide the full derivation. The functional form $f_{T_d}(.)$ may be such that possibly closed forms can be obtained for the predictions. However, one should not be restricted by the need to have a closed form since most computations involve only a one dimensional integration.

4.2 Bridge and Network Condition Assessment

Let N be a network of n bridges, $B_1, B_2, \ldots, B_n$. Let the estimated condition curve for each bridge be $\hat{C}_{B_i}(t)$, $i = 1, \ldots, n$. Each bridge B_i has m_i elements, with corresponding weights $w_{i,j}$. These weights have been assessed according to a procedure using the California Bridge Health Index or a similar approach. Letting $\hat{C}_{i,j}(t)$ be the estimated condition curve for the j^{th} element, the bridge estimated condition is $\hat{C}_{B_i}(t) = \sum_{j=1}^{m_i} w_{i,j}\hat{C}_{i,j}(t) \quad i = 1, \ldots, n$ and the variance of the bridge condition is $V(C_{B_i}(t)) = \sum_{j=1}^{m_i} w_{i,j}^2 \; V(C_{i,j}(t)),\ i = 1, \ldots, n$. Considering a network N of n bridges $B_1, B_2, \ldots, B_n$, and letting the estimated condition curve for each bridge be $\hat{C}_{B_i}(t),\ i = 1, \ldots, n$, the estimation for the overall network condition of the n bridges is $\hat{C}_N(t) = \sum_{i=1}^{n} \frac{1}{n}\hat{C}_{B_i}(t)$. The variance for the network estimated condition is $V(C_N(t)) = \sum_{i=1}^{n}(\frac{1}{n})^2 \; V(C_{B_i}(t))$. Figure 4(a) shows the network condition model, as it typically appears in all cases. It has a concave starting curvature which flexes into a convex tail. It is interesting to note this characteristic form. The shape is similar to that of the reliability function in the case of a univariate lifetime distribution. Figures 4(b) shows the example of a bridge that is part of the network considered in which the bridge estimated condition along with its standard deviation (straight lines) is plotted against the network condition (dashed lines).



(a) Network Estimated Condition (b) Bridge and Network Estimated Conditions

Fig. 4: Network Estimated Condition

5 Conclusion

Pontis methodology has taken hold in the United States and subsequently elsewhere. We introduce a different method. The statistical model we showcase implements a new approach. The main driving force in the work is the fact that a statistical model like the gamma process is better suited for bridge lifetime assessment. We followed the work of van Noortwijk and others in modeling deterioration in civil infrastructure and found support for their approach. We extend some results and develop a new stochastic process for modeling structural deterioration.

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