Neural Networks Analysis and Synthesis of the Multidimensional Signals by More-Equal-Less Logic

Dobilas Kirvelis
Department of Biochemistry and Biophysics, Vilnius University
M.K.Čiurlionio 21/27, LT-03101 Vilnius, Lithuania
Fax: 370-5 2398231; E-mail: dobilas.kirvelis@gf.vu.lt

Abstract
The aim of this paper is to formulate non-formal more-equal-less (M-E-L) logic of the neural networks analysis and synthesis of the multidimensional signals for anticipatory control in living and organized systems. The signals analysis and synthesis structures are necessary for anticipation procedures in the more complex systems decision making. The possibilities of neural nets composed of neurons as the algebraic dot productors of continuously varied impulse frequencies characterized by diode non-linearity $\{N\}$, when informational operations of fuzzy logic are performed is analyzed. According to the facts of neurobiological research the neurons are divided into satellite and pyramidal ones, and their functional-static characteristics are presented. The operations performed by satellite neurons are characterized as qualitative (not quantitative) informational estimations ***"more"***, ***"less"***, ***"equal"***, i.e., they function according to more-equal-less (M-E-L) logic. Pyramidal neurons with suppressing entries perform algebraic signal operations and as a result of them the output signals are controlled by means of universal logical function "***NON disjunction***" (Pierce arrow or Dagger function). It is demonstrated how satellite and pyramidal neurons can be used to synthesize the neural nets functioning in parallel and realizing all logical and elementary algebraic functions as well as to perform the conditional controlled operations of information processing. Such neural nets functioning by principles of M-E-L and suppression logic can perform signals' classification, filtration and other informational procedures by non-quantitative assessment, and their informational possibilities (the amount of qualitative states), depending on the number n of analyzing elements-neurons, are proportional to $n!$ or even to $2^n \cdot n!$, i.e., much bigger than the possibilities of traditional informational automats functioning by binary principle. Such neurostructures of analysis by synthesis carries signal-information procedures on the principal-factor components analysis methods.
Keywords: neural network, fuzzy-like signal, "more-equal-less" (M-E-L) logic, signals analysis, signals synthesis,

1 Introduction

The analysis and synthesis methods are widely used in scientific research and technical decision making, but usually there are used separately. But some theories in cognitive psychology, neuropsychology, neurocybernetics and technological decision making

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state that information systems where procedures of the analysis and synthesis are functionally closed-loop interconnected work much more effectively. Those systems work by a principle of the analysis by synthesis.

Neural networks of the neocortex that function on principle of "Modeling Relation" (Rosen, 1985, Casti, 1989) signals analysisi by synthesis (A-by-S) or Closed-Loop Coding-Decoding (CL-CD) (Kirvelis, 2002-2009) can be these modeling structures capable to create anticipatory models of an environment and to carry out Perceptual Control (Powers, 1956). The neuro-layered structures can be interpreted as neuroinformational processors that can opperat by analog and digital signals. The neuro-structures allow conceptual and mathematical modeling signals transformations where these properties are a simmilar to quasi-holographic complex orthogonal or quasi-orthogonal transformations. The orthogonal transformations of signals allow understanding of possible principles of construction of neuro-models of external environment and internal systems in the neocortex. These transformations are necessary for synthesis of anticipatory control models-plans in CL-CD systems that work on A-by-S principle.

The aim of this paper is to present the neural fuzzy-like conception of neural nets *more-equal-logic* of sensory neocortex. For this purpose the representation of the functional organization of neural structures of the neuro-structures of the analyzis and synthesis.

Neurobiological experiments clearly show that neurons as non-linear summators (logical and algebraic) possess certain linear properties. The possibilities of neural nets composed of neurons – the algebraic accumulators (summators) of continuously varied impulse frequencies characterized by diode non-linearity $\{N\}$, when informational operations of fuzzy logic are performed is analyzed. According to the facts of neurobiological research the neurons are divided into stellate and pyramidal ones, and their functional-static characteristics are presented. The operations performed by stellate neurons are characterized as qualitative (not quantitative) informational estimations ***``more"***, ***``less"***, ***``equal"***, i.e., they function according to M-E-L logic. Pyramidal neurons with suppressing entries perform algebraic signal operations and as a result of them the output signals are controlled by means of universal logical function ***``NON disjunction"*** (Pierce arrow or Dagger function). It is demonstrated how stellate and pyramidal neurons can be used to synthesize the neural nets functioning in parallel and realizing all logical and elementary algebraic functions as well as to perform the conditional controlled operations of information processing. Such neural nets functioning by principles of M-E-L and suppression logic can perform signals' classification, filtration and other informational procedures by non-quantitative assessment, and their informational possibilities (the amount of qualitative states), depending on the number n of analyzing elements-neurons, are proportional to $n!$ or even to $2^n \cdot n!$, i.e., much bigger than the possibilities of traditional informational automats functioning by binary principle.

In addition, there is substantial evidence that in biological networks there is strong feedback (by way of axon collaterals, interneurons and *e.c.*), both between nearby neurons and between bigger structural units of the nervous system (ganglia, nuclei, cortical fields, etc.). It has been emphasized that it is these feedback collaterals that grow and form new synaptic contacts during the life of an organism. It has been pointed out that this feedback is nonlinear, and its significance for the functional properties of the neuronal network has been considered .

These properties of non-linear negative feedback, in the context of the purpose of the nervous system, call attention to a statement by the creator of biometrics, mathematician R. Fischer, in which he proposed that dynamical systems with non-linear feedback may identify an object, or create a model of the object. This means that non-linear neuronal feedback may be one of the most important mechanisms in brain functional organization and functioning. Therefore, it makes sense to formulate a purpose and find a way to synthesize appropriate functions, i.e., a neuronal structure which would generate the needed phase portrait. One should explore the potential of negative neuronal feedback and create a basic memory-endowed network, a continuous neuronal factorial bifurcator, which would be able to remember the permutation that arranges the positive continuous components of an input vector in increasing order.

Therefore, understanding of the synthesis of neuronal structures could explain not only neurobiological facts but also would help create more effective technology for information processing.

2 Sensory Neocortex as Model-Based Anticipatory Perception Neuro-structure or Subsystem Analysis and Synthesis

The perfect object for analyze functional structure of a brain is expedient for carrying out on an example of the visual analyzer. (Fig.1.) The interpretation of visual perception, anatomic and morphological structure of visual systems of animals, neuro-physiological, psychological and psycho-physiological data in the light of a number of the theoretical solutions of image recognition, visual perception processes simulation enable active information processing [Kirvelis. 2004].

The activities in special areas of cortex are as follows: focused attention, prediction with analysis of visual scenes and synthesis, predictive mental images. In the projection zone of visual cortex *Area Striata* or **V1** a "sensory" screen (SS) and "reconstruction" or synthesis screen (RS) are supposed to exist. The functional structure of visual analyzer consist of: analysis of visual scenes projected onto SS; "tracing" of images; preliminary recognition; reversive image reconstruction onto RS; comparison of images projected onto SS with images reconstructed onto RS; and "correction" of preliminary recognition. The closed procedure of analysis by synthesis (A-by-S) correspond to mental images vision procedures. It is supposed, that the quasi-holographical principles of the neuronal organization within the brain of the image "tracing" and reverse image reconstruction perform periodical procedure of coding-decoding.

The functional organization of neuron layers of Brodmann area 16 in primary visual zone of mammals is especially interesting. Morphological, neurophysiologic and computational research had generated most abundant experimental data and had given many theoretical models, but principles of area 16 organization and functioning are very hazy yet. Interpretations of functioning of *Area Striate* can enclear the CL-CDC principles of neocortex in visual thinking and general thinking to.

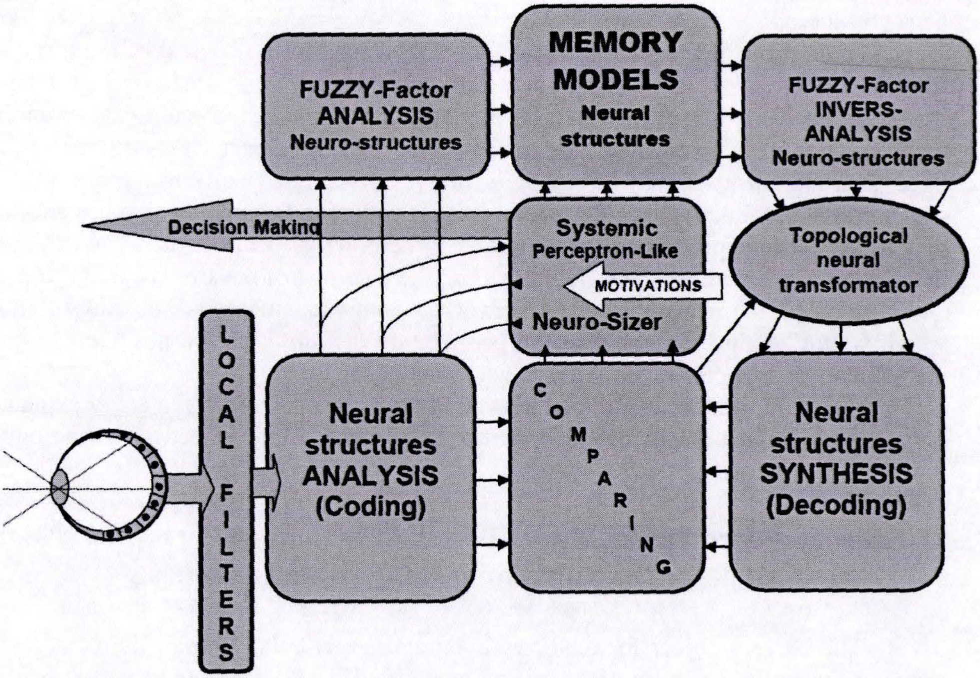


Figure 1: Functional organization neuro-structures of the sensory neocortex for Analysis by Synthesis (A-by-S) or imitative Closed-Loop Coding-Decoding (CL-CD),that's carry out model-based anticipatory perceptions

Receptoric structures code the environment images and their changes and send coded information to primary visual zones of neocortex. Here properties of visual images are analysed in detail Analysis results are used for primary perceptronic recognition, for comparison with reconstructed image from memory and for registering in memory structures. The essential specifics of visual neocortex is that it continually analyses images transferred from retina and compares them with anticipated images that are collected from memory on basis of environmental situation and motivations. This cyclic A-by-S or internal distinctly anticipatory Closed-Loop Coding-Decoding control procedure carries imitative cognitive modeling. The results are used in generation of pragmatic behavior models for current and future actions. All this activities are used:

1) to generate knowledge cognitive models that must correspond to reality as exactly as it is possible. The role of motivation here is to shorten search of most corresponding models in memory.

2) to generate pragmatic models that are used to change reality. The reality is changed that it would correspond to more or less strategic plans generated in mind. The last feature is especially characteristic for human mind.

Maybe this imitative Analysis by Synthesis with Closed-Loop Coding-Decoding feedbacks is an essence of thinking, because thinking is mental creation of schemes (point of view of cognitive psychology) that possibly is implemented in structures of neocortex.

A full dynamical perceptual control system of coding-decoding in visual analyzer is close-looped because an actual image is compared with reconstructed one from memory. Closure of this system is completed as closed-loop space/time coding-decoding structures, that corresponds reflective local quasi-orthogonal Hermite-Laguerre-like coder and Hermite-Laguerre-like decoder structures. This property gives functional sense for information in visual perception system

The interpretation of peculiarities of visual perception and visual systems morphology, of neurophysiological, psychological and psychophysiological data in the light of image recognition theories confirms the idea that active information processing is essential in a visual perception. The activities in visual areas of cortex lead to attention focusing, predictive analysis of visual scenes and synthesis of predictive mental images.

It is supposed that a "sensory" neuronal screen (SS) and "reconstruction" neuronal screen (RS) exist in the projection zone of visual cortex (*Area Striata* or V1). The functioning of visual analyzer consists of following intertwined operations: analysis of visual scenes projected onto SS; quasi-holographic "tracing" of images; preliminary recognition; quasi-holographic image reconstruction from memory onto RS; comparison of images projected onto SS with images reconstructed onto RS; and correction of preliminary recognition. The CL-CD procedure of analysis-by-synthesis corresponds to visual procedures on mental images. It is supposed that the image "tracing" and reverse image reconstruction based on Fuzzy-Factors analysis and special memory mechanisms and principles of brain neuronal organization as periodic CL-CD procedures.

We propose that the neuronal structure implementing the quasi-holographic Fuzzy-Factors analysis-by-synthesis ought to possess at least ten functional layered complexes: (1) the receptor layer where the retinal image is projected; (2) layer of local filtering; (3) local Hermite-Laguerre like analyzer and (4) local Hermite-Laguerre like syntesizer with (5) comparator between them. These structures are looped by quasi-holographic Fazy-Factor memory layered complexes (6, 7) with (8) special memory neural structure controlled by systemic perceptron-like classificator (9) in-between them. The memory traces are extracted by means of the topological transformations structure (10) controlled by signals from the comparator. The comparison block collates actual signal of local analysis and mental image of local synthesis. The synthesis may be accomplished by dedicated predictive structures driven by arbitrary motivations or preliminary expectations of events in environment. Note, that the system described above resembles the closed-loop coding-decoding, similar as classic non-loop communication system of the Shannon information theory, whereby analysis/decomposition and Fuzzy-Factor analysis is equivalent to the encoding step, and the reconstruction/synthesis with invers Fuzzy-Factor analysis corresponds to decoding.

This model is based on both the visual psychophysical and neurobiological data, interpreted in the light of the theoretical solutions of image recognition and visual. The functioning of visual analyzer according this model consists of the following stages:

- projection of retinal image (image arriving from retina) to the local Hermite-Laguerre like analysis. This projection is coupled with analysis of image. The actual images are guasi-holographically Fuzzy-Factor tranformed and recorded to chronological searchable special memory (with systemic search catalogue - sizer).
- projection of mental image from the special fenomenal memory (searched in catalogue, extracted, decoded by inverse Fuzzy-Factor quasi-holographic transformations and topologically transformed) to the local Hermit-Lagerr-like synthesis structure.
- Comparison of real image and hypothetic mental images, and detection of mismatching features. Additional rotation, shift and other topological transformations are used in comparison.
- Decision or recognition is based on the preliminary recognition and involves an iterative formulation of image identity hypothesis, which leads to synthesis of an image of the new object. The iterative procedures last until the correspondence between the actual object and its retrieved hypothetical image is achieved.

So the information system is organized on the principle analysis by synthesis or CL-CDC can to generate the information and control the new technological procedures.

3 Neurons and its Features

Neuromorphological studies show that the structural and functional element of the nervous system, the neuron, has a multitude of synaptic contacts with other neurons and one long process, the axon. The axon branches and impinges on neurons and other cells, making its synapses. This is how neuronal structures and neural networks are formed. Neurons come in different shapes but in most cases they may be divided into "stellate" and "pyramidal" neurons. For the sake of simplicity we assume that our neurons (quasineurons) are summators with many functional inputs and one functional output (Fig. 2, 3).

3.1 Static Neuronal Characteristic

Neurophysiological investigations of the neurons is the collector of signals (impulse frequencies), which has up to ten thousands inputs (synapses) and one functional output (axon) . The latter one can branch and contact through synapses with many other neurons in farther layers and with itself. The neurons' input and output are variable – neuropuls frequency S^{-1} (Fig. 2.)

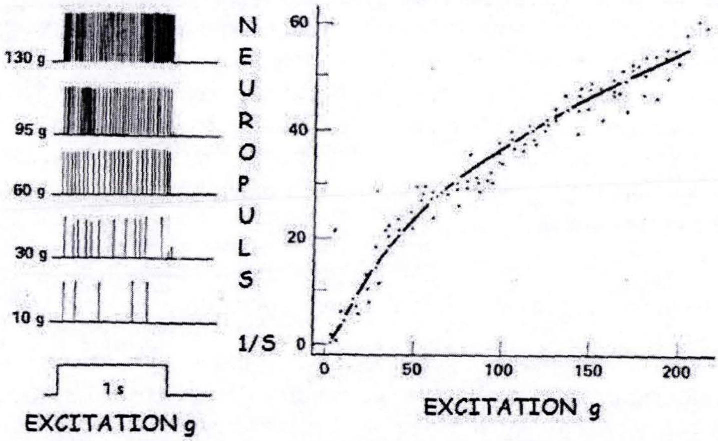


Figure 2: Example neuro-reaction by neuro-puls frequency of the motor-sensor to loading (excitation) in gramme **g**

If neuron is the collector of signals (impulse frequencies) $\mathbf{X_i}$, which has up to ten thousands inputs (synapses) $\mathbf{S_i}$ and one functional output (axon) **Y**. The latter one can branch and contact through synapses with many other neurons in farther layers and with itself (Fig. 3.). The neurons' input $\mathbf{X_i}$ and output **Y** are variable excitements with approximately linear dependence (see the dotted line in Fig.2). Hence usually it is quite acceptable to approximate the static part of signal transfer by straight line.

$$Y = N\left\{\sum_{i=1}^{n} S_i \cdot X_i\right\} = \begin{cases} \sum_{i=1}^{n} S_i \cdot X_i ..if.. \sum_{i=1}^{n} S_i \cdot X_i \geq 0, \\ 0 .... if.. \sum_{i=1}^{n} S_i \cdot X_i \leq 0. \end{cases} \qquad Y = \begin{cases} N\left\{\sum_{i=1}^{n} S_i \cdot X_i\right\} ... if..Z_1.OR.Z_2 = 0, \\ 0 .... if..Z_1.OR.Z_2 \neq 0. \end{cases}$$

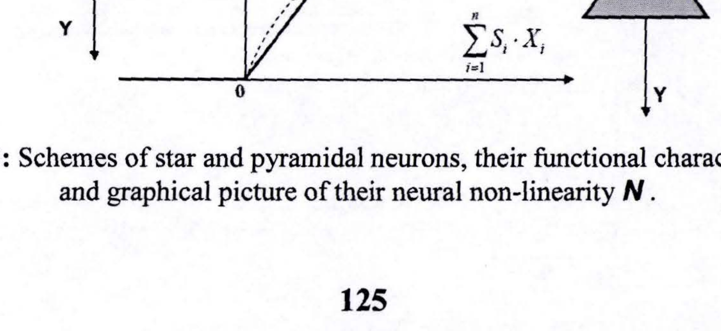
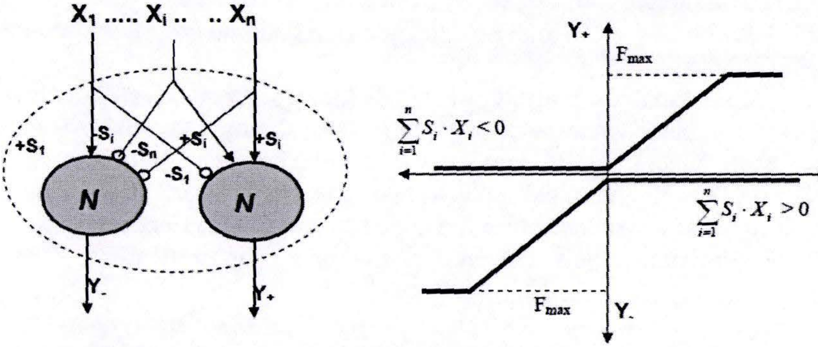


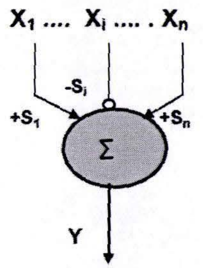
Figure 3: Schemes of star and pyramidal neurons, their functional characteristics and graphical picture of their neural non-linearity ***N***.

The pyramidal neurons that are located in the cerebral cortex have additional possibilities. Their functional organization and mechanisms of action hold maybe the greatest still unrevealed secrets of nervous system. Typically the pyramidal neurons have plenty of synaptic contacts, especially on the dendrites, where the input signals are summed. It seems that the synaptic contacts on the pyramidal neuron's body perform the suppression, which means that the pyramidal neuron transmit the positive sum of signals to the axon and to the other neurons only in case zero impulsation from any ***Z*** entry. Thus the pyramidal neuron responds and forwards the signals unless and until there isn't any signal suppressing on its body. It will realize universal logic operation – Peirce arrow (Dagger function).

3.2 Reciprocal Neurons as Scalar or Dot Product Operator

Neurons in neuron net can not transmit negative signals – trains of negative pulses. The system that produces signals with both signs is a reciprocal pair of neurons. Here pulses of one neuron are considered as negative ones, and pulses of other neuron – as positive ones. (Fig. 4.).





$$Y = \begin{cases} Y_- = \begin{cases} \sum_{i=1}^{n} s_i \cdot x_i, if \sum s_i \cdot x_i \le 0, \\ 0, if \sum s_i \cdot x_i \ge 0, \end{cases} \\ Y_+ = \begin{cases} 0, if \sum s_i \cdot x_i \le 0, \\ \sum s_i \cdot x_i, if \sum s_i \cdot x_i \ge 0. \end{cases} \end{cases}$$

Figure 4: The pair of reciprocal (inverse) neurons – linear algebraic collector of neuropuls frequencies and its characteristics

The neurons of reciprocal pair have the same structure of synaptic connections, just their signs are opposite, i. e. the coefficients of correspondent synapses are equal but have the opposite signs. In biological neural networks the sign of neural signal is defined by type of synapse at end.

Such a pair of neurons is a linear algebraic "space/time integrate" where one neuron of the pair sends signals $\mathbf{Y}_-$ about negative sum of incoming signals $\mathbf{X}$ and the other neuron of the pair sends signals $\mathbf{Y}_+$ about positive sum of incoming signals $\mathbf{X}$. When the sum of synaptic coefficients with the same synaptic sign is less then **1** (e. g. $\Sigma \mathbf{S}_+ \leq \mathbf{1}$ for exciting synapses) neuron's function always is in linear phase but can not be maximally $\mathbf{F}_{\mathbf{max}}$. In that case the pair of reciprocal neurons realizes the scalar product of entry vector $\mathbf{X}$ and entry vector of synaptic connections $\mathbf{S}$ or, in other words, estimates the correlation of these vectors by value $\mathbf{Y}$. When both neurons have the threshold $\mathbf{\Theta}$, this algebraic "space/time integrate" is non-linear and have insensible zone $\pm\ \mathbf{\Theta}$.

4 More-Equal-Less Logics as Fuzzy Logic Neural Networks

The gist of neuroinformatics is realization of logical operations by means of neural structures. In those cases when neurons filter the features of objects reverberated in receptors and estimate their expression by means of continuous values, their logical analysis must be performed by methods of continuous logic. There are a lot of various variants of continuous logic called differently: infinite, continuous, neuronal, analogical, syncretic, fuzzy logic, etc. It is relevant to various hybrid computers, informational technologies as well as neuroinformatics. There are plenty of algebraic algorithms for their realization, e. g. R-functions (Rvachiov, 1967), neural logic (Kirvelis, Pozin, 1967), fuzzy logic (Zadeh, 1969). For the neural structures it is the easiest to apply the neural logic expressed in algebraic terms, which virtually expresses the main features of all mentioned logics.

4.1 Neuronal Fuzzy Logic

The features of reverberated objects filtrated by two neurons' continuous entry signals of opposite signs and their magnitude expressed by intensity of neuron's excitation Y can be further processed by means of fuzzy logic. The merest neuronal structures that realize elementary operations of fuzzy logic are shown in Fig. 5.

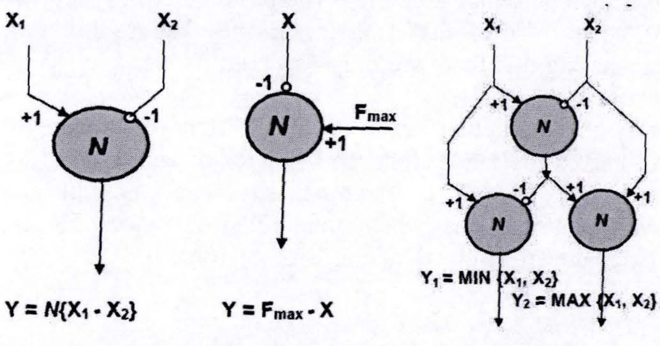


Figure 5: Neuronal structures that realize the main operations of continuous (neuronal, analogical, syncretic, fuzzy) logic

Neuron which has one exciting and one suppressive entry realizes the conditional difference or informs about the elementary inequality saying that $X_1 > X_2$, and presents the expression of this inequality on outlet Y. Such neuron which has one suppressive entry X and one exciting entry with constant F_{max} realizes logical inversion $Y = F_{max} - X$ which corresponds to one of the most important operations of traditional logic, namely the negation $Y = notX$.

The structure of two similar neurons can perform other principal elementary operations of continuous or fuzzy logic – conjunction (logical multiplication) or disjunction (logical addition). In neuronal fuzzy logic they are expressed as follows:

$Y = MIN\{X_1, X_2\} = N\{X_1 - N[X_1 - X_2]\} = N\{X_2 - N[X_2 - X_1]\}$ (conjunction),
$Y = MAX\{X_1, X_2\} = N\{X_1 + N[X_1 - X_2]\} = N\{X_2 + N[X_2 - X_1]\}$ (disjunction).

Selection of minimal value is understood as fuzzy conjunction and selection of maximal value is understood as fuzzy disjunction. It is obvious in the logical analysis of two features; however the same conception is also applied in the fuzzy logical analysis of multiple featured neuronal structures.

As stated above, the principal operation performed synthesizing neuronal fuzzy logic's ***MIN*** and ***MAX*** structures is $N\{X_i - X_k\}$ and it is realized by separate neuron. It is seen clearly when minimum and maximum separation procedures of multiple entries' neurons are written down:
$MIN\{X_1, X_2, ... X_n\} = N\{X_1 - N[X_1 - N(X_2 - N(X_2 - ....)]\}$,
$MAX\{X_1, X_2, ... X_n\} = N\{X_1 + N[X_1 - N(X_2 - N(X_2 - ....)]\}$.

It is obvious that the same neurons are necessary to realize the negative of multiple entries, namely inversion, when neuron-invertor is set for every entry. (It is worthy to note that complete neural network has such invertors only in the primary receptor structures where only the positive signals dominate. In the further neuroinformational procedures the inverted and non-inverted signal vectors function in parallel.)

It all goes to show that the basic neuronal fuzzy logic's operator is element-neuron which performs the "more" and "less" comparisons. It is understandable that various schemes performing any functions of fuzzy logic can be synthesized from such neuronal structures. There can be even synthesized such schemes as "uncertain", "equal", "indefinite" and similar ones that are disclaimed by traditional categorical logic stating that there are only two possible variants "yes" or "no", and no third variant is possible.

It is this particular feature which differentiates categorical logic from fuzzy logic and makes the latter closer to behavior of animals and humans. It can be easily interpreted by schemes of neuronal structures' possible functioning. It is undoubtedly well demonstrated when conditional operators ***IF*** used in computer programming language are realized by means of neurons.

4.2 Neuronal Structure – the Operator of Arithmetical Conditions

The operator of arithmetical condition used in programming languages is expressed as follows:

IF (arithmetic-algebraic function) m_1, m_2, m_3

It means that if performing computational procedures the arithmetical value calculated according to algebraic expression is negative, the further operation will be performed considering the address m_1 indicated in the program, if the value is positive, the operation will be performed considering the address m_3, and if it is equal to zero, the operation will be performed considering the address m_2. This conception exhibits such actions as *"less"*, *"more"*, *"equal"*, which are naturally performed by neurons or unsophisticated structures of some neurons. Even the pair of reciprocal neurons in Fig. 5 performs "less" ($\Sigma < 0$) and "more" ($0 < \Sigma$) operations when the first neuron sends signals to one group of neurons and the second neuron sends signals to another group of neurons. They can not generate the signals and act on the same groups of neurons simultaneously.

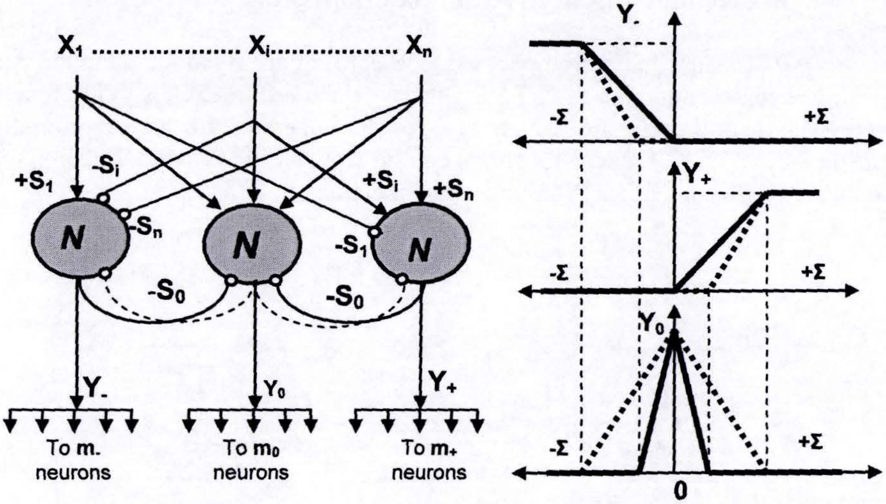


Figure 6: Neural structure – the operator of arithmetic condition
IF (Arithmetic expression $\Sigma s_i x_i$) m_-, m_0, m_+

demonstrates the complete neuronal operator of arithmetical condition designed for perpendicular net of neurons. The neuron located between the reciprocal neurons of the pair will be excited only if both the reciprocal neurons are still, i. e. both of them fulfill condition $\Sigma s_i x_i = 0$. If one of the reciprocal neurons is excited, the middle neuron will be extinguished by intense suppression $-S_0$ of one of the reciprocal neurons.

The middle neuron which has zero or indefinite identification status can also suppress both of the reciprocal neurons. Such an interaction of three neurons realizes the function given below:

$$Y = \begin{cases} Y_-, IF \sum S_i \cdot X_i < 0, \\ Y_0, IF \sum S_i \cdot X_i = 0, \\ Y_+, IF \sum S_i \cdot X_i > 0. \end{cases}$$

The features of fuzzy logic are apparently demonstrated by the presented diagrams of all three neurons' reactions. The excited neuron Y_- means "less", Y_+ – "more" and Y_0 – "uncertain" or "equal".

4.3 Neuronal Structure – the Operator of Logical Conditions

The operator of logical condition used in programming languages is expressed as follows:

IF (Logical function) Arithmetic-algebraic expression

It means that arithmetic value according to algebraic function will be given only if logical function is "YES". Otherwise the arithmetic-algebraic function is ignored. The neural structure which realizes the operator of logical condition is demonstrated in Fig. 6. Logical functions can be realized by pyramidal neurons which are connected with each other by inhibitory connections.

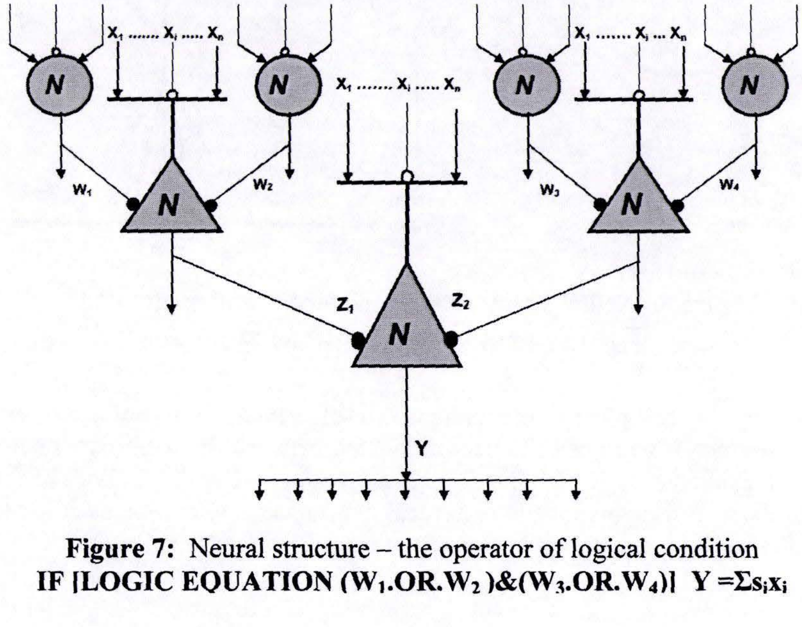


Figure 7: Neural structure – the operator of logical condition
IF [LOGIC EQUATION (W_1.OR.W_2)&(W_3.OR.W_4)] $Y = \Sigma s_i x_i$

Two pyramidal neurons connected in series by inhibitory connections realize the double negation, subsequently as a result of it forms the proposition. Several suppressions converged in one pyramidal neuron realize the Pirs arrow (Dagger function), which is expressed as follows:

$$.NOT.\ [Z_1.OR.\ Z_2] = .NOT.Z_1\ \&\ .NOT.Z_2$$

If star neuron transmits information about the features of reverberated object by signal batches ***W*** and transfers them to the suppressive entries of primary pyramidal neuron, which transfers them further to the suppressing entries of following pyramidal neuron, then such a neuron will realize the described function:

$$Y = \begin{cases} N\left\{\sum_{i=1}^{n} S_i \cdot X_i\right\}, when[W_1.OR.W_2]\ \&\ [W_3.OR.W_4] = .YES., \\ 0, when[W_1.OR.W_2]\ \&\ [W_3.OR.W_4] = .FALSE. \end{cases}$$

The excited neuron ***Y*** will act on corresponding groups of neurons by its connections and perform the selective procedure of information processing and transmission. Generally speaking (in conception of computer technique) such neuron is called the neuronal microprocessor functioning by principles of hybrid computer as pyramidal neuron allows consonantly integrate analogical and logical operations. It can form much more complex concepts than star neuron.

Star neurons form initial concepts filtering according to the principle "more", "less", "equal", whereas pyramidal neurons interconnect those concepts by logical "suppression" or Pirs arrow's functions and thus form superior concepts. Since star neurons-accumulators operate by analogical signals, their switchover from suppression to excitation and vice versa is not pronounced and such feature enables to attribute them to fuzzy sets and fuzzy logic.

Examples of the simplest neural structures given here help to understand the possibilities of much more complex neural nets, *i. e.* the functional organization of parallel neural structures operating by multidimensional signal vectors.

5 Neural Network – the Analyzer of Multidimensional Signals

As mentioned above, nervous system is the net of many thousands of neurons functioning in parallel and the abundance of parallel channels of information which start at receptors and end at effectors with collateral informational interactions as well. Therefore, after getting to know the possibilities of elementary neural net, it is essential to design the possibilities of neural analyzer with more complex and numerous entry signals. The neuroscheme of three-feature fuzzy analyzer given in Fig. 8. also fairly clear demonstrates the possibilities of **n**-dimensional analyzer of neural signals. When multidimentional positive-value signal from n+1 entries is analyzed, first of all it is recomented to compose neuron analyzer of *[(n+1)n]/2* elementary reciprocal pairs of neurons, which would analyze tehe signals interrelationships of every entry pairs by principle "more-less", i.e., the excited would become only that neuronwhich receives more intense signal to its exciting synapsis. It is obvious that altogether there can be *(n+1)!*

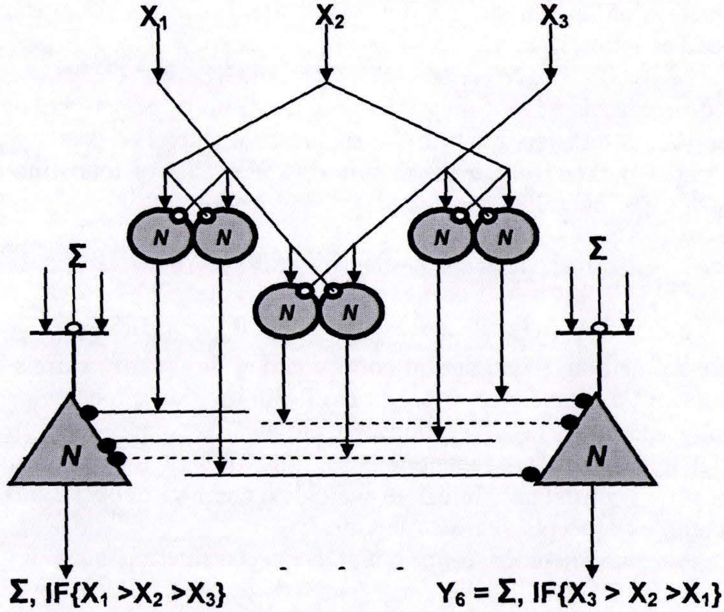


Figure 8: Fuzzy neural network which identifies positive three-dimensional entry vectors according to the more-equal-less logic (3! = 6 possibilities)

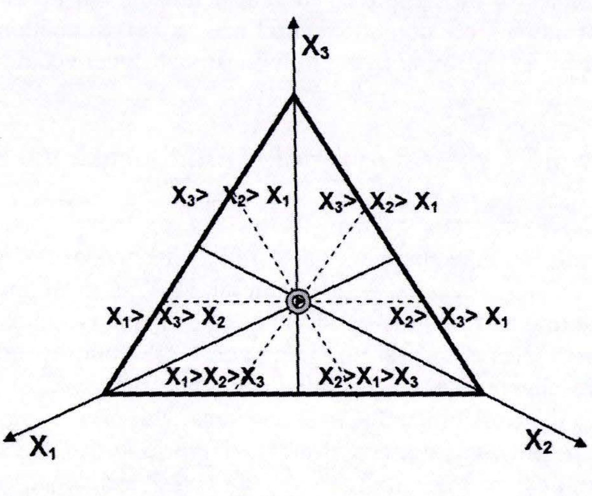


Figure 9: Three-dimensional positive-feature Fuzzy vectors situated according to *more-equal-less* fuzzy logic picture. In middle X_1=X_2=X_3

States of analyzing structures which corresponds to the permutational combinations of the quantity of entry signals and to the respective number of reciprocal chains of mary neural analyzer. This analyzer is differenciator and every $N\{X_i - X_k\}$, $N\{X_k - X_i\}$, neuron of its reciprocal pair "cuts" the space of entry signals by hyperplane to two symmetric pieces which pass through the central axis "all equal" $\{X_1 = X_2 = ... X_i = ... = X_{n-1}\}$, devide the plane X_i=X_k through its middle and pass throgh all other axes as well (Fig. 9.). Such neuronal analyzer subdivides all the positive quasi-octant to (n+1)! symmetric sectors and every of them match the direction of entry signals' vectors, which in its turn fulfills the corresponding inequable alignment according to the value of signal $X_k > X_i > X_{m-1} > ... > X_n > X_j$.

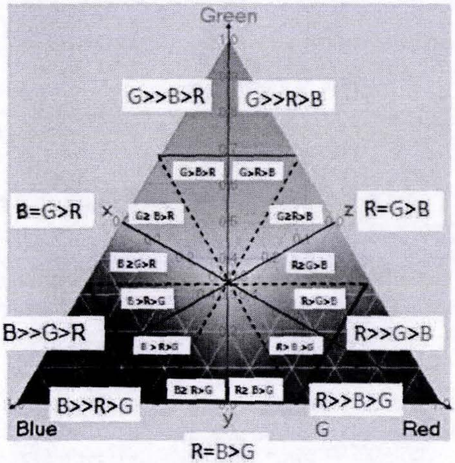


Figure 10: Example of the tree-dimentional Maxwel collours coding by fuzzy more-equal-less logic

Hereafter the neural net can be organized by means of pyromidal neurons and their inhibitory connections in such way that the corresponding pyromidal neuron would be excitaded only in that case if entry signals' vectors lies in that sector. It is posible to form $N_1 = (n+1)!$ pyramidal neurons wich would identify a single concept.Likewise, one can synthesize a three-dimensional, four-dimensional, and, in the general case, **n**-dimensional switch, which would remember one of the **n!** symmetric states of an **n**-dimensional input vector. Such a structure would be made of n pyramidal neurons, and **3n+2** interneurons, realizing **n+1** intersecting hyperplanes. One hyperplane would divide the hypercube of the phase space by a diagonal hyperplane perpendicular to the hyperline "all equal", i.e., **X1=X2=...=Xi=...=Xn**. All the other **n*(n-1)** hyperplanes, parallel to the hyperline "all equal" and moved to every coordinate axis, which would be away from them by **k*Xm**, (k<1) in the positive direction. That would create **n!** absolutely symmetrical intersection points, **n!** "potential pits", in the **n**-dimensional space, every of which would indicate a certain permutation of the vector **U** components (arrangement in increasing order). Depending on the values of the **U** components, the interneurons (acting by way of feedback) would push the system into one of these "pits". The general functional diagram of such a neuronal structure is shown in Fig.9 and as the tree-dimentional Maxwel collours coding by fuzzy more-equal-less logic in Fig. 10.

The neural structures of the brain consist of thousands or millions neurons. Therefore appropriate information capacity Q = log n! may be employed Stirling's approximation (or Stirling's formula).

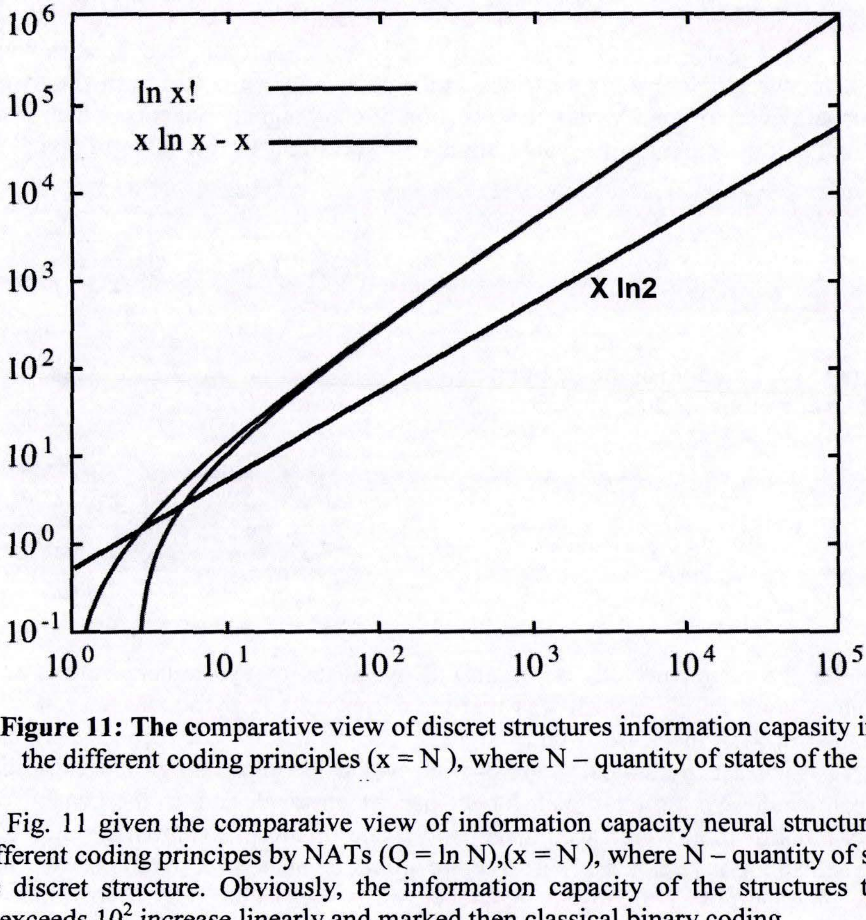


Figure 11: The comparative view of discret structures information capasity in the different coding principles (x = N), where N – quantity of states of the

In the Fig. 11 given the comparative view of information capacity neural structures in the different coding principes by NATs (Q = ln N),(x = N), where N – quantity of states of the discret structure. Obviously, the information capacity of the structures that's states exceeds 10^2 increase linearly and marked then classical binary coding.

It is easy to see that similar methods may be used to synthesize **n**-dimensional dynamical structures with a rather complex phase portraits. We can call them ***factorial switches***. If the binary logic is used to analyze the states of a neural net, then n neurons can have **2^n** states, whereas the factorial logic of analog neurons can see as many as **$M=n!*2^n$** states. Every hyperquandrant of the phasic space can have **n!** stable states. It can be attained by virtue of the feedback nonlinearity of analog interneurons.

The factorial neuron switches may by organized as neuro-structures of analysis, synthesis, comparing, different coding and decoding, also as analysis by synthesis.

6 Analysis and Synthesis

As mentioned above, the analysis and synthesis methods are widely used in scientific research and technical decision making, but usually there are used separately. Some theories in cognitive psychology, neuropsychology, neurocybernetics and technological decision making state that information systems where procedures of the analysis and synthesis are functionally closed-loop interconnected work much more effectively. Those systems work by a principle of the analysis by synthesis multidimentional signals and fuzzy vectors. The principal structure of the neuro-analizer fuzzy vectors presented in Fig. 12.

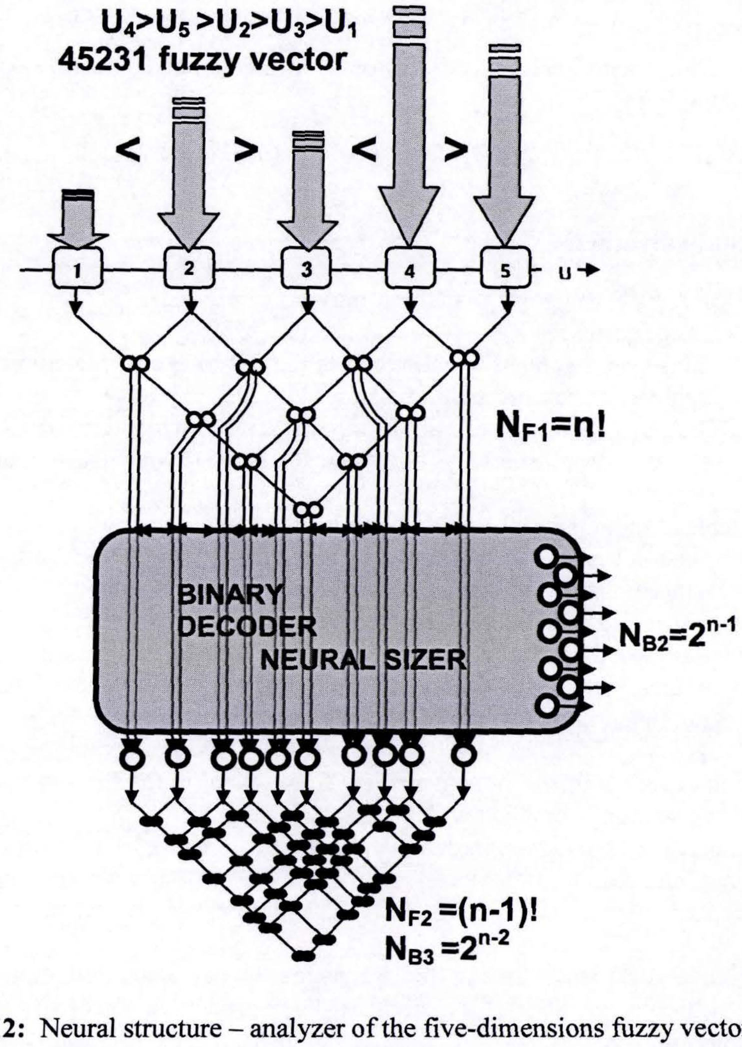


Figure 12: Neural structure – analyzer of the five-dimensions fuzzy vectors 45231 by more-equal-less logic. In example: $n=5$; $N_{F1}=120$; $N_{B2}=16$; $N_{F2}=24$; $N_{B3}=6$;

n-dimentional discrete analyzer of pozitive signal vectors can classify the fuzzy vectors – components of the alignment procedure a hierarchy. Where **n** is the multidimentional components, the theoretically maximum possible **n-1** perform levels. The first level, where all the sensors provide only positive signals analog values, the neural analyzer shall appoint one of the

$\mathbf{N_F = n!}$ **fuzzy vectors**

The second level is to perfor two stages of analysis:

- first, binary estimation of the primary differences in status $\mathbf{N_B = 2^{n-1}}$, and
- second, the fuzzy vector of one of the $\mathbf{N_F = (n-1)!}$.

Then the second level of $\mathbf{N = N_B\, N_F = 2^{n-1}\,(n-1)!}$.

n-1 levels in the hierarhical analysis in total n-dimentional neuro-fuzzy analyzer can distinguish one from

$$\mathbf{N= 2^{n-1}\,(n-1)!\;2^{n-1}\,(n-2)!\;2^{n-2}\,(n-3)!\;2^{n-3}\,(n-3)!\ldots 2^{3}\,(3)!\;2^{2}\,(2)!\;2^{1}\,(1)!} = \prod_{i=1}^{n} 2^{i-1}\cdot(i-1)!.$$

6.1 Analysis by Synthesis

The idea of A-by-S suggests that there may be two parallel screens in the sensory projection areas of cortex:

1. The sensory screen, which receives the 2D image of the environment from peripheral receptors and translates it into the subjective sensory scene;
2. The adjacent reconstruction/synthesis screen, which reproduces the image retrieved from memory that represents the hypothesis about the object identity.

Such organization of screens helps to understand a number of psychopathological conditions of vision that are accompanied by visualizations or hallucinations, as well as visual experience during dreaming. The hypothetical model of visual analyzer organization is consistent with the A-by-S hypothesis.

This model is based on both the visual psychophysical and neurobiological data, interpreted in the light of the theoretical solutions of image recognition and visual processes. The functioning of visual analyzer according this model consists of the following stages:

1) Projection of retinal image (image arriving from retina) to the sensory screen (SS).
   - This projection is coupled with analysis of image.
   - The actual images are recorded to chronological memory where they can be used in systemic search catalogue.
2) Projection of mental image from the visual memory to the reconstruction screen (RS).
   - Retrieval of mental image for RS is based on prediction that depends on motivation and earlier experiences and predictive recognition of earlier recognition cycle.

3) Comparison of real image and hypothetic mental image, and detection of mismatching features.
   – Rotational, shift and other topological transformations are used in comparison.
4) Decision or recognition is based on the preliminary recognition and involves an iterative formulation of image identity hypothesis, which leads to synthesis of an image of the new object. The iterative procedures last until the correspondence between the actual object and its retrieved hypothetical image is achieved.

The functional organization of the analysis ant synthesis proposed herein may be organized on the base of neuro-informational layers structures.

6.2 Neural Layer as Basic Neuro-Informational Structure of the Brain

The neuron layer with feedforward and feedback connections and with logical inpute for control of the information processing is presented in Fig. 13. The sequences of appropriate synaptic communications, values of weights $\mathbf{S_i}$ and constants $\mathbf{T_i}$ determine function of a layer.

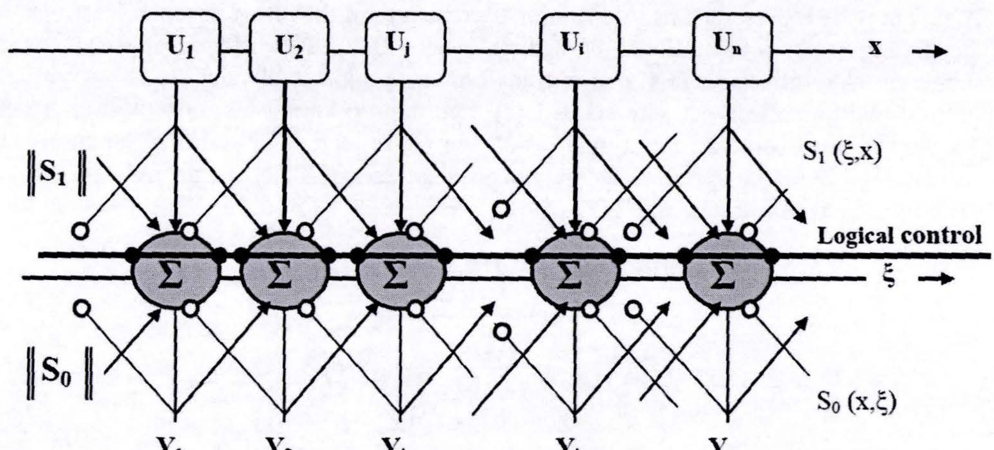


Figure 13: Neuron layer with feedforward and feedback connections and logical inputes for control of the information processing

The function of a layer can be determined by formulas:

$$\vec{R} = \begin{cases} \vec{Y} = \dfrac{\|S_1\|}{I - \|S_0\|} \cdot \vec{U}, IF(Logic.function) = .YES., \\ 0, IF(Logic.function) = .NOT. \end{cases}$$ for discrete structures,

and

$$R(\xi)=\begin{cases} Y(\xi)=\int_{-\infty}^{+\infty} Y(x)\cdot S_1(\xi,x)dx-\int_{-\infty}^{+\infty} Y(x)\cdot S_0(\xi,x)dx, IF(Logic.function)=.YES., \\ 0, IF(Logic.function)=.NOT. \end{cases}$$

for continuous structure.

Transformations of 2D signals by 2D neuron layers with special matrixes of connections include all possibilities of optical systems of noncoherent linear and coherent nonlinear optics. This quasi-optical approach allows theoretical analysis and computer simulation of neurophysiological data of the neurons receptive fields Abstract mathematical analysis and synthesis lead to understanding of information processing of variousmultidimentional signals in neuron layers analysis, synthesis, quasi-holographic, orthogonal and other transformations.

6.3 Neural structures of the orthogonal analysis and synthesis

The neuro-structures of the brain analysis and synthesis, analysisi by synthesis or CL-CD may be represented as informational procedure of the integral transformations. The coding is an informational procedure, where the real world or physical space (subsystem **X**) with the states **u(x)** is transformed to another physical space as virtual neural world ξ (reflection) with states **U(ξ)** according to kernel function **Φ(x, ξ)**. Then decoding is an inverse informational procedure (re-reflection or re-transformation) of the states of memory space **U(ξ)** on the physical space **X** according inverse kernel function **Ψ(x, ξ)** as the state **u(x')** (Fig.14.)

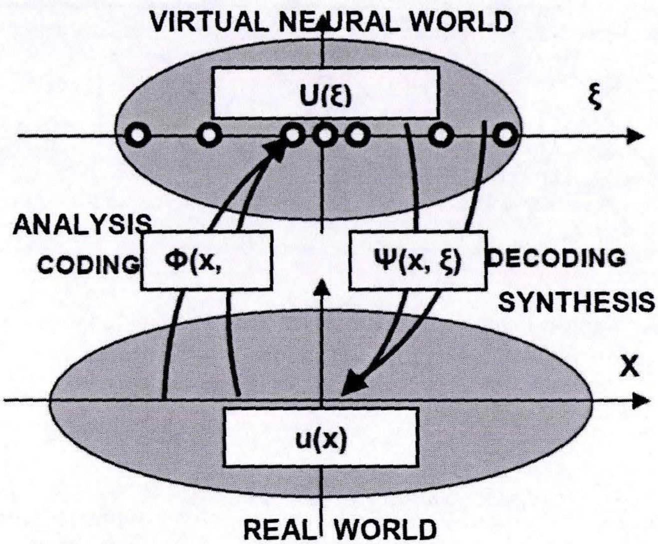


Figure 14: Analysis and synthesis as inversible orthogonal transformations

Ideal coding-decoding must satisfy orthogonal set of the kernel functions

$$\int_{-\infty}^{+\infty} \Phi(\xi, x) \cdot \Psi(\xi, x') d\xi \approx \delta(x - x').$$

Then coding-decoding as integral transformations may be expressed mathematically:

Equations of analysis

$$U(\xi) = \int_{-\infty}^{+\infty} u(x) \cdot \Phi(\xi - x) dx, \qquad U(\xi) = \int_{-\infty}^{+\infty} u(x) \cdot \Phi(\xi \cdot x) dx,$$

$$u(x) = \int_{-\infty}^{+\infty} U(\xi) \cdot \Psi(x' - \xi) d\xi \qquad u(x) = \int_{-\infty}^{+\infty} U(\xi) \cdot \Psi(x' \cdot \xi) d\xi$$

Equations of synthesis

The first equations represents non-homogeneous procedures, and recend is convolution integrals for homogeneous ones. This integral coding-decoding may be realized in discrete form, when kernel functions are expressed as matrix-operators [Φ] and [Ψ] that satisfy orthogonality condition

$$[\Phi] \cdot [\Psi] \approx \mathrm{I}.$$

For example, Fourier-like functions represent non-homogeneous transformations, and Frehnel-like represents the homogeneous ones. These functions represent technology of coding-decoding. Fourier and Frehnel coding-decoding transformations are known for their quasi-holographic features and their kernel functions are complex. The organization of neural structures of complex orhhogonal tranansformators represented in Fig. 15.

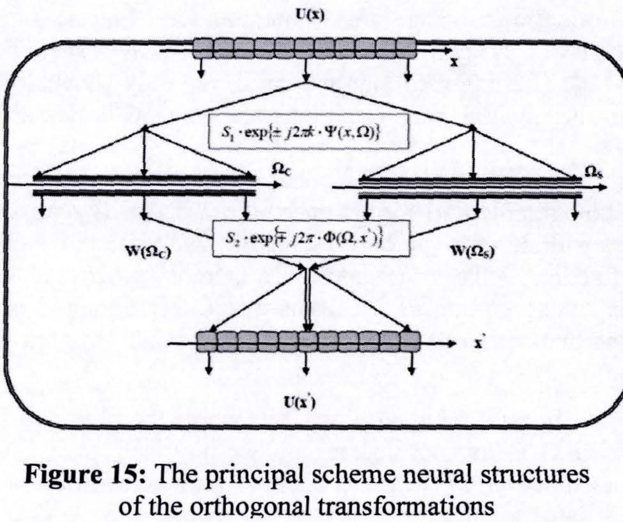


Figure 15: The principal scheme neural structures of the orthogonal transformations

The principle schema of complex orthogonal neural net structure consists of three layers:

- receptors layer where environment influence are reflected and signal image **U(x)** is generated;
- neural layer where environment image **U(x)** is transformed through the special connection function $\mathbf{S_1(x, \Omega)}$ and written to layer that contains the transformed image traces;
- reconstruction layer where image is reconstructed from memory traces by inverse transformation that is realized by connection function $\mathbf{S_2(\Omega, x)}$.

These transformations for 1D neural structures can be mathematically expressed as integral transformations with complex kernels $\mathbf{S_1(x, \Omega)}$, $\mathbf{S_2(\Omega, x)}$.

$$W(\Omega) = W_C(\Omega_C) + jW_S(\Omega_S) = \int_{-\infty}^{+\infty} U(x) \cdot S_1 \cdot \exp\{\pm j2\pi k\Psi(x,\Omega)\}dx,$$

$$U(x') = \int_{-\infty}^{+\infty} W(\Omega) \cdot S_2 \cdot \exp\{\mp j2\pi\Phi(\Omega, x')\}d\Omega$$

Integral of complex kernels product satisfies the condition of orthogonality or quasi-orthogonality when this product is **δ(x)** function.

$$\int_{-\infty}^{+\infty} S_1 \cdot S_2 \cdot \exp\{\mp j2\pi k \cdot [\Psi(x,\Omega) - \Phi(\Omega, x')\ \}d\Omega \cong \delta(x, x')$$

In this case the most information rich signal **δ(x)** can be dissipated and ideally reconstructed. The structure of second layer is not infinite and has a final number of summator neurons, so reconstruction can not be ideal.

The nature of connections **Ψ(x, Ω)** and **Φ(Ω, x)** determines the orthogonal nature of transformations. The transformation is Fourier quasi-holographic transformation when **Ψ(x, Ω)** and **Φ(Ω, x)** ~ **xΩ**, and it is a Fresnel one when **Ψ(x, Ω)** and **Φ(Ω, x)** ~ $\mathbf{(x\text{-}\Omega)^2}$. These transformations can be not only 1D but 2D, 3D and many-dimensional also. The integral transformations are realized in neural nets as discrete ones.

The signals transformation by Hadamard matrixes, Walsh functions and other are kind of quasi-holographic transformations and can be easily realized in neural nets. Such integrated transformations can be presented as analysisi and synthesis or CL-CD procedure, i.e. reflection of the information in a memory space with ability to extract initial information. Interpretation on the basis of CL-CD principle is a possibility to understand systems that realize A-by-S principle (the expanded concept of perception and perceptual control).

The coding is an informational procedure, where the physical space (subsystem **X**) with the states **u(x)** is transformed to another physical space ξ with states **U(ξ)** according to kernel function **Φ(x, ξ)**. Decoding is an inverse informational procedure (re-reflection or re-transformation) of the states of memory space **U(ξ)** on the physical space **X** according inverse kernel function **Ψ(x, ξ)** as the state **u(x)**

ANALYSIS-CODING **SYNTHESIS-DECODING**

$$W(\Omega) = \int_{-\infty}^{+\infty} U(x) \cdot S_1(x,\Omega) dx \qquad U(x) = \int_{-\infty}^{+\infty} W(\Omega) \cdot S_2(\Omega, x) d\Omega$$

The closed-loop coding-decoding may be represented as an informational procedure of the integral discrete and continual different integral transformations. Orthogonal Hermite-Laguerre-like transformations represent special interest, especially at perception visual and acoustic signals.

The analysis of mathematical kernel or synaptic functions neural structures demonstrate, that function analysis and synthesis are identically or inverse-identically. It's facilitate the understanding of the synthesis neuro-structures organization

The problems is how the neural structures may be organized for synthesis multidimensional fuzzy vectors signals. Obviously as, inverse-kernel operation of fuzzy vectors analysis structures that are constructed on the principle differentiation, the neuro-structures of synthesis mast be constructed on the principle integration. The neural layer for integration of signals presented in Fig. 16. The neural layer with special feedback synaptic functions operate integration:

$$Y(\xi_i) = \int_{1}^{+i} X(x_i) \cdot dx$$

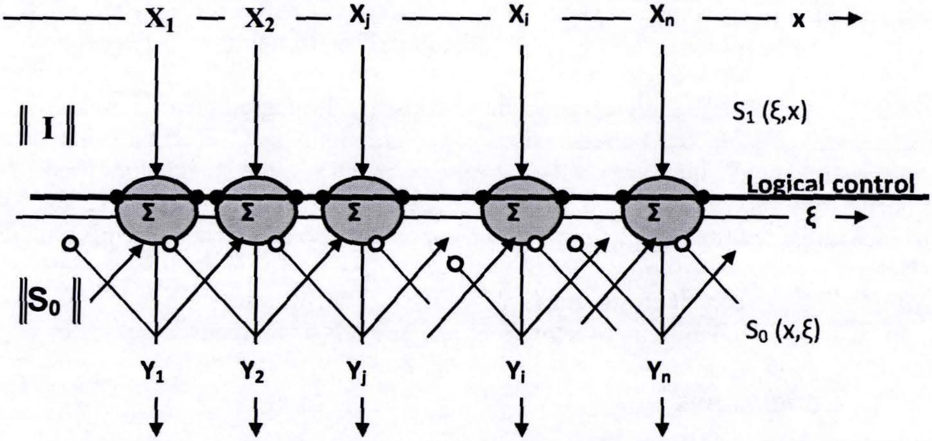


Figure 16: Neuron layer as **X** integrator or syntheser with asymmetric feedback connections and logical inputs for control of the information

It's supposed that similar organization of neural structures will solve problem of synthesis fuzzy vectors for analysis by synthesis that's for systems of anticipatory cognitions.

The essence of fuzzy-vector analysis lies in the vector components sort by neuro-signal pulse frequency activity. This shows that fuzzy logics neuroanalysis constructed by principal and factor component analysis methods and procedures. The neuronal analytical procedures based on autonomous signal differentiation in separates post-pair discret-differenciations. (Fig. 4; Fig17;)

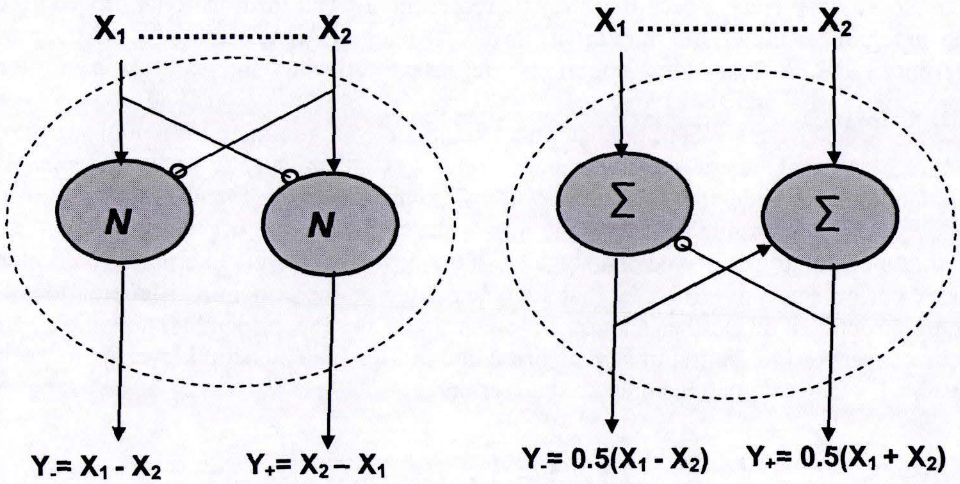


Figure 17: The neuronal autonomous signal post-pair discret-differenciator and post-pair discret-integrator

The fuzzy-vector synthesis consists of integrating structures. Since fuzzy-analytical solution is a paired individual differentiations - differentiate signals between neural reactions, the fuzzy-vector synthesis is the opposite integral procedures and decreasing sort by neuronal activity fuzzy-vectors. (Fig. 17) Analysis by synthesis procedures shall be completed fuzzy-vector comparisons matches and mismatches in the detection.

Such neurostructures of analysis by synthesis carries signal-information procedures on the principal-factor components analysis methods.

7 Conclusions

1. The brain neural networks in neocortex sensory structures of the human beings and mammals can function on the principles fuzzy-like more-equal-less (M-E-L) logic.
2. The brain neural networks process information by multidimensional fuzzy-like vectors.
3. The brain neocortex carry out anticipatory information processing by multidimensional analysis end synthesis neuro-structures that's operating on fuzzy-like vectors on the principal-factor components analysis methods.

4. The given neural networks models for the fuzzy-like vectors information procedures by more-equal-less logic is the theoretical test to understand brain and construct "neurocomputers" as new information technologies.

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