Universal Rewrite and Self-Organization

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Abstract
The conventional approach to computer programming using rewrite systems can be extended to create a universal rewrite system, which provides a computational approach to both physics and mathematics, based on the idea of a zero totality alphabet. Mathematics emerges from the system in the form of a Clifford-type algebra, while physics takes the form of nilpotent quantum mechanics (NQM). The most significant characteristic of NQM is that the quantum system (fermion state) and its environment (vacuum) are mathematical mirror images of each other. So a change in one automatically leads to corresponding changes in the other. We have used this characteristic as a model for self-organization, which has applications well beyond quantum physics. The nilpotent structure, seen as emerging from two commutative vector spaces, has a number of identifiable characteristics which we can expect to find in systems where self-organization is dominant; a recent case involves the neurons in the visual cortex. We expect to find many complex systems where our general principles, based, by analogy, on NQM, will apply.
Keywords: universal rewrite system, self-organization, nilpotent quantum mechanics, renormalization group, Berry phase

1 Introduction

Three main developments form the background to this work. The first is a universal rewrite system, which is a scale-independent and fractal computational process of generating zero-totality alphabets, with seemingly very general application. The most immediate applications of the rewrite structure have been found in physics and biology, which brings us to the second development. Nilpotent quantum mechanics is a form of relativistic quantum mechanics / quantum field theory which can be derived from the rewrite system and which minimalises the whole quantum apparatus to a single operator acting on a universal environment, which is its mirror image. The third development is that both the rewrite structure and nilpotent quantum mechanics require a combination of two vector spaces, each dual to the other, which provides a powerful model for self-organization.

Nilpotent quantum mechanics is the most immediately successful application of the universal rewrite system and serves as an almost perfect model for other applications. It is not so much that these applications derive directly from nilpotent quantum mechanics, rather that they derive from the structure which makes this form of quantum mechanics possible. Many characteristics can be described as identifiers of both the rewrite and the nilpotent structures, whether at the quantum mechanical level, or

International Journal of Computing Anticipatory Systems, Volume 27, 2014
Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-930396-16-4

applicable in mathematics, chemistry, biology or other areas of physics. We have proposed a number of such features as being detectable in systems of very different kinds and as thus being signatures of quantum-like organization or behaviour, especially where self-organization is dominant, and we have identified a new one in the organization of the neurons in the visual cortex.

2 The Universal Rewrite System

Computer programming, as it has been developed since the mid-twentieth century, has been based on the Turing machine, digital logic, and the rewrite or production system. A conventional rewrite system has 4 fixed components:

alphabet
rewrite rules (productions)
a start 'axiom' or symbol
stopping criteria

An example (suggested by our colleague, Bernard Diaz) is a process for generating Fibonacci numbers. There are two rules: p1: A → B; p2: B → AB. We begin with generation 0, and a single symbol A. Rule p1 then tells us to replace A with B. In the next generation, B becomes AB, and the process then repeats indefinitely:

N=0	A	length of string	1
N=1	→B		1
N=2	→ AB		2
N=3	→ BAB		3
N=4	→ ABBAB		5
N=5	→ BABABBAB		8
N=6	→ ABBABBABABBAB		13
N=7	→ BABABBABABBABBABABBAB		21 ...

It may be significant that the rules A → B and B → AB seem to be suggesting the structure of 3-dimensional (quaternion) algebra

$$\boldsymbol{i} \rightarrow \boldsymbol{j} \qquad \boldsymbol{j} \rightarrow \boldsymbol{i}\boldsymbol{j} = \boldsymbol{k}$$

and that a string like BABABBABABBABBABABBAB appears to be creating a fractal-like structure in 3-dimensional space, but situated in the AB or ***ij*** plane, as in holography. The logarithmic spiral becomes a way of expressing 3-dimensionality in the plane, with the increasing length of the intervals substituting for penetration into the third dimension.

Unlike a conventional rewrite system, a universal rewrite system[1-3] is one in which all four elements – alphabet, start object, rules, and stopping criteria – can be varied. The assumption here is that the universe, or any alphabet which it contains, is always a zero totality state, with no unique description, and so infinitely degenerate. In this system, we have to continually regenerate the alphabet, in such a way that it is always new, but there is no limit or stopping criteria, as the new state created is always another nonunique zero totality. It is a kind of *zero attractor*. A non-zero deviation from 0 (say *R*) will always incorporate an automatic mechanism for recovering the zero (say the 'conjugate' *R**), but the zero totality which results, say (*R*, *R**), will not be unique, and will necessarily lead to a new structure.

The characteristics of the rewrite process include self-similarity, scale-independence, duality, bifurcation, and holism. The universal process differs from other rewrite processes in having no fixed starting or ending point, and an alphabet and production rules that are endlessly reconstructed during the process. The self-similarity is a necessary consequence of the lack of a fixed starting point. It suggests that, if there are physical applications at one level, then there are likely to be applications also at others. As we scale up from small to larger systems, we can imagine that some principle such as the renormalization group takes effect to maintain the form of the structures generated by the rewrite process.

Now we ensure that a structure or alphabet is new by defining the position of all previous structures within it as subalphabets. The process will then continue indefinitely. Effectively, the process involves defining a series of *cardinalities*. Successive alphabets absorb the previous ones in the sequence, so creating a new cardinality. The cardinalities are like Cantor's cardinalities of infinity, but are cardinalities of zero instead. From the point of view of the observer, i.e. someone 'inside' the system ('universe', 'nature', 'reality'), we have to start from (R, R^*), which is the minimum description of a zero totality alphabet (or of a zero totality universe in physical terms). The successive stages are all zeros, as we go from one zero cardinality or totality to the next, and we ensure that they are cardinalities by always including the previous cardinality or alphabet. So (R, R^*, A, A^*), for example, includes (R, R^*). We may start at any arbitrary zero-totality alphabet but there is no natural beginning or end to the process. Because all the stages are cardinalities or zero totality alphabets, the process is always holistic. We have to include everything.

A convenient though not unique way of representing the process is by a 'concatenation' or placing together, with no algebraic significance, of any given alphabet with respect to either its components or subalphabets or itself. Since an alphabet is defined to be a cardinality, then anything other than itself must necessarily be a 'subalphabet' and the concatenation will produce nothing new. Only concatenation of the entire alphabet with itself will produce a new cardinality or zero totality alphabet. It is convenient to represent these two aspects of the process by the symbols $\Rightarrow$, for *create*, in which every alphabet produces a new one which incorporates itself as a component, and $\rightarrow$, for *conserve*, which means that nothing new is created by concatenating with a subalphabet.

conserve: (subalphabet) (alphabet) $\rightarrow$ (alphabet) *there is nothing new*

create: (alphabet) (alphabet) $\Rightarrow$ (new alphabet) *a zero totality is not unique*

The process is simultaneously recursive, creating everything E (all symbols) at once, and iterative, creating a single symbol only. It is also fractal and can begin or end at any stage. Since it describes or creates both time and 3D space, it can be thought of as prior to both. The create and conserve aspects must also be simultaneous; we only know which new alphabet will emerge when we have ensured that all possible concatenations with subalphabets yield only the alphabet itself.

Suppose, we then start with a zero totality alphabet with the form (R, R^*). Of course, we have to assume that this is not necessarily the beginning, though it is the point where we as observers start from. So, this already 'bifurcated' state will have started from a previous alphabet, which we assume we can't access directly, because we have no structure for it. If we describe this as R, then the * or R^* character creates the doubling process. Before we create (R, R^*), we have to assume that (R) is a zero totality alphabet, but it is a zero to which we have no access. In effect, we are trying to posit an ontology that exists before the epistemology or observation, begins with (R, R^*). So, we assume that it must happen without being able to observe it.

Applying the conserve process $(\rightarrow)$ to concatenate (R, R^*) with its subalphabets should produce nothing new. No concept of 'ordering' is needed in this process, but each term must be distinct. So

$$(R)\,(R, R^*) \rightarrow (R, R^*)$$
$$(R^*)\,(R, R^*) \rightarrow (R^*, R) \rightarrow (R, R^*)$$

It follows immediately that these concatenations lead to rules of the form:

$$(R)\,(R) \rightarrow (R)\,;\ (R^*)\,(R) \rightarrow (R^*)\,;$$
$$(R)\,(R^*) \rightarrow (R^*)\,;\ (R^*)\,(R^*) \rightarrow (R)$$

The next stage is to show that the zero-totality alphabet (R, R^*) is not unique, and that a concatenation with itself will produce a new zero-totality alphabet. Our first suggestion might be something like (A, A^*), but, with the terms undefined, this is indistinguishable from (R, R^*), and so the only way to ensure that the new alphabet is distinguishable from the old is by incorporating the old one, and we need to do this in such a way that ensures that the subalphabets yield nothing new. So we try

$$(R, R^*)\,(R, R^*) \Rightarrow (R, R^*, A, A^*) \qquad (1)$$

Having used the 'create' mechanism, we now apply the conserve operation $(\rightarrow)$ to this new alphabet, and concatenate with the subalphabets. So

$$(R)\,(R, R^*, A, A^*) \rightarrow (R, R^*, A, A^*) \rightarrow (R, R^*, A, A^*)$$
$$(R^*)\,(R, R^*, A, A^*) \rightarrow (R^*, R, A^*, A) \rightarrow (R, R^*, A, A^*)$$
$$(A)\,(R, R^*, A, A^*) \rightarrow (A, A^*, R^*, R) \rightarrow (R, R^*, A, A^*)$$
$$(A^*)\,(R, R^*, A, A^*) \rightarrow (A^*, A, R, R^*) \rightarrow (R, R^*, A, A^*)$$

As before the order of the terms is different for each operation, as we require, but the total is the same, and we soon quickly realise that (R, R^*) and (A, A^*) can only be different if

$$A\,A \rightarrow R^*, \text{ etc., while } R\,R \rightarrow R.$$

Duality is intrinsic to the process. The operation $()\,() \Rightarrow (\,,\,)$ describes how we go from one zero totality alphabet – or description of the universe – to the next one up. The $(\,,\,)$ is a kind of 'doubling' or 'bifurcation'. So we could write the result of (1) in the form $(R, R^*,\quad A, A^*)$, and represent $(R, R^*)\,(R, R^*)$ as a kind of doubling, to create a new cardinality (R, R^*, A, A^*), almost like transforming the second (R, R^*) into (A, A^*).

The next stage presents a new problem, for

$$(R, R^*, A, A^*)\,(R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*)$$

would fail the application of the conserve mechanism $(\rightarrow)$ by introducing new concatenated *terms* like AB, AB^*, which lie outside the alphabet. This means that we must include these in advance, as in

$$(R, R^*, A, A^*)\,(R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*).$$

The question that remains is: does this new alphabet satisfy all our requirements, when we concatenate separately with (*R*), (*R**), (*A*), (*A**), (*B*), (*B**), (*AB*), (*AB**)? The process is straightforward for the first six concatenations:

$$(R)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$(R^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (R^*, R, A^*, A, B^*, B, AB^*, AB)$$
$$(A)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (A, A^*, R^*, R, AB, AB^*, B, B^*)$$
$$(A^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (A^*, A, R, R^*, AB^*, AB, B^*, B)$$
$$(B)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (B, B^*, AB, AB^*, R^*, R, A, A^*)$$
$$(B^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (B^*, B, AB^*, AB, R, R^*, A^*, A)$$

But concatenations of the *concatenated terms*, (*AB*) and (*AB**), on themselves and on each other appear, to leave us with two options, which we can describe as 'commutative' and 'anticommutative':

$$(AB)\ (AB) \rightarrow (R) \qquad (commutative)$$
$$(AB)\ (AB) \rightarrow (R^*) \qquad (anticommutative)$$

In fact, however, there is no choice, for *only the anticommutative option* produces something new. Labelling is arbitrary in the rewrite structure, and so the labels *A* and *B* alone cannot distinguish these terms from each other – this can only be done if they produce distinguishable outcomes. The commutative option leaves *A* and *B* indistinguishable except by labelling, and so does not extend the alphabet. We are obliged to default on the anticommutative option, which means that the last two concatenations become:

$$(AB)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (AB, AB^*, B, B^*, A, A^*, R^*, R)$$
$$(AB^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (AB^*, AB, B^*, B, A^*, A, R, R^*)$$

This solution for *A* and *B* cannot be repeated to include new terms, such as (*C*), (*D*),..., when we extend alphabet to higher stages because an inconsistency will always reveal itself at some point in the analysis. Anticommutativity produces a closed 'cycle' with components (*A*, *B*, *AB*) and their conjugates (*A**, *B**, *AB**), and prevents further terms, such *C*, *D*, etc., from anticommuting with them in a consistent manner. However, successive cycles of the form (*A*, *B*, *AB*), (*C*, *D*, *CD*), etc., can be introduced into the structure, if they commute with each other, and this can be continued indefinitely. All of the terms then have a unique identity *because they each have a unique partner*, and the successive alphabets can be seen as a regular series of identically structured closed anticommutative cycles, each of which commutes with all the others. This structure is familiar to us in the form of the infinite series of finite (binary) integers of conventional mathematics, each alphabet representing a new integer. We can regard the closed cycles as an infinite ordinal sequence, and so establishing for the first time in this process both the number 1 and the binary symbol 1 of classical Boolean logic as a conjugation state of 0, with the alphabets structuring themselves as an infinite series of binary digits. Mathematics and digital logic become emergent properties of a rewrite process which has no specific defined starting point, and can be reconstructed endlessly in a fractal manner, with self-similarity at all stages and a zero attractor related to the Golden number 1.618 ... .

3 An Algebraic Structure

The universal rewrite system is a pure description of process with many representations. It can, for example, be presented as a series of 'doublings' or 'bifurcations', analogous to the initial creation of (R, R^*), even when they represent 'complexification' (the introduction of a new anticommutative cycle) or 'dimensionalization' (the closing of the cycle) rather than conjugation (the zeroing process used in the first alphabet). So, one way of writing

$$(R, R^*, A, A^*)\,(R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*)$$

would be as

$$(R, R^*, A, A^*)\,(R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, \quad B, B^*, AB, AB^*)$$

in which we retain the old alphabet (R, R^*, A, A^*) and a dual (B, B^*, AB, AB^*) formed by some process, as we did with R and R^*. In practical terms, we introduce a new character (B).

A significant representation is the mathematical one, for the effective creation of a discrete integer system means that, by applying this to the original terms (R, A, B, etc.), we can also generate an entire arithmetic and an algebra. Once we have generated integers, the rest of the constructible number system will follow automatically, along with arithmetical operations. At the same time, application of the constructed number systems to the undefined state with which the process began suggests that this state, which is not intrinsically discrete, can be interpreted in terms of a continuity of real numbers in the Cantorian sense.

In principle, discreteness appears in the construction only when we introduce anticommutativity or 'dimensionality', and specifically 3-dimensionality. Physically, 3-dimensionality, or anticommutativity, becomes the ultimate source of discreteness in a zero totality universe, and we observe, in all cases, that 3-dimensionality requires discreteness, and discreteness requires 3-dimensionality.

The rewrite process can be represented in symbolic form in the table:

	0	Δ_a	Δ_b	Δ_c	...	Δ_n
0	00	$0\Delta_a$	$0\Delta_b$	$0\Delta_c$		$0\Delta_n$
Δ_a	$\Delta_a 0$	$\Delta_a\Delta_a$	$\Delta_a\Delta_b$	$\Delta_a\Delta_c$		$\Delta_a\Delta_n$
Δ_b	$\Delta_b 0$	$\Delta_b\Delta_a$	$\Delta_b\Delta_b$	$\Delta_b\Delta_c$		$\Delta_b\Delta_n$
Δ_c	$\Delta_c 0$	$\Delta_c\Delta_a$	$\Delta_c\Delta_b$	$\Delta_c\Delta_c$		$\Delta_c\Delta_n$
:						
Δ_n	$\Delta_n 0$	$\Delta_n\Delta_a$	$\Delta_n\Delta_b$	$\Delta_n\Delta_c$		$\Delta_n\Delta_n$

Here, the Δ symbols represent the alphabets:

Δ_a (R)

Δ_b (R, R^*)

Δ_c (R, R^*, A, A^*)

Δ_d (R, R^*, A, A^*, B, B^*, AB, AB^*)

Δ_e (R, R^*, A, A^*, B, B^*, AB, AB^*, C, C^*, AC, AC^*, BC, BC^*, ABC, ABC^*) ...

The rewrite process is not restricted to any specific mathematical interpretation, but incorporates digital logic and binary integers, while a convenient consequence is the algebraic series:

$$
\begin{aligned}
&(1, -1) \\
&(1, -1) \times (1, \boldsymbol{i}_1) \\
&(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \\
&(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \\
&(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \\
&(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3) \ldots
\end{aligned}
$$

The anticommutative pairs *A*, *B*; *C*, *D*; *E* ... now become successive quaternion units, $\boldsymbol{i}_1, \boldsymbol{j}_1$; $\boldsymbol{i}_2, \boldsymbol{j}_2$); $\boldsymbol{i}_3$ ..., each of which is commutative to all the others. By the fourth stage, we have repetition, which then continues indefinitely. An incomplete set of quaternion units (for example, $\boldsymbol{i}_3$ in the sixth alphabet) becomes equivalent to the algebra of complex numbers. Mathematically, we can see the process of the creation of the zero totality alphabets as one of conjugation, followed by repeated cycles of complexification and dimensionalization.

At the point where the cycle repeats, we have what can be recognised as a Clifford algebra – the algebra of 3-D space, where the vectors **i**, **j**, **k** are constructed from $\boldsymbol{i}_1\, \boldsymbol{i}_2, \boldsymbol{j}_1\, \boldsymbol{i}_2, \boldsymbol{i}_1\, \boldsymbol{j}_1\, \boldsymbol{i}_2$, and $\boldsymbol{i}_1, \boldsymbol{j}_1, \boldsymbol{i}_1\, \boldsymbol{j}_1 = \boldsymbol{k}_1$ and $\boldsymbol{i}_2, \boldsymbol{j}_2, \boldsymbol{i}_2\, \boldsymbol{j}_2 = \boldsymbol{k}_2$ are (mutually commutative) quaternion algebras of the form $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$.

$$
\begin{aligned}
&(1, -1) \\
&(1, -1) \times (1, \boldsymbol{i}) \\
&(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \\
&(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i}) \\
&(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i}) \times (1, \mathbf{j}) \\
&(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i}) \times (1, \mathbf{j}) \times (1, i) \ldots
\end{aligned}
$$

In this algebra, the unit vectors **i**, **j**, **k** have the multiplication rules

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = 1$$

$$\mathbf{ij} = -\,\mathbf{ji} = i\mathbf{k} \;;\; \mathbf{jk} = -\,\mathbf{kj} = i\mathbf{i} \;;\; \mathbf{ki} = -\,\mathbf{ik} = i\mathbf{j}$$

which are essentially those of complexified quaternions, with multiplication rules

$$(i\boldsymbol{i})^2 = (i\boldsymbol{j})^2 = (i\boldsymbol{k})^2 = 1$$

compared to those for pure quaternions

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1$$

In the Clifford vector algebra, there is a full product between vectors **a** and **b** which combines vector and scalar products

$$\mathbf{ab} = \mathbf{a.b} + i\mathbf{a} \times \mathbf{b}$$

It has been shown by Hestenes[4] and others, that using a Clifford vector algebra is a natural way of incorporating spin into quantum mechanics as an automatic consequence of the vector structure of space and momentum. The units are, significantly, isomorphic to those of Pauli matrices.

Clifford vector algebra produces three subalgabras from the products of its basic units. Bivectors (for example, area and angular momentum, in physics) are products of two orthogonal vector units; they are also called pseudovectors and are isomorphic to quaternion units. Trivectors (for example, volume) are products of three orthogonal vector units, and are also called pseudoscalars; their full algebra is that of complex numbers.

vector	$\mathbf{i}, \mathbf{j}, \mathbf{k}$		
bivector	$i\mathbf{i}, i\mathbf{j}, i\mathbf{k}$	pseudovector	quaternion $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$
trivector	i	pseudoscalar	complex algebra
scalar	1		

Standard Clifford vector algebra notably produces these subalgebras in the reverse order to the universal rewrite system, which generates, in its first four alphabets, scalars, pseudoscalars, quaternions and vectors, along with the scalar subalgebras of pseudoscalars and quaternions.

Significantly, if we take all these algebras as *independently true*, and hence commutative, as the rewrite structure seems to suggest we should, since each is a complete description of zero totality, then we require an algebra that is a commutative combination of vectors, bivectors, trivectors and scalars, or vectors, quaternions, pseudoscalars and scalars. This turns out to be equivalent to the algebra of the sixth alphabet, a group structure of order 64 with elements:

$\pm \mathbf{i}$	$\pm \mathbf{j}$	$\pm \mathbf{k}$	$\pm i\mathbf{i}$	$\pm i\mathbf{j}$	$\pm i\mathbf{k}$	$\pm i$	± 1
$\pm \boldsymbol{i}$	$\pm \boldsymbol{j}$	$\pm \boldsymbol{k}$	$\pm i\boldsymbol{i}$	$\pm \mathbf{i}\boldsymbol{i}$	$\pm i\boldsymbol{k}$		
$\pm \mathbf{i}\boldsymbol{i}$	$\pm \mathbf{i}\boldsymbol{j}$	$\pm \mathbf{i}\boldsymbol{k}$	$\pm i\mathbf{i}\boldsymbol{i}$	$\pm \mathbf{ii}\boldsymbol{i}$	$\pm i\mathbf{i}\boldsymbol{k}$		
$\pm \mathbf{j}\boldsymbol{i}$	$\pm \mathbf{j}\boldsymbol{j}$	$\pm \mathbf{j}\boldsymbol{k}$	$\pm i\mathbf{j}\boldsymbol{i}$	$\pm \mathbf{ij}\boldsymbol{i}$	$\pm i\mathbf{j}\boldsymbol{k}$		
$\pm \mathbf{k}\boldsymbol{i}$	$\pm \mathbf{k}\boldsymbol{j}$	$\pm \mathbf{k}\boldsymbol{k}$	$\pm i\mathbf{k}\boldsymbol{i}$	$\pm \mathbf{ik}\boldsymbol{i}$	$\pm i\mathbf{k}\boldsymbol{k}$		

These generate an algebra which is isomorphic to that of the gamma matrices of the Dirac equation, as used in relativistic quantum mechanics.

4 Application to Physics

It would seem that we have an application of the system to mathematics, with binary arithmetic, digital logic and Clifford algebra appearing as consequences of the system rather than starting assumptions. The fact that the system becomes repetitive with the algebra of 3-D space also suggests that it might apply to the physical world as well. For this to be true we would expect each successive alphabet to have a physical meaning, and for each to be independently valid at the same time.

In applying to physics, we note that the universal rewrite creates successive models for a zero-totality universe. This is what we mean by *physical parameters*. We can recognize the algebras of the fundamental parameters mass, time, charge and space as being, respectively, scalar, pseudoscalar, quaternion and vector, exactly as is generated by the first four alphabets of the universal rewrite system. (Here, mass is the source of gravity and includes energy, while charge is a term used to represent the sources of the three nongravitational interactions.)[3] The four alphabets seem to be independent descriptions of the universe which must be simultaneously true, as should all subsequent alphabets following these.

i	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	1	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$
time	*space*	*mass*	*charge*
pseudoscalar	*vector*	*scalar*	*quaternion*
trivector	*vector*	*scalar*	*bivector*

Now, if we combine the algebras of these quantities, we obtain the 64-part algebra isomorphic to the Dirac gamma algebra. But the 8 units of time, space, mass and charge are not the minimum number of starting units to generate this algebra. This is, in fact, 5 and its construction always involves the breaking of the symmetry of one of the two 3-D components, space or charge. Typically, we 'combine' one of the units of charge with each of those of time, space and mass, to obtain:

$i\boldsymbol{k}$	$\boldsymbol{i}\mathbf{i}, \boldsymbol{i}\mathbf{j}, \boldsymbol{i}\mathbf{k}$	$\boldsymbol{j}$
energy	*momentum*	*rest mass*
$i\boldsymbol{k}E$	$\boldsymbol{i}\mathbf{i}p_x, \boldsymbol{i}\mathbf{j}p_y, \boldsymbol{i}\mathbf{k}p_z$	$\boldsymbol{j}m$
pseudoscalar	*vector*	*scalar*

The units $i\boldsymbol{k}$, $\boldsymbol{i}\mathbf{i}$, $\boldsymbol{i}\mathbf{j}$, $\boldsymbol{i}\mathbf{k}$, $\boldsymbol{j}$ correspond to the five base units of the gamma algebra, and the new terms, energy, momentum and rest mass, can be seen to take on aspects of the original charge units, together with the pseudoscalar, vector and scalar properties of their other parent quantities. Now, when combine the momentum terms into a single vector $\mathbf{p}$ and take the complete package $(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ to represent the properties of a fundamental physical unit (particle or fermion), we find that we can immediately solve the problem of indefinite extension of the alphabets in this case, since $(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ is a *nilpotent* or square root of zero. The equation

$$(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = E^2 - p^2 - m^2 = 0$$

is then simply the relativistic and quantum mechanical conservation of energy and momentum. So, if we take $(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ as incorporating all the alphabets needed to create a repetitive sequence, when we seek to generate the next alphabet by squaring, we will find that it is zeroed automatically, so zeroing all higher alphabets which incorporate it, and we can describe the world through an indefinite succession of such units.

Simultaneously with the creation of the concepts of energy, momentum and rest mass, the combination breaks the symmetry between the weak, strong and electric charges, which then take upon the algebraic characteristics of their associated parameters:

ik	***i*i, *i*j, *i*k**	***j***
weak	*strong*	*electric*
pseudoscalar	*vector*	*scalar*

The reduction or 'compactification' of the original 8 units to the composite set of 5 is, in fact, a characteristic symmetry-breaking operation in nature, which is found also in mathematics, chemistry and biology as well as in several aspects of physics. In all these areas, 5 seems to be the number at which symmetry is necessarily broken, which is one of the reasons for the significance of the Fibonacci numbers and the golden section in nature .

5 Nilpotent Quantum Mechanics

Nilpotent quantum mechanics is founded on a nilpotent operator which can be expressed in the form $(\pm ikE \pm i\mathbf{p} + jm)$, which is an abbreviated expression for a row or column vector, whose 4 terms encompass the four sign variations in E and $\mathbf{p}$. We can use a canonical quantization procedure to replace E and $\mathbf{p}$ as operators by $E \rightarrow i\partial / \partial t$, $\mathbf{p} \rightarrow -i\nabla$, for a free fermion, or by covariant derivatives such as $E \rightarrow i\partial / \partial t + e\phi$, $\mathbf{p} \rightarrow -i\nabla + e\mathbf{A} + \ldots$, for a fermion constrained by any number of potentials of any type or even by curvature terms. The structure of the operator then determines both the complete quantum behaviour of the fermion and also that of its environment or 'vacuum', by defining a unique phase term which, when operated on, produces an amplitude which squares to zero:

$$(\text{operator acting on unique phase term})^2 = \text{amplitude}^2 = 0$$

The process incorporates both Dirac and Klein-Gordon equations in the form

$$(\pm ikE \pm i\mathbf{p} + jm)\,(\pm ikE \pm i\mathbf{p} + jm) \rightarrow 0$$

where $(\pm ikE \pm i\mathbf{p} + jm)$ can stand for either operator or amplitude. This would make the Dirac equation for a free fermion

$$\left(\mp k \frac{\partial}{\partial t} \mp ii\nabla + jm\right)(\pm ikE \pm i\mathbf{p} + jm)e^{-i(Et-\mathbf{p}.\mathbf{r})} = 0$$

For a fermion under the constraint of a potential or any number of potentials, the phase factor would take a different form but the result would still be a term with the same structure as $(\pm ikE \pm i\mathbf{p} + jm)$ being squared to zero.

Nilpotent quantum mechanics is relativistic and is concerned with fermions.[3] It shares all the standard characteristics of relativistic quantum mechanics using the more conventional formalisms of the Dirac equation, and can be easily transformed into these formalisms using the one-to-one correspondence between the algebraic operators and gamma matrices. However, it also has some characteristics which only become apparent in this mathematical form, but which are necessary for understanding how the process can be scaled up in higher order systems.

Spin ½ and *zitterbewegung* are among the shared characteristics and can be easily derived using variants of the standard formalisms. Chirality (or the intrinsic left-handedness of fermions and right-handedness of antifermions) emerges in the same way. Fermion uniqueness or Pauli exclusion is obvious in the nilpotent formalism, as any combination of identical fermions will automatically vanish. However, the nilpotent also creates a completely new meaning for the concept. Because the totality of experience is defined always to be zero, if we take a fermion in any state, say $(\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, subject to any number of constraints that can be built into its operator, and imagine that we can create it from absolutely nothing, then the 'vacuum' which defines the rest of the universe for that fermion, must be a kind of mirror image, $-(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, so that both the superposition and the combination of vacuum and fermion remain at zero:

$$-(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) + (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0$$
$$-(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0$$

To maintain this zero totality in all circumstances, any change in either the fermion or its environment must be reflected in a corresponding change in the other. In effect, this creates a principle of self-organization which can be imagined on systems on a much larger scale.

Another significant aspect of quantum mechanics is that it involves both locality and nonlocality. The distinction between the two processes is clear in the nilpotent form. Everything *inside* the bracket is local; everything *outside* the bracket is nonlocal. So the conservation laws of energy and angular momentum are local; superpositions and combination states and interactions with vacuum are nonlocal. Both processes, however, are holistic in requiring the cooperation of the entire universe, and each produces consequences which affect the other. In nilpotent quantum mechanics, the individual fermion conserves its energy only with respect to the rest of the universe. The fermion is an open system and intrinsically dissipative. The first law of thermodynamics must be accompanied by the second.

The nilpotent structure here is very specific, with many consequences which will become significant in later sections, but the nilpotent outcome has an interesting generic relationship with Spencer-Brown's Laws of Form,[5] where the mark $\rceil$, distinguishes between *inside* and *outside*, producing two laws of transformation, similar, in principle, to idempotent and nilpotent properties. (Idempotents are used in the nilpotent quantum mechanics as partitions of the vacuum; it is also possible to write the nilpotent equations in idempotent form.[3]) The law of calling uses two marks, neither inside the other, to produce a single mark

$$\rceil\,\rceil = \rceil$$

while the law of crossing puts one mark inside the other to produce an unmarked state, equivalent to nothing

$$\rceil\!\rceil =$$

Fermions are, in a very fundamental way, incomplete. They have half-integral spin, are only observable when interacting in a pairing with other fermions, and are square roots of algebraic operators which only have meaning when multiplied with other objects of the same kind. In the nilpotent formalism, bosons of spin 1 and spin 0 are formed from fermion-antifermion combinations of the form $(\pm ikE \pm \mathbf{ip} + jm)\ (\mp ikE \pm \mathbf{ip} + jm)$ and $(\pm ikE \pm \mathbf{ip} + jm)\ (\mp ikE \mp \mathbf{ip} + jm)$, while a fermion-fermion combination can exist in the form $(\pm ikE \pm \mathbf{ip} + jm)\ (\pm ikE \mp \mathbf{ip} + jm)$ in Cooper pairs, Bose-Einstein condensates and other applications of Berry phase. All these expressions become scalars when multiplied out. All the tendency for aggregation in nature can be seen as stemming from the need for fermions to acquire partners to remove this incompleteness, and it can be linked to the action of a harmonic oscillator, of which the *zitterbewegung* is a special instance. The same pattern emerges at higher levels, suggesting that the nilpotent model applies well beyond the direct application of quantum principles. It is very likely that a major role in providing a 'staircase' that leads up from the smallest systems to the largest will be provided by the renormalization group procedure.

6 Nilpotents Using Discrete Variation

Reversing the usual procedure, we have shown how quantum mechanics can arise from a computational process. However, the application to processes at the purely quantum level is only one possible consequence. A discrete or anticommutative differentiation process, developed by Kauffman,[6] offers us a way in which we can make a link between quantum and classical worlds, with more general application than to the purely quantum state. Here, we define a discrete differentiation of the function F, which preserves the Leibniz chain rule, by taking the commutators:

$$\frac{\partial F}{\partial t} = [F, H] = [F, E] \quad \text{and} \quad \frac{\partial F}{\partial X_i} = [F, P_i] \tag{2}$$

with $\mathcal{H} = E$ and P_i representing energy and momentum operators, and make the further assumption that, with velocity operators not needed, we may use $\partial F / \partial t$ rather than dF / dt. We now define a nilpotent amplitude

$$\psi = ikE + \mathbf{i}iP_1 + \mathbf{i}jP_2 + \mathbf{i}kP_3 + jm$$

and an operator

$$\mathcal{D} = ik\frac{\partial}{\partial t} + \mathbf{i}i\frac{\partial}{\partial X_1} + \mathbf{i}j\frac{\partial}{\partial X_2} + \mathbf{i}k\frac{\partial}{\partial X_3},$$

Using (2) for $F = \psi$ and some basic algebraic manipulation, we obtain
$-\mathcal{D}\psi = i\psi(ikE + \mathbf{i}iP_1 + \mathbf{i}jP_2 + \mathbf{i}kP_3 + jm) + i(ikE + \mathbf{i}iP_1 + \mathbf{i}jP_2 + \mathbf{i}kP_3 + jm)\psi - 2i(E^2 - P_1^2 - P_2^2 - P_3^2 - m^2)$.

When ψ is nilpotent, then

$$\left(ik\frac{\partial}{\partial t} + \mathbf{i}\nabla\right)\psi = 0.$$

Over four solutions:

$$\left(\pm k\frac{\partial}{\partial t} \pm i\mathbf{i}\nabla\right)\psi = 0.$$

This is a discrete version of the nilpotent Dirac equation, in which we have eliminated the mass term from the operator, and also the phase factor. The equation is also valid, where nilpotency is a fundamental condition, in discrete classical as well as in quantum contexts, the factor i or $i\hbar$, as applied to the operator, being optional.

7 Dual Space and the Holographic Principle

The most significant aspect of the nilpotent structure is that it incorporates two full vector spaces with the full Clifford algebra of each. The 64-part algebra requires a Clifford vector algebra for space commutatively combined with its three subalgebras, representing time, mass and charge. If we take these three subalgebras together, we find that they have the mathematical characteristics of another vector space, entirely commutative to the first. This 'space', however, as a composite of three other parameters, is not an observable quantity. So, the nilpotent structure emerges from combination of two vector spaces, only one of which is observable.

We can call this unobserved space 'vacuum space', and its effects are immediately apparent in spin ½ and the 4-component structure of the fermion wavefunction. Here, the fermion also includes two terms associated with antifermion states. These are a manifestation of the fermion's vacuum, and are responsible for the fermion spending half its time as a real particle and half as a vacuum particle (*zitterbewegung*), which is also one of many ways of accounting for the fermion's ½ spin.

Another way of looking at this is to relate it to Berry phase, and to attribute this to the fact that the fermion is a singularity with respect to ordinary space. As is well known, Berry or geometric phase can be described in purely topological terms. If we parallel transport a vector around any complete circuital path in ordinary or simply-connected space, we can expect it to leave the vector pointing in the same direction at both beginning and end of the circuit. However, if the space of the circuit contains a singularity or is multiply-connected, then the vector will gain a phase change of π and end up pointing in the opposite direction from its starting position.

Spin ½ could be seen as indicating that the fermion singularity rotates in its *own* multiply-connected space. So, we can attribute the same effect to the fact that the fermion is defined as a singularity and that it is defined by a nilpotent connection between two spaces, leading to the conclusion that the dual space structure is actually *responsible* for the existence of discrete matter in the form of physical singularities. In our understanding, the Berry phase / spin ½ / *zitterbewegung* is that, defining a localised point particle occurs simultaneously with defining the nonlocalised vacuum that determines its relation to the rest of the universe, and that carries the information about its future evolution. We can consider Berry phase to be a particularly significant indicator of the presence of some kind of dual space, nilpotent-related behaviour, especially in systems subject to self-organization.

The nilpotent dual spaces are genuinely dual, in that they contain precisely the same information, though in different forms. This duality between two spaces (which should be distinguished from algebraic duals to vector spaces) has many manifestations. For example, the uniqueness of the nilpotent ($\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$) and Pauli exclusion could be determined by the 'direction' of a line drawn from the origin if E, p and m as represented as coordinates on the quaternion axes $\boldsymbol{k}$, $\boldsymbol{i}$ and $\boldsymbol{j}$. Alternatively, we could express Pauli exclusion by the more conventional method of defining fermion wavefunctions as antisymmetric. This leads to a truly remarkable result if we take $(\psi_1 \psi_2 - \psi_2 \psi_1)$ for two fermions in the nilpotent formalism:

$$(\pm \boldsymbol{ik}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)(\pm \boldsymbol{ik}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2) - (\pm \boldsymbol{ik}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)(\pm \boldsymbol{ik}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)$$

$$= 4\mathbf{p}_1\mathbf{p}_2 - 4\mathbf{p}_2\mathbf{p}_1 = 8\, i\, \mathbf{p}_1 \times \mathbf{p}_2,$$

for this only has a nonzero value if the fermion spins are oriented in different directions.

In effect, the entire information about a fermion state is contained in its instantaneous spin direction, or in the plane to which this is perpendicular. In principle, the orientation of the fermion in real space and in the 'vacuum space' created by the quaternion axes $\boldsymbol{k}$, $\boldsymbol{i}$ and $\boldsymbol{j}$ carries the same information. Exactly the same duality occurs in the derivation of spin ½ either from the anticommutativity of the momentum operator, which uses real space, or from the Thomas precession, which uses vacuum space, and the duality again informs the holographic principle.

The holographic principle, in which the entire information about a system is found on the bounding area, is thought to be a significant organizing principle for many systems. We have already considered it as 'a characteristic signature of a nilpotent, self-organizing system with its planar fractality'.[7] Essentially, it uses the information coded in the E and $\mathbf{p}$ terms of the operator $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, that is, in two components of the vacuum space, as nilpotency makes the third term redundant. This then becomes equivalent to using the information coded in two components of the dual real space. Significantly, this can also be coded in one dimension of space and one of time, which would be equivalent to using the vacuum space. As space and momentum are conjugate variables, area is also a conjugate of angular momentum, and $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ is recognizably an angular momentum operator, with the E term determining the handedness, $\mathbf{p}$ the direction and m the magnitude. Since any system which conserves angular momentum or which operates according to the holographic principle (for example, galaxies acting collectively under gravity) can be expressed in this form, then any such system can be seen as a direct analogue of the nilpotent fermion, even though it is not intrinsically quantum or relativistic.

The application of the nilpotent operator to the holographic principle also suggests that it can itself be regarded as a quantum hologram, with phase $\boldsymbol{ik}E$, amplitude $\boldsymbol{i}\mathbf{p}$ and reference phase $\boldsymbol{j}m$. Again, we can recover the entire structure from just two terms, for example, phase and reference phase. Quantum holography has now been officially recognized as occurring in the case of 'quantum holographic encoding in a two-dimensional electron gas',[8] but the work of Walter Schempp has already shown that it has extensive practical application in Magnetic Resonance Imaging based on harmonic analysis on the 3D Heisenberg Lie group.[9] The universal rewrite system shows that the repeating unit that we need for the description of a quantum or quantum-like system is a double vector space. The two three dimensional spaces 'relate to the 3D Heisenberg Lie Group and its nilpotent Lie algebra and their dual / inverses, and make quantum holography possible via Fourier transform action'.[7]

Quantum holography, unlike classical holography is nondegenerate, and so shows how the uniqueness of nilpotency becomes the source of the uniqueness of the semantic logic which becomes possible with the physical description. Only one universal condition can apply at any given instant, and this is described by a unique birthordering among nilpotent fermionic states.

The holographic paradigm is particularly significant in that the wavefunction is defined only up to an arbitrary fixed phase, which provides a simple physical understanding of the quantum vacuum in quantum field theory, in allowing only relative phases, which encode the 3+1 relativistic space-time geometries, to be measured. This phase becomes the arbitrary fixed measurement standard for all subsequent measurements, and acts 'as the holographic basis for a self-organized universal quantum process of emergent novel fermion states of matter' in which, after each emergence, 'the arbitrary standard is re-fixed anew so as to provide a complete history / holographic record or hologram of the current fixed past, advancing an unending irreversible evolution', as perceived by our senses. The universal rewrite system 'implies that the inseparability of objects and fields in the quantum universe is based on the fact that the only valid mathematical representations are all automorphisms of the universe itself, and that this is the mathematical meaning of quantum entanglement'.[10]

8 Self-Organization

The significance of nilpotent functions is that they operate in such a way that the system becomes a mirror image of its environment, changing in response to it and also directly changing it in a precisely defined way. Though quantum mechanics provides the most perfect example, nilpotency is not confined to quantum physics, or even to physics. A number of characteristic mathematical patterns suggest that there are applications in biology and living systems, especially in relation to the genetic code. They emerge when we apply universal rewrite in a semantic view of nature as opposed to the more purely syntactic view generated by mathematics.

Of particular significance is the need for dual spaces to explain the nilpotent structure, for these are what provide all self-organizing systems with their dynamic evolution, one space being that of real observation, and the other (the 'vacuum' space) being the source of the potential changes. Nilpotency emerges only when these are made dual, so that they incorporate the same information, though the symmetry-breaking ensures that they are packaged in different ways. One space can be considered as describing the system's effect on the environment, and the other as describing the environment's effect on the system. The only requirement is that the system is described discretely; it does not have to be specifically 'quantum'.

The universal rewrite system appears to predict a 'staircase' of emerging structures in which nilpotents at one level become units at the next. From the perfectly self-organizing fermion, we move to structures composed from fermions, such as bosons, nuclei, atoms and molecules which are described by the same quantum mechanics, and so self-organize in the same way. Of course, though overall structures may prevail where there is coherence (as in crystals), coherence is quickly lost when many different components interact, possibly even with chaotic consequences. Nevertheless, it seems that a self-similar order tends to emerge at higher levels, based on the universal Golden attractor. An example of such self-organization in condensed matter is provided by linked chains of magnetic atoms of cobalt, one atom wide, which transform, under a magnetic field applied at right angles to an aligned spin into a quantum critical state.[11] This is a quantum version of a fractal pattern, with resonances occurring in the golden ratio of 1.618…, and an underlying E_8 symmetry.

A very significant area where self-organized systems interact with environments occurs, of course, in biology, and a very extensive range of work in this area is to be found in the papers of Hill and Rowlands.[12-14] The human brain is also very likely organised according to the 'natural' logic which emerges from the universal rewrite structure, thus explaining its semantic, as well as syntactic, capabilities. In Vitiello's quantum field model, the brain is a dissipative system in which positive energy output by the brain is mirrored by a negative energy output by the environment.[15] A nilpotent wavefunction would provide an ideal mathematical structure for modelling this process. Various large-scale physical processes, as well as quantum ones, also appear to generate structures which are similar to those which emerge in biology, and which appear to be explicable in terms of the rewrite process. One is a helix-shaped nebula near the centre of the Milky Way, twisted into shape by a powerful magnetic field, which looks very like the classic image of a DNA molecule.[16] Another is a demonstration that particles in a plasma can undergo self-organization into helical strands, again resembling those of DNA, which can replicate and even 'evolve' by natural selection.[17]

The self-organizing systems found in biology and chemistry frequently show order emerging from chaos. In the universal rewrite system, we have shown that 'quantum chaos' leads directly to nilpotence, 'where this is the quantum criterion for the universal fractal behaviour and quantum holographic patterns and signal pattern recognition, via the phase parameterized universal fractal golden number attractor where every such nilpotent pattern exhibits uniqueness and other features'.[7] The rule of successive cardinalities in the rewrite procedure is indicative of an irreversible procedure, and this is reflected in the thermodynamic properties of the nilpotent structure and an open, and therefore entropic system. The unique birthordering of nilpotent states or systems, which is the ultimate consequence of the universal rewrite procedure, additionally suggests that another characteristic indicator of its presence is the Quantum Carnot Engine extended model of thermodynamic irreversibility,[18] where a small amount of quantum coherence / entanglement remains at the elementary particle level to constitute new emergent fermion states of matter.

Following the accumulated evidence presented in this and the preceding sections, a completely new application has emerged. Our previous work has indicated that geometrical or Berry phase is a particularly significant identifier of nilpotent-like behaviour in a system that need not necessarily be quantum In an earlier paper, we wrote, concerning nilpotent structure: 'each X^2 signifies a return (in terms of a corresponding unique dual Dirac annihilation operator) to the quantum mechanical vacuum state which takes the form of a universal attractor of fractal dimension 2 …, where the uniqueness of each of the nilpotent quantum mechanical Dirac operators is carried by means of quantum phase, in the form a unique gauge invariant Berry / geometric phase able to encode the requisite relativistic 3+1 space time geometric information about the unique fermion state vector, and is 'scale free'.'[19]

Now, new research, by Kaschube *et al.* shows that the neurons in the visual cortex in the brain of three distantly-related mammals have a quasiperiodic structure. Orientations of the neurons in the flat sheets of the cortex change continuously, repeating over a length known as the 'map period' (λ), while appearing to converge on centres known as 'pinwheels', while the pinwheel density per λ^2 appears to equal π to within a few percent.[20] According to Kenneth J. Miller, writing in the same issue: 'The result offers insight into the development and evolution of the visual cortex, and strongly suggests that key architectural features are self-organized rather than genetically hard-wired.' Miller also says that 'The universality of self-organizing behavior provides a simple and compelling explanation for the arrival of widely divergent evolutionary lines at this common design.'[21]

In our interpretation, the appearance of π in the density of the squared 'map period' of the neurons in the visual cortex strongly suggests the presence of a geometrical phase term in the spatial structure of the system. If we regard the 'pinwheel' as the equivalent of a singularity in the space, then a *double* circuit through the cycle of orientations or 'map period' would be needed to return to the original phase state. So the singularity would correspond to a double map period (2λ) in any given direction of the two-dimensional cortical sheet, and each pinwheel singularity would situate itself in a circle with radius length λ in this two-dimensional space, creating a pinwheel density / λ^2 of π. This would coincide directly with our proposal that a characteristic structure for the space of self-organizing systems at all levels of complexity results from a dual vector system, or equivalent, for which a geometrical phase of π becomes an identifying feature.

The universal rewrite system provides a blueprint for self-organization mediated through the nilpotent relation between the defined system and the rest of the universe, which emerges in this form of quantum mechanics. The particular characteristics of nilpotent quantum mechanics provide us with a number of identifiers which we have already linked to self-organization, citing specific examples, and which include double 3-dimensionality; a 5-fold broken symmetry; geometric phase; spin ½ or equivalent double helical structure; uniqueness of the objects and unique birthordering; irreversibility; dissipation; chirality, harmonic oscillator mechanism, *zitterbewegung*; fractality of dimension 2; holographic principle and quantum holography.

Of course, it would be interesting to see if we could create *artificial* self-organizing systems operating in the rapidly changing environments that have led to the evolution of natural ones. Digital logic gives us the correct syntactic structures for creating new 'artificial intelligence', but lacks an organizing principle. If, however, we consider that this principle is provided by universal rewrite, and, in particular, its nilpotent manifestation, then it would be possible to approach digital construction using the discrete version of nilpotency, represented in section 5. Digital logic allows us to deal with amplitudes much readily than phases, and commutators more easily than differentials, so it might be possible, using this calculus, to design a digital system based on amplitude changes, with an asymptotic approach to nilpotent conditions. In addition, the *E* and *P* terms which represent 'energy' and 'momentum' in the physics application, can be regarded, in a more general context as the parts of the system responding to respective changes of the time and space coordinates. Possible ways of approaching such a construction might, for example, use the cellular automata promoted by Wolfram,[22] the discrete anticipatory computation of Dubois,[23-24] the theory of the cybernetic and intelligent machine based on Lie commutators of Fatmi, Jessel, Marcer and Resconi,[25] or Fatmi and Resconi's New (semantic) Computing Principle.[26] In an earlier paper,[27] we made the case to explain 'how intelligence evolved' in a self organized universe through the universal rewrite process, and it may be that we can now see a route to developing both theory and applications relating to this process.

9 Conclusion

We have shown that a universal rewrite system, based on zero totality, can generate both a mathematical structure, related to Clifford algebra, and a physical picture of the universe, based on the principle of nilpotency, in which a system and its environment show a variation, defined by the phase, which preserves, through any number of changes, a dual mirror-image relationship between the two. As we have demonstrated, there is already a large body of strong evidence which supports the application of this nilpotent universal rewrite approach to quantum physics. The dual relationship between fermion and environment can be seen, in addition, as a perfect example of how self-organization might work in nature, and we have suggested that the universal rewrite mechanism might be considered as a computational model for the whole class of self-organizing systems. Many examples of recent discoveries in physics, chemistry and biology have been proposed as supporting this model, based on various properties that might be taken as indicators. These examples include a new application to the structure of the visual cortex, which appears to follow our prediction that the Berry phase as a strong indicator of the dual space structure which the model requires. From the point of view of future practical application, we have suggested that a nilpotent representation based on a discrete differentiation procedure might offer a new means of using existing digital computational technology to approach the construction of an artificial self-organizing system, possibly linking this with the computational methods used by other workers.

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