Locality, Weak or Strong Anticipation and Quantum Computing. I. Non-locality in Quantum Theory

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Abstract The universal Turing machine is an anticipatory theory of computability by any digital or quantum machine. However the Church-Turing hypothesis only gives weak anticipation. The construction of the quantum computer (unlike classical computing) requires theory with strong anticipation. Category theory provides the necessary coordinate-free mathematical language which is both constructive and non-local to subsume the various interpretations of quantum theory in one pullback/pushout Dolittle diagram. This diagram can be used to test and classify physical devices and proposed algorithms for weak or strong anticipation. Quantum Information Science is more than a merger of Church-Turing and quantum theories. It has constructively to bridge the non-local chasm between the weak anticipation of mathematics and the strong anticipation of physics.

Keywords : non-locality, anticipatory systems, quantum computing, pullback-pushout, Dolittle diagram.

1 Weak and Strong Anticipation

Computation whether performed by humans or machines from the abacus to current computers, is an activity of an anticipatory system. For these all behave as models of arithmetic, the logic of numbers. Numbers are fundamental epistemological components of locality that can be constituted into more complicated structures in geometry and text which includes algebra. According to Rosen [80] systems that follow their normal behaviour, for instance physical systems acting under the principles of mechanics, operate according to a reactive paradigm. How these systems behave can be predicted by anticipatory systems. Machines that follow algorithms can calculate the behaviour in advance as anticipatory systems [28, 29]. This is weak anticipation. Classical computers rely on the statistically predictable behaviour of matter in bulk to give the right answer on average. Usually this is a phenomenon at one level arising from a limit at another level.

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In anticipating arithmetic, computation is a local version of some more widely distributed system with a global presence. Because of the general principle of adjointness [33, 67], any category can anticipate any other category. Category theory [67, 9] is used for reasons to be given throughout the rest of this paper. This general applicability is an expression of the free functor F which selects category A as an anticipatory system of category C as shown in Figure 1. The unit of adjunction η expresses the creativity of the selection [81]. The co-free functor G gives the characteristics and the counit ϵ is a measure of the quality [40] of the characterisation. The category A is in general an anticipatory system giving weak anticipation of category C. It is in this sense that computation is an anticipatory model. For anticipation in time G will have a temporal component. Category theory is able to give a definitive abstract form of adjointness which appears in many guises often very specialised versions like the Galois connection [15] and Stone-C̃zech compactification ([33] para 1.814), ([56] section IV 2) which are useful in physics.

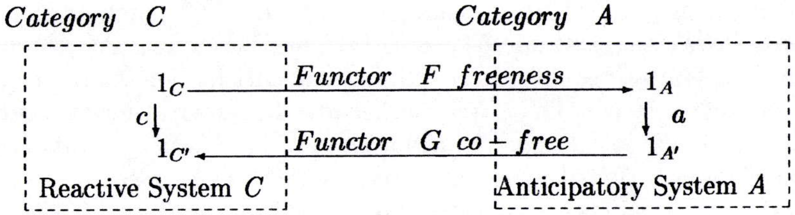

Fig. 1: Nature of Anticipatory Systems: a anticipates c

Figure 1 gives typical arrows $c : 1_C \longrightarrow 1_{C'}$ and $a : 1_A \longrightarrow 1_{A'}$ for the model. The anticipation of c by a arises from the adjointness: the identity functors 1_C, 1_A describe respectively the type of objects C and A. The double bar indicates inference and its converse.

$$\begin{array}{c} 1_C \leq GF \\ \hline\hline FG \leq 1_A \end{array}$$

GF is the functorial composition of applying functor G to the result of applying functor F to category C. FG is the corresponding application of functor F to the result of applying functor G to category A. The symbol $\leq$ is the usual reflexive transitive ordering. This expression gives a formal description for anticipation (Dubois [29, 30]). For strong anticipation A is a subcategory of C. Note that this means that not only its objects but also its arrows are to be found in C. If the objects and arrows in A are isomorphic to those in C then A is a reflective subcategory. Reflective subcategories provide the characteristics of notions like fractals or the monads of Leibnitz. Local quantum objects like photons and electrons exhibit the properties of non-local states. Because of the innate non-local structure of the quantum world (represented by functors F and G), Figure 1 can represent weak anticipation in classical models and strong anticipation as in quantum systems. If the word model is applied to these it is not quite in the same sense. A quantum theory example is the Einstein-Podolsky-Rosen [31] effect (EPR) which gives rise to Bell pairs C and A. The types 1_C and 1_A are of opposite parity. Expression 1 shows that observation of A anticipates the parity of C so that each of the pair anticipates the other giving rise to superluminal correlation without superluminal communication. This phenomenon has now been verified experimentally over 10km and confirms the reality of non-locality [94]. The only freedom is in the selection of F which uniquely determines G. If F is an identity, so is G. If F is an isomorphism, so is G. Only when F and G are both identities, will the model be wholly accurate. In practice there will always be inaccuracies but these will be of no consequence if the system is operating within some bounds of statistical significance for external observation with analogue systems or some internal reductionism in digital systems. In this lies the great power of numbers and the use of arithmetic but this does not assume the existence of the natural numbers which is an approximation for digital computers. However, the inaccuracy comes with the system. This is usually tolerable although not always, for instance when the computer system crashes altogether. Classical computers exhibit strong anticipation. This is viewed as a disadvantage giving problems in reliability and reproducibility. Reproducibility is usually taken as an advantage in classical systems because it provides scientific verification. However replication does not exist in quantum systems in a precise form although approximate cloning is possible and can be put to good use [18]. The tolerance can be improved by appropriate engineering depending on the selection of the functor F. According to Leibniz [61] each monad mirrors the universe from its own point of view, every monad differs from the other. The counit ϵ is a qualitative measure of this.

Full computation as a physical process is not accessible by local methods. This is the message of quantum theory. The bounds of approximate classical methods are themselves now being reached with miniaturisation and increasing numbers of chips on smaller-scale surfaces. It is not fully clear what will be the effect in nanotechnology at hybrid quantum/classical boundaries where it is envisaged that very small classical machines are to operate in environments where they may exhibit quantum phenomena like tunnelling electrons [7, 22, 51]. Little seems to be known of physical behaviour and the operation of the correspondence principle for the boundary conditions in the zone where quantum and classical phenomena meet. Developments with the engineering of nano technology should provide a physics nano laboratory where experimental data can be collected. The same question of the division between classical and quantum computing seems also to arise in biocomputing such as *in vivo* gene construction using DNA techniques [1, 4, 17].

In reality, all computing is quantum computing. This is because computation is a physical process and all physical processes are fundamentally governed by quantum mechanics. Non-locality is a general property of nature [23, 72]. Quantum systems are non-local. Therefore true quantum computation is beyond the local model. Quantum computation is still an operation of an anticipatory system but it is now strong anticipation because it must arise from within the system itself and not locally from some separate model. Strong anticipation involves the system providing its own self-prediction. In categorial terms the functors in the diagram in Figure 1 are endofunctors. In the non-local version of category C there is no other category than category C the system itself. However Category A is still a reflective subcategory of C (Freyd & Scedrov [33] at section 1.813).

Modern computers with a von Neumann architecture are physical devices viewed from a preliminary classical mode and whose operations model classically what in its true form is quantum. As anticipatory systems [80], these computing machines have weak anticipation. Non-locality arises in any form of parallel computation. Challenges of a non-local character may be a prime cause of the limited impact of parallel hardware devices beyond von Neumann that have been pursued since the 1980s like the distributed array processor [54] and artificial neural networks [69]. However, now quantum computing is an application within Quantum Information Science where challenges of non-locality need to be faced head-on for substantial progress to be made.

2 Non-locality

Quantum systems are non-local. Non-local is not the same as universal but a holistic approach is needed to encompass non-locality where local systems fail. A related property is openness. Locality is not open, non-local systems are open. As a weak anticipatory system a model therefore is as we have seen normally local. A universal model is a contradiction in classical terms for a proper model gives only a partial representation. In the classical world, trivially, the only true model of an object is the model itself with identity adjoint functors. Weak anticipation is local. Strong anticipation, however, as we have seen [41, 42] can be non-local by being embedded in the system itself.

First-order methods are local. Classical models provided by set-theoretic methods are mostly local because there is usually (although there does not have to be) a specificity invoked under the axiom of choice as arising from an operation of the left adjoint free functor. These classical models include the use of number systems so far as the natural, real, and complex numbers and quaternions are concerned but possibly not so far as the non-associative octonians and Cayley numbers. In particular the statistical methods then will include all models derived from them and predominantly the concept of probability. However not all mathematical methods are local. Category theory can provide a mathematical work space which is global and non-local. Table 1 shows corresponding alternative descriptions respectively for non-local and local anticipation.

The left-exact ontology is the quantum world of reality, the right-exact epistemologies is the world as we see it, the world of the observer, the world that exists in the mind, a local reflective subcategory of the world itself. The corresponding equivalence of the items in the list on the right of the table are mainly verifiable empirically from the fact that sets exist only in the mind. Most of the items on the right are based on the concept of *set*. Some have been proved formally as the proof by Diaconescu [26] and Myhill [71] that Boolean is equivalent to the axiom of choice. Some concepts that lie between left and right have been omitted like topology. Topology has some of the characteristics of the left-class because of openness and the nature of homeomorphisms. However, the basis in sets anchors topology very firmly to the right-class, although these touches of the left-class have made topology a useful modelling language for quantum theory for example in the use of *knot* theory [59]. Artificial neural nets [69] have been included on the left as a kind of hardware equivalent of a statistical model they would be on the right. However, it depends on how they are applied, that is the interpretation of their input/output data and the nature of any inherent feedback. Because they are 'massively parallel' these have some non-local features. For instance the way they can handle embedded data types. The earlier perceptron [79] with its simple feedback loop is on the right.

Stone duality could be placed on both sides of Table 1 because it appears to subsume both. Taylor [91] and Johnstone [56] discuss this important relation of Boolean algebras and lattices advanced by Marshall Stone [89]. Taylor has a current manifesto [92, 93] for an Abstract Stone Duality relying on Paré's theorem from 1973 that the monadicity of the contravariant powerset functor classifies subobjects in an elementary topos. Taylor‘s manifesto attempts to re-axiomatise mathematics on the basis of the monadic adjunction between the dual categories of frames and spaces, rather than on finite intersections and arbitrary unions.

It may be observed that there is a once and for all exercise of the axiom of choice for all entries on the left of the table. However, that is at a higher level namely at the level of the universe itself which is local. The *axiom of choice* can only be local and is included solely in the right-hand list. For the same reason *universal* (a local concept) is on the right in contrast to its interior, that is *global*, which is non-local and on the left.

These considerations seem to be an essential prerequisite for the subject of QIS (Quantum Information Science) but seem to be neglected in current texts [25, 74, 16, 47] even when the aim is the construction of practical information systems. If a quantum computer is to be built it will have to be designed to operate non-locally. Conventional software engineering in standard design would seek to map on to computer structures the models of reality. Three objectives are maximal cohesion, loose coupling and low-energy performance [88]. Usually overall guidance is provided by heuristics. Is the same approach needed in quantum information systems or is it even possible?

Table 1: Local Epistemology and Non-local Ontology

left-class: non-local ontology	right-class: local epistomologies
strong anticipation	weak anticipation
neutral model	proper model
systemic	component
global	universal
hyperincursion	incursion
	cardinals, ordinals, partially-ordered set (POS)
	axiomatic system, axiom of choice
intuitionistic	positivism
‘informal‘ e.g. natural language	formal e.g. formal language
topos	set, concrete categories
	graph theory
Heyting	Boolean
	sheaf theory
	fuzzy logic
octonians	natural numbers, integers
Cayley numbers	quarternions
geometry	arithmetic
co-algebra	algebra
	dimensions
randomness	probability
	quantum logic
Berry‘s geometric phase	gauge theory
open	closed
parallelism	
higher-order	first-order
	automota, Petri nets
	model theory
	measure theory
	Shannon information theory
	noise
holography	spectral theorem
	superposition
self-organization	chaos theory
emergence	complexity theory
	metric spaces
large categories	concrete categories
	natural number object (NNO)
2-categories	n-categories
Stone duality	tertium non datur

We can see from the study of anticipatory systems that the critical question lies in the word *model.* Is it possible to formulate a model for quantum computing that is implementable? The nature of quantum theory raises a doubt. There is a big jump here between theory and practice. Quantum mechanics was developed in the first half of the twentieth century by considering departures from classical mechanics in the behaviour of objects at a very small scale. Powerful theories emerged by the use of formal tools in the hands of pioneers but they are theories that all appear to be local and classical models of non-classical phenomena. Also at around the same time as quantum theory was under development, workers like Church and Turing were engaged on a theory of computation mainly using the classical logic of the day. Both groups relied on axiomatised theory including the axiom of choice. The mathematical environment of both quantum theory and the logic of computation was predominantly algebraic. Geometric perspectives were hardly explored although these were available at the time in areas like differential geometry and topology where concepts of non-locality and openness can be found. This algebraic influence has persisted and still dominates both the field of quantum theory and that of theoretical computer science.

3 Implementing Quantum Computers

The concept of the quantum computer was realised during the last two decades of the twentieth century and as might be expected drew heavily on standard quantum theory and computational theory to postulate an analogous Church-Turing hypothesis for quantum computing. However realising the concept of a quantum computer is not the same as realising a quantum computer. A quantum information system is a physical system. The current quantum paradigm is mathematical. The effect is to view the quantum computer as a suite of classical models. But a classical model of quantum computing is not quantum computing, it is classical. The literature on the quantum computer is mainly bottom-up replicating the path of the classical computer in the last half-century. The qubit corresponds to the bit, quantum logic to propositional logic, quantum algorithms like those of Shor [87], Grover [37] and Deutsch-Jozsa [24] are alternative to NP methods. Shor and Deutsch-Jozsa algorithms are rather like a quantum version of the fast Fourier transform requiring only n^2 steps rather than $n * 2^n$ steps. The review by Aharonov [2] notes that the Grover iteration can be understood as a product of two reflections. Deutsch-Jozsa [24] applied a quantum algorithm to determine whether an unknown mathematical function is constant or balanced (for instance as many 1's as 0's). These methods all appear to be local in essence. However, in the context of anticipatory systems Makarenko ([63] para 6.6) has raised the question of non-locality and proposed the term ***hobit*** (for a holistic bit) instead of the qubit.

Maurer and colleagues [66] have used conventional computers to calculate how quantum computers would cope with a well-chosen portfolio of programs for solving

NP-complete problems. Quantum parallelism is claimed at least to double the speed or be up to ten times faster with a single program. In another study by Chuang employing nuclear magnetic resonance to carbon-13 in chloroform molecules dissolved in acetone, it was estimated that a quantum computer on average required one evaluation for a function compared to 2.25 for a classical computer [19].

In a recent experimental realization of quantum games on a nuclear magnetic resonance quantum computer, Jiangfeng [55] has generalised the quantum prisoner's dilemma by the case of non-maximally entangled states. Results suggest the existence of two thresholds partitioning these regions classical, quantum and in-between. This may prove very significant for exploring the nano/quantum interface.

The full significance of QIS for the construction of future information systems and the role of quantum algorithms in databases will also need to be addressed in a future paper. It is a question of the structure inherent in information. A database scheme utilises this in the construction and storage of the data. Tree constructions with lexicographical ordering may give the order of log N comparisons. So some elementary structuring (B-trees) can give faster conventional systems [20] than by the use of Grover's algorithm. B-tree searching enables one record in one million to be retrieved in five disk accesses [84]. Grover‘s algorithm for quantum searching of structured databases is an example of the classical-quantum gap.

Bhattacharya, on the other hand, with colleagues at the Van der Waals' Zeeman Institute have implemented [11] a quantum search algorithm using classical Fourier optics showing that classical waves can search a N-item database just as efficiently. It is claimed that although the lack of quantum entanglement limits the database size, entanglement is neither necessary for the algorithm itself, nor for its efficiency.

A classical computer can be programmed to model the operation of a quantum computer with Grover‘s algorithms [38] but only as a weak anticipatory system and therefore very inefficiently. It is still to be shown whether any of these published algorithms are really only in the same anticipatory class and so lack the capability of non-local operation.

The non-locality point runs deeper and is really a fundamental problem for the use of mathematics in physics. Axiomatic methods have dominated mathematics for the last three hundred years but the crunch time may have come with the challenge of the quantum computer. For this widespread interest and effort that is being devoted to how to program quantum computers can at present only be directed towards mathematical representations (and only local weak anticipatory versions at that) of such machines. If these algorithms are to be realisable on physical machines we need to re-examine the various interpretations of the physics to be found in quantum theory in order to be satisfied that they can be converted into constructable systems.

These theoretical advances of Deutsch in the 1980s followed by the algorithms of Shor and Grover in the 1990s have led to some fervent activity in developing quantum physical devices that could form the components to realise future quantum computers. These have to be examined closely. There may now be a temptation for any researcher on small-scale materials to hope their results have potential in quantum information science [75]. The test would seem to be whether the significance of operation lies within the left-class in the first column rather than only within the local right-class in the second column of Table 1.

Physics is not axiomatic as demonstrated by the failure of David Hilbert's sixth of his 22 mathematical problems for the twentieth century [45, 46, 21]. Axiomatic mathematics works quite well by test of experiments, prediction, etc, that is as an anticipatory system. Experimental verification of theory has formed the main path for the development of science for the second-half of the last millennium. This is realisation of the anticipation of an anticipatory system. It confirms that the local solution in the normal holds in the wider world. Whether the wide-world in this context is still local depends on the experimental conditions. If things in the mind are normally local, the non-local validity of thought experiments has a question mark hanging over it until it is realisable in the universe. This is particularly so for the non-local procedures of quantum computation. For realisability as in QIS there is a need for extra constructive power. The essence of the technology is a macroscopic variable under the control of microscopic energy [10]. One of the earliest physical devices proposed as a quantum-processing element is the Super conducting Quantum Interference Device (SQUID [96]) consisting of a super conducting ring interrupted by a Josephson junction.

In 1998 Hey [44] reviewed possible technologies up to that date including SQUIDs, trapped ions, nuclear magnetic resonance and solid-state devices concluding that 'most of the current proposals are little more than a wish-list with a critical step missing' usually in scaling-up. In the last year or two there have been further developments. The ability drastically to slow down pulses of light not just in gases but also in solids like yttrium-based crystal where the pulses can be trapped and released has been suggested as the basis for a high-density information storage in quantum computing [95]. This could possibly be non-local as could the control states in the Earth's gravitational field which have been observed by Nesvizhevsky [73] The proposal in laser science to search simultaneously a whole database with what is termed a 'database wave' would be non-local [27]. Others seem to be more directed at the localised qubit. Sackett and co-workers at NIST [83, 82] have obtained entanglement of a 4-qubit system using beryllium ions held in an electromagnetic wave trap. More recently the entanglement has been extended to a 7-qubit system [62].

Much of the activity on potential quantum devices is in nanotechnology [8] where as previously mentioned it is not yet clear what is happening at the cross-over boundary from classical to quantum phenomena. Dekkar [7] has produced logic circuits built from carbon nano tubes as single 'transistors'. Lieber has been able to get these to self-organise and grow with controlled dimensions into various arrays including a potential random-access cell [22].

It might be noted that silicon is still trying to fight back. Kane [58] reports the design of a silicon-based quantum computer. Some report direct observation of quanta phenomena like the experimental long-lived entanglement of two macroscopic objects in Julsgaard [57]. The control of Bose-Einstein condensates seems non-local and possibly 'hobit' in character, not just qubit. Reichel [39] with colleagues at Ludwig-Maximilians University in Munich have used a lithographic technique to create Bose-Einstein condensates and manipulate them using so-called *atom chips.*

Holography is an alternative approach applicable to quantum computation (Marcer [65], Schempp [85], Mützel [70]) also in the hobit mode of operation. Quantum dots can generate non-classical light with 'tunable' photon statistics and can easily be embedded in solid-state systems [77, 60] where they can exhibit properties of atoms, like discrete energy spectra. There is recent evidence (Meschede [68]) that ultra-small photonic crystals mught be constructed with complex nanostructures by the positioning of atoms with a laser to build an optical quantum computer which could operate possibly in either qubit or hobit mode.

Nevertheless at first sight most methods of building a quantum computer bottom-up appear to belong to the right-class of local methods (Table 1) but if the laboratory techniques involve self-assembly [64] this could be non-local. However, there appears to be a problem arising from quantum chaos. According to Bertrand Georgeot and Dima Shepelyansky of the Université Paul Sabatier in Toulouse, France:

> This affliction is basically caused by excess of choice. The interactions between elements of the array give them so many possible, and virtually equivalent, ways to arrange themselves that they lose the ability to pick one and stay with it. Instead, the system plunges into uncontrollable disorder [34].

This has all the hallmarks of a weak anticipatory system. Developments therefore both in the software and the hardware for implementing quantum computers suggest that we need better understanding of what is involved in implementing a strong anticipatory system and therefore to re-visit the fundamentals.

4 Interpretations of Quantum Theory

Quantum theory is far from singular. It is composed of many interpretations which raises the problem of how to implement it. It is not as though there are many valid versions each of which could give rise to a different type of quantum computer. Does a choice have to be made then between the various interpretations? This is one reason for using category theory as it can subsume the different interpretations.

Table 2: Interpretations of Quantum Theory matched with Category Theory
(a). Plato-Aristotle - Suppes

Name	**Quantum concept**	**Category theory concept**
Plato-Aristotle	freewill	free functor
Aristotle-Kolmogorov	chance	underlying (co-free) functor
Leibniz-Huygens	monad	reflective subcategories
	coherency	locally cartesian closed
Planck	correspondence principle	pushout functor
de Broglie	wave particle duality	Stone duality arrow/object identity
	probability wave	subobject in a topos
	superposition	co-limit
Pauli	canonical conjugate variables	contravariant duality
Heisenberg	uncertainty principle	co-equalizer
	reduction of wave packet	pushout
von Neumann	operator calculus in Hilbert space	endofunctor
	projection postulate	pushout
Born	state vector	pre-order
	statistical interpretation	subobject classifier
	probability as an intermediate physical reality	category of co-limits
Bohr	complementarity	duality
	indivisibilty of quantum of action	limits
	individuality of elementary processes	existential Σ, half-bits, quantifier Π
	interaction of object and instrument	pullback functor Δ
Schrödinger	quantisation as eigen values	quotient partial order
Bell	inequalities	unit of adjunction
	no quantum local hidden variables	adjointness
Bohm	implicate order	adjunctions
	wholeness in the undivided universe	monadacity
von Neumann-Birkhoff	orthocomplemented modular lattice	set-valued natural transformation
Mittelstē̈dt	hidden variables	adjunctions
	tertium non datur	(non) Heyting logic
Suppes	probabilistic logic	NNO valued POS functor

Table 2: Interpretations of Quantum Theory matched with Category Theory
(b). Einstein - Calderbank-Shor-Steane

Name	**Quantum concept**	**Category theory concept**
Einstein	statistical ensemble	topos NNO subobject classifier
	determinism	adjointness
	relativistic space-time	Dolittle diagram
	relativistic space-matter	Dolittle diagram
	superluminal correlation	adjointness
Mackey	axiomatic groups in Hilbert space	small category of monad
Einstein-Dirac	superstring	free functor
Everett	many worlds interpretation	co-limit quotient partial order
Feynman	integral path parallelism	hom category (exponential)
Nielsen-Chuang	postulates for quantum computing:-	
	1 physical system state vector (Hilbert) space	1. Dolittle diagram
	2 evolution of closed quantum system as unitary transformation (Schrödinger time eqn)	2. Pullback
	3 quantum measurement (e.g. qubit) as operator with state probability /post-measurement	3. Pushout
Bennett	quantum cryptography	adjoining adjoints
	superdense coding	covariant adjoint composition
	teleportation	covariant adjoint composition
Calderbank-Shor-Steane	quantum error correction	contravariant adjunction

Table 2 lists some of the names of the well-known persons and their theories showing the diversity of interpretation of quantum theory that has to be physically realised. On account of space restrictions and because they are well-known, citations to the persons and concepts in Table 2 have been omitted but they can be found in texts like Jammer [52, 53]. From the old quantum theory at the beginning of the 20th century through the Copenhagen interpretation of the twenties, through Heisenberg and von Neumann's in the succeeding decades and through the many worlds interpretation of the late 20th century is quite a diverse path to what seems to be the dominant quantum information interpretation of the turn of the millennium as currently represented by Nelson and Chuang, leading finally to the main application area of quantum cryptography and error correction.

The correspondence principle that quantum mechanics must reduce to classical mechanics in the world was implicit in Planck's work on modern physics in the old quantum theory and was taken over by Bohr early on to be gradually transformed into the new theory (See Jammer [52] p109-118). However, with the application of mathematics of wave mechanics to quantum phenomenon, this correspondence principle itself rather became lost sight of and emerged in various forms of the complementarity principle. Bohr saw it to begin with as a complementarity between space-time description and the wave-mechanical formulations but seems later to have applied complementarity to wave particle duality which had earlier come from de Broglie who had applied it to canonical conjugate variables. This was the sense of complementarity used by Pauli. Positivism was a prevalent mid-twentieth century philosophy that influenced both Bohr and Heisenberg. For Heisenberg who took the view that there was no other metaphysical level at work, the state vector held all the 'knowledge of the system'. This is in effect the notion of the wave function as a strong anticipatory system. In Hilbert space these can all only be examples of weak anticipation. However, Heisenberg's concept of the reduction of the wave packet is like Bohr's collapse or reduction of description and von Neumann's projection postulate. Yet alternative to the complex Hilbert Space, what about quantum theory as a real Hilbert space, a quarternion Hilbert space (Jammer [52] p205, references at footnote 17) and in terms of the more generalised [3] non-associative quarternion version with Cayley numbers as scalar multipliers [36]?

Bohr was heavily influenced ([32] p37) both on 'quantum-leap' and on the observer postulate by the earlier Danish philosopher of religion Kierkegaard who might be considered as a post-modernist before his time. The main exponent of Kierkegaard was Høffding (a close friend of Bohr's father) a positivist [50] who stressed that life has no bystanders. One is always a participant never an impartial observer ([48] p66). There are exceptions today like the discussion of Shimony [86] and Gernert [35] who recognizes the position of the internal observer and the distinction between endo- and exo-physics but generally the role of the observer now tends to get neglected in any formal representation. For quantum computing it seems essential to include the observer in the design.

There is a long history beyond *tertium non datur*, at least from the time of Zeno's paradox. Aristotle queried the indeterminate truth value of the outcome of a future sea battle in his *de Interpretatione* ($19^a27 - 19^b4$ [5]) and claimed an intermediate between justice and injustice that is neither just nor unjust in his *Categories* (11^b38 [5]). This was rejected by the stoic Chrisippus of Soli (280-210 BC). Various versions of the 'third way' were considered by the mediaeval Arabian philisopher Averroes with his concept of 'double truth' and by Peter de Rivo in the 15th century. In more recent times these have been followed by H MacColl, C S Peirce, Duns Scotus, L E J Brouwer, N A Vassil'ev, Z Zawirski, J Łukasiewicz (Jammer [53] p 341-346). F Zwicki advocated it as a new logic for microphysics [97]. Reichenbach [78] developed a more extensive three-level logic on quantum theory. The mainstream fathers of quantum theory however did not follow down this path. Max Born called Reichenbach's proposal 'a game with symbols', entertaining but with no gain in playing it. Born also observed ([14] p104-105) that Reichenbach could only explain three-valued logic by the use of two-valued logic.

For an application of his operation calculus to a thought experiment John von Neumann sought assistance from Garett Birkhoff who had been doing postgraduate work on lattices at Cambridge and who went on to write the standard work on lattices [13]. Together they replaced the Boolean lattice logic of classical mechanics with an orthocomplemented modular lattice where distributivity is replaced by the weak modular identity [12]:

$$x \cup (y \cap c) = (x \cup y) \cap c \text{ for } x \subseteq c \text{ instead of } (x \cup y) \cap c = (x \cap c) \cup (y \cap c)$$

The non-distributive modular lattice structure of subspaces represents relations between measurement of different observables. It means that these measurements of different observables are apt to interfere with each other (Jammer [52]) (at p376). Yet to disturb the concept of the distributive law of classical propositional calculus seems to breach the correspondence principle. The Birkhoff-von Neumann paper was challenged as logically inconsistent by Popper [76]. The journal *Nature* received replies suggesting that Popper too was inconsistent but it seems these were never published ([53] p353). Another approach was the probability logic of Suppes [90] where non-classical logic arises from the probability assigned to every event and conjunction of event. Mittelstëdt explicitly refuted the denial of *tertium non datur*. The arguments of Peter Mittelstëdt including separate objections to them by both Kurt Hübner and Hans Lenk are discussed in Jammer ([53] p395-399).

The moral of all this for quantum computers seems to be that these different interpretations of quantum theory are weak anticipatory systems. Problems remain unresolved like the nature of hidden variables (Hess & Phillips [43]) and the mainstream consensus that has emerged must be viewed against the variegated backdrop of the other different interpretations (see Audi [6]). The current paradigm that supports QIS is exemplified in Table 2 by the postulates of Nielsen and Chuang [74]. These amount to a substantial simplification of various possibilities arising from the earlier interpretations and therefore whether they are sufficient foundations for QIS remains to be seen.

The third column suggests a concept of category theory that corresponds to each quantum theory concept by attempting to match the categorial equivalence of the classical methods used by the originator named in column 1. This shows the advantage of the use of category theory as a high-level mathematical workspace that can tolerate differing lower-level (weak anticipatory) interpretations. The claim for category theory that it is a strong anticipatory system rests on its constructive character. If there is a mismatch the rigour of category theory will sort it out in the application. For instance category theory by means of the pullback is able to give a full formal explanation of Everett's many world's hypothesis. Yet category theory makes Everett's theory vacuous. Moreover category theory does not need to linearize, nor be restricted to superposition as only a local condition.

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