Topological Spaces in the Systems Theory

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Abstract
The continuity is a common concept, often used by people. It is not the case of topology, although behind the phenomenon of continuity always a topology must be understood. In a continuous process, if two possible causes remain close, that is in a certain neighborhood, they will produce close effects. The set of possible causes which lead to a studied effect is organized as a topological space. Researchers can deal with metric spaces, with norms or semi-norms, but it is important to establish the notion of neighborhood, or to define a system of open sets. In the study of dynamic systems with infinite memory a locally convex topology was introduced. The present paper reviewed results obtained when the inputs were continuous and indefinite derivable functions from minus infinite to the present moment, then it pass to the case when the inputs are known only as rows of values. In the last part the informational topology is exposed.
Keywords: norms, semi-norms, locally convex space, informational topology

1 Introduction

Different topologies were proposed to modeling some systems since more than 70 years. In 1972 in a study entitled "Topological Concepts in the Mathematical Theory of General Systems", published in the volume *Trends in General Systems Theory*, edited by George J. Klir, Joseph V. Cornacchio analyzed this manner to approach the modeling of systems. He related about a wide range of disciplines where one or other sort of topology was used, with or without a mathematical formalization, and accounted Engineering, Computer science, Social and Behavioral Sciences (here Anthropology, Economics, Social Psychology are quoted), Physics (especially in theoretical studies of quantum-mechanical particles scattering). Finally, Cornacchio presented functional systems, with a topology related to inputs which are bounded and Lebesgue measurable functions. The neighborhoods of this sort of inputs-function are given by integral formulae. The works quoted by Cornacchio, in which some topologies are used in the systems analysis, gone back to the year 1936, when K. Lewin published in New York the *Priciples of Topologycal Psychology* (Cornacchio, 1972).

An explanation of the success of the topological model for the studies of the dynamic systems is the modality in which the evolution of the causes determines the state of a system. Indeed, the causes of different phenomena unfold in time, their effects are considered at a fixed moment. In a mathematical approach, we deal with vector functions on a real variable (the time) which describe the history of inputs of the dynamic system and a vector functional to make correspondence from the inputs-function to the state of the system. The numerical values of this vector functional are the parameters of the system state at the considered moment. In a general study, we discuss about causes which can be closer or remoter between them, about greater or smaller effects of these causes, the speed of modification of effects or causes. The set of functions which represent the evolution in time of causes is first of all organized with an algebraic structure, that of a vector space. On the other hand, we must have a metrics of the space, to can speak about the distance between two possible causes. A topology establishes neighborhoods of a function. When we speak about the continuity of the relation from causes to effect, we understand that if two causes remained in a certain neighborhood, they produce very close effects. A very used topology is given by some *norms*. The *distance* between two continuous functions, defined on a finite interval, is the norm of the point wise difference of these functions. The norms the most used for the continuous systems are given by definite integrals from the absolute values of the functions on the limited interval of time considered.

A topology having *semi-norms* instead of norms is used to structure the space of histories of system inputs, when these inputs are considered on an infinite interval of time. These are the *locally convex spaces*. A structure of locally convex topology is used in the theory of distributions. One kind of differentiability in topological locally convex spaces was introduced by Gheorghe Marinescu (Marinescu, 1963) generalizing the Frèchet's derivative in normed spaces. A locally convex topology used in the mathematical modeling of systems with infinite fading memory and the Frèchet-Marinescu's differential calculus led us to demonstrate some possibilities to represent by integral formulae the relations between the history of inputs and the system present state. Using integral formulae, systems of integro-differential equations could be used in the theory of the pairs of systems conjugate with anticipation and retardation.

A different topology, *informational topology*, explains phenomena of social life (Otlacan, 2006).

2 Locally convex topology for continuous systems with infinite memory

Systems with infinite memory are considered systems whose state is the effect of the inputs coming from an indefinite interval of time prior to the present moment.

The paper "The Synergy and the Chaos Identified in the Constitutive Equation of a Dynamic System" (Otlacan, 2004) explained why the representation of the state $x(t)$ of a system with infinite memory, at the present moment t, can be given by the value of a vector functional F, defined and continuous on a space of functions Ω^t , this vector space having a locally convex topology. This topology has a family of semi-norms $|\,.\,|_\lambda$ with a real parameter $\lambda > 0$.

The following formulae were considered:

$$x(t) = F\{[g(\tau); \tau \leq t]\} \tag{2.1}$$

$$\Omega^t = \{g \mid g \in C^\infty, g : (-\infty, t] \to R^m\} \tag{2.2}$$

$$|g|_\lambda = \sup_{\tau \in [t-\lambda, t]} \left(\sum_{k=1}^{m} g_k^2(\tau) \right)^{1/2} \tag{2.3}$$

Here g_k , k = 1,2,...,m are the components of the vector function g. The constitutive functional F is understood as a vector functional, $F = (F_1, F_2, \ldots, F_n)$ and $F_i : \Omega^t \to R$, i=1,2,...n are continuous real functionals in the sense of the locally convex topology of the set Ω^t. A neighborhood of the function $g \in \Omega^t$ depends on two numbers, λ>0, μ>0, containing all the functions $h \in \Omega^t$ satisfying the inequality $|g-h|_\lambda < \mu$:

$$V_{\lambda,\mu}[g] = \{h \in \Omega^t; |g-h|_\lambda < \mu\} \tag{2.4}$$

In the quoted paper the differentiability definition of the functional F in Fréchet-Marinescu's sense, in a point $g \in \Omega^t$ is introduced. This means that there a number λ>0 and a functional $\delta F(g| \; h)$ depending on g and h, but linear only on its second argument $h \in \Omega^t$ exist, such that the following equalities hold:

$$F\{g(\tau)+h(\tau); \tau \le t\} - F\{g(\tau); \tau \le t\} = \delta F\{g \; (\tau)| \; h(\tau); \tau \le t\} + \omega(g \; ;h) \tag{2.5}$$

$$\lim_{h \to 0} \frac{| \; \omega(g \; ;h) \; |}{|h|_\lambda} = 0 \tag{2.6}$$

The demonstration of the possibility to represent the functional $\delta F(g_0;h)$ by an integral formula is found in the Annex of the quoted paper (Otlacan, 2004). This formula is the following:

$$\delta F\{g| \; h(\tau); \tau \le t\} = \int_{t-\lambda}^{t} \sum_{k=1}^{m} a_k(\tau) h_k^{'}(\tau) d\tau \tag{2.7}$$

According to this result, the transition from the system state $x(t-\lambda)$ to the state $x(t)$, with a function-input h on the interval of time $[t - \lambda, t]$ can be calculate by the following formula:

$$x(t) - x(t-\lambda) = \sum_{j=1}^{m} a_j(t) h_j(t) - \int_{t-\lambda}^{t} \sum_{j=1}^{m} a_j^{'}(\tau) h_j(\tau) d\tau + \omega(g_0; h) \tag{2.8}$$

Putting aside the term $\omega(g_0;h)$, that has the property 2.6, the integral calculus is justified. This formula emphasizes the importance of the present moment t and introduces the vector function a, that can be determined only experimentally and not mathematically, this being an existential theorem.

In our article (Manzatu, 1983) a theorem for the derivative of the constitutive equation of a dynamic system with fading memory is demonstrated. According to this theorem the following formula

$$x^{'}(t) = -\delta F\left[g| \frac{dh}{d\tau} \right] \tag{2.9}$$

The formulae 2.7 and 2.9 will give the following integral representation:

$$x^{'}(t) = -\int_{t-\lambda}^{t} \sum_{k=1}^{m} a_k(\tau) h_k^{''}(\tau) d\tau \tag{2.10}$$

This integral representation is not approximate one. That is why we propose this formula to be considered in the study of anticipative systems, using anticipate future values for the inputs $h_k(\tau)$, connected with the state of the system x or with the state of another system y. Other formulae of representation could be deduced, to complete those proposed and solved till now (Dubois, 2003; Otlacan, 2008). In some cases, with $h_k(\tau)$ known and $x'(t)$ imposed, so also known, the formula 2.10 can be an integral equation for the functions $a_k(\tau)$.

Other results, regarding the representation by double integral, can be also found in the quoted paper (Otlacan, 2004).

3 A Locally convex topology on the set of inputs known as rows of values

We consider the system inputs known at the moments:

$$t = t_0 > t_1 > t_2 > t_3 > \ldots > t_\alpha > \ldots$$

and given by vector rows:

$$g = [g(t_\alpha)]_{\alpha \in \mathbf{N}} = [g_1(t_\alpha), g_2(t_\alpha), \ldots, g_m(t_\alpha)]_{\alpha \in \mathbf{N}} \quad (3.1)$$

where $g(t_\alpha)$ are m-dimensional real vectors for every natural number α.

The Cartesian product S or S(− ∞, t] will be a set of rows obtained in the following mode:

$$\mathrm{S} = \prod_{j=1}^{m} S_j = \mathrm{S}_1 \times \mathrm{S}_2 \times \ldots \times \mathrm{S}_j \times \ldots \times \mathrm{S}_m, \text{ where } \mathrm{S}_j = \{g_j = [g_j(t_\alpha)]_{\alpha \in \mathbf{N}} ;\ g_j(t_\alpha) \in \mathbf{R}\} \quad (3.2)$$

The set S_j of numerical rows will have a locally convex topology with a sufficient family Λ_j of semi-norms depending on a real positive number λ:

$$s_{j\lambda}(g_j) = \max_{t_\alpha \in [t-\lambda, t]} |g_j(t_\alpha)|, \quad s_{j\lambda} \in \Lambda_j, \lambda > 0 \quad (3.3)$$

Sometimes it is possible the following semi-norms are more convenient:

$$p_{j\lambda}(g_j) = \sum_{t-\lambda \le t_\alpha \le t} |g_j(t_\alpha)| \quad (3.4)$$

We define on the Cartesian product S two topologies, in a concrete case using one or the other of them:

A. The locally convex topology with a <u>dominant component</u> (let it j) is given by the following semi-norms:

$$\overline{s}_{j\lambda}(g) = s_{j\lambda}(g_j), s_{j\lambda} \in \bigcup_{j=1}^{m} \Lambda_j \quad or \quad \overline{p}_{j\lambda}(g) = p_{j\lambda}(g_j) \quad (3.5)$$

B. The locally convex topology with <u>cumulative effect</u>, that uses all the components of the vector $g(t_\alpha)$ and that is given by the following semi-norms:

$$|g|_\lambda = \max_{t-\lambda \le t_\alpha \le \lambda} \left(\sum_{j=1}^{m} g_j^2(t_\alpha) \right)^{1/2} \quad (3.6)$$

In the two cases, the neighborhoods of the origin of the space S are the following:

$$\overline{V}_{j\lambda,\delta}[0] = \{[g(t_\alpha)]_{\alpha\in N}; \overline{s}_{j\lambda}(g) < \delta\} \tag{3.7}$$

$$V_{\lambda,\delta}[0] = \{[g(t_\alpha)]_{\alpha\in N}; |g|_\lambda < \delta \tag{3.8}$$

The *n*-vector functional F: S $\rightarrow \boldsymbol{R}^n$ is continuous in a point (row) $g \in$ S if $(\forall)\varepsilon > 0$ $(\exists)\delta>0$, $\delta=\delta(\varepsilon)$ and also $(\exists)\lambda=\lambda(\varepsilon)$ so as we have, with the norm from $\boldsymbol{R}^n$:

$$|F(g) - F(h)| < \varepsilon \tag{3.9}$$

with the condition $h \in \overline{V}_{j\lambda(\varepsilon),\delta(\varepsilon)}[g]$, respectively if $h \in V_{\lambda(\varepsilon),\delta(\varepsilon)}[g]$.

In the case A, the inequality 3.9 is verified if for only one index j we have:

$$\max_{t-\lambda(\varepsilon)\le t_\alpha \le t} \left|g_j(t_\alpha) - h_j(t_\alpha)\right| < \delta(\varepsilon) \tag{3.10}$$

The case B asks for all j= 1,2,…, m the satisfaction of inequality 3.10 in order to realize the condition 3.9.

The dynamic system with the constitutive functional F has the state $x(t)$ at the present moment t, and the formula can be written as follows:

$$x(t) = \underset{\alpha\in N}{F}\left\{[g(t_\alpha)]_\alpha; t_\alpha \le t, \lim_{\alpha\to\infty} t_\alpha = -\infty\right\} \tag{3.11}$$

The continuity of the functional F on the locally convex space S gives to the system the property of *fading memory*: the contribution to the value $x(t)$ of the terms from the row $[g(t_\alpha)]_{\alpha\in \mathbf{N}}$ decrease when the index $\alpha \in \mathbf{N}$ increases.

The <u>definition</u> of the *differentiability* of the functional F in the Fréchet-Marinescu's sense, in a point $g = [g(t_\alpha)]_{\alpha\in\mathbf{N}}$ from S, asks the existence of a natural number λ [the semi-norm $\overline{s}_{j\lambda}(g) = s_{j\lambda}(g_j),\ s_{j\lambda} \in \bigcup_{j=1}^{m} \Lambda_j$ *or* $\overline{p}_{j\lambda}(g) = p_{j\lambda}(g_j)$],

and of a functional $\delta F(g)$: S $\rightarrow \mathbf{R}^n$, linear and continuous in the respective topology, so that we have:

$$\underset{\alpha\in N}{F}[g(t_\alpha) + h(t_\alpha)] - \underset{\alpha\in N}{F}[g(t_\alpha)] = \underset{\alpha\in N}{\delta F(g)}[h(t_\alpha)] + \omega(g;h) \tag{3.12}$$

and according the case A or B, the last term realizes the conditions:

$$\frac{|\omega(g;h)|}{\overline{s}_{j\lambda}(h)} < \varepsilon \ \ if \ \ h \in \overline{V}_{j\lambda,\delta_\varepsilon}[0] \tag{3.13}$$

respectively:

$$\frac{|\omega(g;h)|}{|h|_\lambda} < \varepsilon \ \ if \ \ h \in V_{\lambda,\delta_\varepsilon}[0] \tag{3.14}$$

THEOREM. If $\delta F(g)$: S $\rightarrow$ **R** is the differential in the Frèchet-Marinescu'sense in the point g of the real functional F: S $\rightarrow$ **R,** S being the vector space defined by the formula 3.2 and having a locally convex topology with the semi-norms 3.5 or 3.6, and if the function $h\in$S, is the row $h = [h(t_\alpha)]_{\alpha\in\mathbf{N}}$ and $h(t_\alpha) = 0$ for $t_\alpha < t - \lambda$, and if

$$t = t_0 > t_1 > t_2 > t_3 > \ldots > t_\beta > t - \lambda > t_{\beta+1}, \ldots$$

then β+1 vectors $(a_{1\alpha}, a_{2\alpha}, \ldots, a_{m\alpha})$, $\alpha \in \{0,1,2,\ldots\beta)$ exist, so as the following

equality is valid:

$$\delta F(g)[h]=\sum_{j=1}^{m}\sum_{\alpha=0}^{\beta}a_{j\lambda}h_j(t_\alpha) \qquad (3.15)$$

(To read δF which depends on the input g is applied to the input h).

Demonstration: We consider the number $\lambda > 0$ resulting from the condition of the differentiability in Frèchet-Marinescu's sense of the functional F, so that the formulae 3.12, 3.13, respectively 3.12, 3.14 are accomplished. Also we consider the inputs $h = (h_1, h_2,\dots h_m)$ known at the moments $t = t_0, t_1, t_2, t_\beta, t_{\beta+1},\dots$ with $t_\beta < t-\lambda < t_{\beta+1}$, and $h_j(t_\alpha) = 0$ if $\alpha > \beta$, $j = 1,2,\dots, m$. This row $[h_j(t_\alpha)]_{\alpha\in \mathbf{N}}$ is an element of the subspace

$S^{0j}(\beta) = \{\xi_j = (\xi_{j\alpha})_{\alpha\in \mathbf{N}}, \xi_{j\alpha} = 0 \text{ if } \alpha > \beta\}$.

This space $S^{0j}(\beta)$ is a Hilbert's space and has the following scalar product:

$$<\xi_j, \eta_j>=\sum_{\alpha=0}^{\beta}\xi_{j\alpha}\eta_{j\alpha}$$

Let consider $S^{0j}(\beta), j=1,2,\dots, m$ and the Cartesian product:

$S^0(\beta) = S^{01}(\beta)\times S^{02}(\beta)\times\dots\times S^{0m}(\beta)$

Two arbitrary elements ξ, η from $S^0(\beta)$, $\xi = (\xi_1, \xi_2,\dots, \xi_m)$, $\eta =(\eta_1, \eta_2,\dots, \eta_m)$ have the scalar product given by the following formula:

$$<\xi,\eta>=\sum_{j=1}^{m}\xi_j\eta_j=\sum_{j=1}^{m}\sum_{\alpha=0}^{\beta}\xi_{j\alpha}\eta_{j\alpha}$$

The norm of the Hilbertian space $S^0(\beta)$ will be:

$$\|\xi\|_\beta=[\sum_{j=1}^{m}\sum_{\alpha=0}^{\beta}(\xi_{j\alpha})^2]^{1/2}$$

The input $h = [(h_1(t_\alpha),\dots, h_m(t_\alpha)]_{\alpha\in\mathbf{N}}$, $h_j(t_\alpha) = 0$ if $\alpha > \beta, j=1,2,\dots,m$ belongs to $S^0(\beta)$; its Hilberian norm is the following:

$$\|h\|_\beta=\{\sum_{j=1}^{m}\sum_{\alpha=0}^{\beta}[h_j(t_\alpha)]^2\}^{1/2}$$

We shall prove now that the linear functional $\delta F(g)[h]$, is continuous in the locally convex topology defined above, is also continuous in the topology of the Hilbertian space S^0. This means we can find out a number $\mu > 0$, so that:

$|\delta F(g)[h]|<1$ if $\|h\|_\beta<\mu$

We have $|\delta F(g)[h]|<1$ if $h\in V_{\lambda_1,\delta_1}[0]$,

That means that if the following condition is realized:

$$\max_{t-\lambda_1\le t_\alpha\le t}|h_j(t_\alpha)|<\delta_1 \quad \text{or} \quad \sum_{\alpha=0}^{\beta}|h_j(t_\alpha)|<\delta_1$$

It is possible to choose $\lambda_1<\lambda$, this $\lambda>0$ being asked by the Frèchet-Marinescu's differentiability, and it is the number that defines the number β.

Let us calculate the Hilbertian norm:

$$\|h\|_\beta^2 = \sum_{j=1}^{m}\sum_{\alpha=0}^{\beta}[h_j(t_\alpha)]^2 = \sum_{j=1}^{m}\sum_{t-\lambda}^{t} h_j^2(t_\alpha) \geq \sum_{j=1}^{m}\sum_{t-\lambda_1}^{t} h_j^2(t_\alpha) \geq \sum_{j=1}^{m}\sum_{\alpha=0}^{\beta_1} h_j^2(t_\alpha)$$

Here β_1 is the biggest integer α for which $t_\alpha \geq t-\lambda_1$.
We use now the Holder's inequality:

$$\sum_{n=1}^{\infty}|\xi_n\eta_n| \leq \left(\sum_{n=1}^{\infty}|\xi_n|^p\right)^{1/p}\left(\sum_{n=1}^{\infty}|\eta_n|^q\right)^{1/q} \quad for\ \frac{1}{p}+\frac{1}{q}=1$$

With p=2, q=2, $\eta_n = 1$, $\xi_j = h_j(t_\alpha)$, the above finite sum will satisfy the following inequality:

$$\sum_{j=1}^{m}|h_j(t_\alpha)| \leq m^{1/2}\left[\sum_{j=1}^{m}h_j^2(t_\alpha)\right]^2 .$$

So the calculus about the norm of h gives:

$$\|h\|_\beta^2 \geq \max_{0<\alpha\leq\beta_1}\sum_{j=1}^{m}h_j^2(t_\alpha) \geq \frac{1}{m}\max_{0<\alpha\leq\beta_1}\left[\sum_{j=1}^{m}|h_j(t_\alpha)|\right]^2 \geq$$

$$\frac{1}{m}\left[\max_{0<\alpha\leq\beta_1}|h_j(t_\alpha)|\right]^2 \geq \frac{1}{m}[\overline{S}_{j\lambda_1}(h)]^2$$

From the condition: $\|h\|_\beta \leq \dfrac{\delta_1}{\sqrt{m}}$, we have $\overline{S}_{j\lambda_1} < \delta_1$ and so $|\partial F(g)[h]| < 1$,

That means the continuity of $\partial F(g)$ in the sense of Hilbertian norm. But, as a linear and continuous functional, from the Riesz's theorem, it results the existence of an element a belonging to the space $S^0(\beta)$, $a = (a_{1\alpha}, a_{2\alpha}, ..., a_{m\alpha})$, so that we have the scalar product $\partial F(g)[h] = < a, h >$. Using the formula of scalar product, the formula of representation 3.15 results and the theorem is demonstrated.
A corollary of this theorem is the possibility to express the difference between the system states from two close moments:

$$x(t) - x(t-\mu) = \delta F(g)[h] = \sum_{j=1}^{m}\sum_{\alpha=0}^{\beta} a_{j\lambda}h_j(t_\alpha) + \omega(g;h) \tag{3.16}$$

In an approximate calculus the formula is written without the term $\omega(g;h)$; β is the biggest natural number α for which $t_\alpha \geq t-\mu$.
Observation: Other way to reach this formula could use the approximate calculus for the integral formula 2.8.

4 About the informational topology

This is a concept inspired by the contemporary social life, in the social system being introduced a mathematical topological structure. Many branches of the complex globalisation phenomenon could be explained through relations (vector functions) defined on the world population, considered as a topological space.

It is possible to outline a hierarchy of the topological structures that framed the human society during the history, the paramount element being the possibility to circulate information. Every historical stage of scientific and technical development of mankind is structured according to his own informational topology. The informational topology is the basis for imagining the evolution functions of the economic and political life, from ancient times to our days, including the globalisation period.

The basic notion of a topology on an abstract set is the concept of *neighbourhood*, every point of this set having its system of neighbourhoods. More topologies can be defined on a set, each of them having its system of neighbourhoods. The elements of the set are named "points", irrespective of their nature. In this sense when we refer to the world population, named the set W, its points are human beings, as individuals or communities.

The topologies may be compared. Let be T and T' be two topologies defined on the same set *W*. The topology T' will be finer than T if for a point *P* from W and any neighbourhood of this point, $V[P]$ belonging to the topology T, a neighbourhood $V'[P]$ of the same point *P*, but belonging to the topology T', exists, so that $V'[P]$ is included in $V[P]$; we write T<T'. If T<T' and T'<T, the two topologies are equivalent.

In the *geometrical topology* defined on the set *W* (representing the human beings from all over the world) each person, considered as an element $P \in W$, has his geometrical neighbourhoods containing all the people from disks centred in *P*. A geometrical neighbourhood represents the human society that lives around *P* on a circular surface with a radius $r>0$ centred in *P*. The system of geometrical neighbourhoods of each person *P* from the world *W* constitutes the *geometrical topology* GT of this set *W*.

Definition. Let the geographical co-ordinates (θ,φ) of a point *Q* from the set *W* and (θ_0,φ_0) those of the point *P* from the same set; the *geometrical distance* $\delta_G(P,Q)$ between *P* and *Q* can be as in the following formula:

$$\delta_G(P,Q) = [(\theta-\theta_0)^2 + (\varphi-\varphi_0)^2]^{1/2} \tag{4.1}$$

$Q(\theta,\varphi)$ will belong to a neighbourhood $V_r[P]$ with the radius $r>0$ of the point $P(\theta_0,\varphi_0)$, and we write $Q\in V_r[P]$, if the *geometrical distance* $\delta_r(P,Q)$ between *P* and *Q* is less than $r>0$: $\delta_G(P,Q)< r$.

The individual *P*, or a group of persons so marked, often acts on the basis of *information* that circulates from *Q* to *P*. Thus the transmission speed of information plays a decisive (determining, prominent) part in all the human actions.

Definition. The *informational distance* between P and Q is a function $\delta_I(P,Q)$ whose value on the pair (P,Q) is equal to the length of the *interval of time* necessary to an useful information that leaving Q arrives at P.
This informational distance, satisfies the same conditions as the geometrical distance: it is positive, i.e. $\delta_I(P,Q)>0$ for every different persons P and Q, accomplishes the axiom of triangle: $\delta_I(P,Q) \leq \delta_I(P,S)+ \delta_I(S,Q)$ for every three persons (or group of persons) P, Q, S, and the axiom of symmetry, $\delta_I(P,Q)= \delta_I(Q,P)$, thinking of the cases when the telephone or internet are used.
Information has always circulated in human society, but in different ways in different epochs of technical development. First information propagates in compact groups, in a restricted circle around a person P. The value of the informational distance in these cases is proportional to the geometrical distance:

$$\delta_I(P,Q) = \delta_G(P,Q)/v \text{ , respectively } \delta_G(P,Q) = v.\delta_I(P,Q) \quad (4.2)$$

Definition. A set (community) of persons $V_\lambda[P]$ constitutes a *neighbourhood* of $P \in W$ in the informational topology *Inf.*T on the world W if the *length of the time interval* $\delta_I(P,Q)$ that is necessary to transmit the information from any person Q belonging to the set $V_\lambda[P]$ to P is less than a positive number λ:

$$V_\lambda[P] = \{Q \in W | \delta_I(P,Q) < \lambda\} \quad (4.3)$$

A neighbourhood of a person P is the informational medium that he masters (dominates, reigns). This is the set of persons with which P communicates in an interval of time with a length determined by the number λ.

The information topology unites the space and the time like as two variables co-operating in the frame of the same structure. The theory of the relativity – "where the space and the time were unified in a continuum space-time" (Iancu, 2002) is suitable not only to describe the world of sub-atomic particles, but also to describe the economic and social life at a planetary level, where the matter, energy and information are unified.

The position of Q in a certain neighbourhood $V_\lambda[P]$, $\lambda>0$, may be characterized by the expression: "Q is close to P". How much? The answer is the following: Q is so close to P as a piece of information comes to P if it started from Q or comes to Q if it stated from P in an interval of time less than λ. In other words, the information crosses (goes over) the geometrical distance between P and Q in an interval of time less than the value λ.

The development of the communication ways, naval, terrestrial or aerial, changed the contents of the expression *close to*, though the metrics of the Euclidian space, therefore the geometrical distance between two points, was preserved. In some way, we could say that, till the employment of radio waves, the geometrical topology structured the activity of all human communities on W. But the radio waves introduced a new topology on W, finer than the geometrical topology. The information coming from remote corners of the world can enter in every geometrical neighbourhood of the person (group of persons) P. "Close to P" means now not only what is included in a material circle around P, but also everything intercepted by radio, telephone or television, because P reacts upon the information that the electromagnetic waves bring to him. The person *P* is in the middle of his neighbourhoods in this finer topology. Nowadays the finest topology on our Planet is introduced by Internet.

It is known, the concept of continuous function is built on the notion of topology. A function defined on a certain set W, which is continuous according to a topology T, can lose the continuity if a weaker topology was considered on W. That is the case of the informational topology *Inf.*T and the geometrical topology GT, weaker than *Inf.*T.

We are interested on the continuity in connection with the stability and the possibilities to rule over the social events.

The degree of stability decreases when on the set (community) *W* a finer topology was installed. The minimum of the stability results when the finest informational topology, the topology of communications by internet TCI, was installed. The assertion is based on the fact that, in these new conditions, the control or the domination of the situation only on a compact geographical area cannot ensure the stability of this area. It must have the control in a neighbourhood of the informational topology. That means to possess the informational instruments to become able to manage the economic, political and social activity and to avoid catastrophe.

The concept of informational topology is important to understand that the process of globalisation will act in the favour of the whole human society only when all the people have the informational means and know what to do with them. In addition, this situation must be prepared (preceded) by scientific and moral education.

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