The Inapplicability of the Concept of Subjective Probability

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Abstract
Controversy over the Bayesian approach has diminished and one can't but admire the elegance of the definition of subjective probability. But it is still a hard sell in practice. This is a fact gathered through engineering experience. When the effect of the prior washes off quickly, the issue is irrelevant and one enjoys the probabilistic updating algorithm. When there is little data, one seriously questions the validity of the Bayesian approach. Very little has been achieved in developing means to support the existence of a prior, assess it and calibrate the person providing the opinion. We review the theory and show a case in bridge maintenance where the likelihood of the expert can be assessed. We state the reason for its inapplicability, take a step back to the days of Kolmogorov and reflect on Bayesian theory.
Keywords : Statistical Foundations, Probability, Expert Opinion.

1 Introduction

In most engineering situations, a reasonable certainty can be achieved in the construction of a solution. For example, a bridge structure is built to sustain a predetermined load. But even in the most sophisticated engineering solutions, uncertainty prevails. We live with some comforting illusion of control, but we do accept the inevitability of uncertainty in almost every aspect of our lives. In 1654, letters written by Blaise Pascal [1623-1662] and Pierre de Fermat [1601-1665] discussed a gambling problem posed by a French nobleman. The problem provided the two French mathematicians with a reason to investigate and consequently lay down principles that formed the genesis of the theory of probability. In 1812, Pierre de Laplace [1749-1827] introduced a formalism in his book, *Théorie Analytique des Probabilités*, and applied probabilistic ideas to scientific problems. Today, the theory of probability is applied in countless engineering problems. Perfection is hard to achieve and environments are hard to predict. Often, a solution is given in probabilistic terms.

The problem has not been solved completely. The difficulty remains in the exact definition of probability. In 1933, the Russian mathematician Andrey Nikolaevich Kolmogorov introduced an axiomatic approach, turning the probability concept into a modern mathematical theory. While no one questions the application of the theory as a mathematical tool, the meaning attributed to probability remains an issue. In the past centuries, schools of thoughts emerged. The most notable ones are the *Frequentists* and the *Bayesians.* In 1763, a theorem by Reverend Thomas Bayes [1702-1761], a British mathematician, was published that gave birth in the 20th century to the 'subjective' or 'personal' interpretation of probability, with the work of de Finetti (1937) and Savage (1954). The subjective probability approach keeps the axiomatic laws of the theory of probability but enlarges the scope of application. It is part of the larger class of Bayesian methods and it is often referred to as the Bayesian approach.

Subjective probability is a hard sell in practice. We come to this conclusion through experience in engineering. In some cases, the effect of the initial subjective input washes off quickly as more data are gathered and the issue is irrelevant. However when there is little or no data, one seriously questions the validity of the Bayesian approach. Not much has been achieved in support of the existence of a prior probability. We review the concept of *Expert Opinion Elicitation* used to build a prior probability distribution. We show a case in bridge maintenance optimization where the likelihood model for the expert can actually be developed and state the reason for the inapplicability of the approach. This makes us take a step back to the days of Kolmogorov and reflect on Bayesian theory.

2 Probability

There are many approaches used in modeling uncertainty. Probability theory is regarded as a sound theoretical approach. There are other models. Fuzzy Logic theory and the Dempster-Shafer theory are the most notable other directions. Let E be a random event or uncertain event. Probability theory assigns a number between 0 and 1 to that event and denotes it $P(E)$. E can be any possible or imagined event, such as 'rain tomorrow', 'Shakespeare wrote the plays' or the 'image belongs to class C'. E represents the truth of a proposition and $P(E)$ the probability of that proposition being true. There are three laws upon which probability theory is built; (1) Convexity: $0 \leq P(E) \leq 1$, (2) if E_1 and E_2 are mutually exclusive, that is they both cannot occur together, then $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$ and (3) Multiplication: $P(E_1 \text{ and } E_2) = P(E_1|E_2)P(E_2)$, where $P(E_1|E_2)$ is known as the *conditional probability* of event $E1$ given (assuming) event E_2 has occurred. Based on these simple laws, a whole body of explanatory science, inference and prediction is built. While probability theory is laid down with mathematical rigor and requires definitions upon which the three laws are based, in essence, it is these three simple laws that are at the heart of any probabilistic modeling. Two other laws derived from these are; (i) *Bayes' Theorem*, due to Thomas Bayes and rediscovered by Laplace in 1774 and (ii) the *Law of Total Probability* also derived by Laplace. The axiomatization of Kolmogorov is accepted by most and is the foundation of the mathematics of probability, by which the probabilities of complicated events of interest are calculated from probabilities of simpler events.

2.1 Subjective Probability

The theory of probability is based on axioms and is not the subject of controversy as a mathematical theory. In 1933, the Russian mathematician Andrey Nikolaevich Kolmogorov introduced the axiomatic approach, *Grundbegriffe der Wahrscheinlichkeitsrechnung* (Kolmogorov, 1933), turning the probability concept into a mathematical theory. Kolmogorov organized a theory Emile Borel [1871-1956] had created many years earlier by combining countable additivity with classical probability. In Kolmogorovs work, they were traces of the work of many others such as that of Borel, the work of Maurice Frechet [1878-1973], and that of Francesco Cantelli [1875-1966], Alexander Chuprov [1874-1926], Paul Levy [1886-1971], Wladyslaw Steinhaus [1887-1972], Stanislaw Ulam [1909-1984] and von Mises [1883-1953] (Shafer and Vovk, 2006). The problem starts when one questions the meaning attributed to probability. If a problem is dissected into its most basic events and these events can't be divided further or conditioned, then one is faced with having to provide a probability for the basic events. The question arises as to what is meant by the assigned numbers. The issue is important and has been controversial over the history of probability theory. Several schools of thoughts dominated at different times. The importance of the issue lies in that different interpretations lead to different methodologies and very different answers at times.

The meaning of probability took many forms over the course of centuries. From James Bernoulli's [1654-1705] notion of probability, to Laplace's [1749-1827] definition of probability, to Venn [1834-1923] and Von Mises [1883-1953] frequentist interpretation, to de Finetti [1906-1985] and Savage [1917-1971] subjective probability, the meaning of probability has been constantly questioned. In the example of a flip of a coin, the probability 0.5 of a face could be arrived at through three possible reasonings. The first one uses the symmetry of the coin. Assuming a perfect symmetry, the argument can be made in favor of the value 0.5. In a second reasoning, probability is taken to be the limit of the frequency of the outcome if the coin is flipped infinitely in similar conditions. This provide the basis for the frequentist view of probability. While this definition of probability prevailed for a long time, it has to share the stage these days with another view, the subjective probability. The frequency concept is liked by many in different scientific fields for its rigorous definition but it does not apply to all situations. It is hardly ever possible to replicate the same conditions for an experiment. Even in the coin toss example, one cannot assume that the conditions are the same each time the coin is tossed. Diaconis et al. (2007) have shown that a coin toss can be predicted if all information about the toss is available. Also, the frequentist argument does not apply for one off events such as 'Mr. X is guilty of the crime' or 'shakespeare wrote the play'. The idea of a *personal* probability, or *subjective* probability saw the day with the work of Frank Ramsey [1903-1930] and Bruno de Finetti early in the 20th century in Europe, and that of Leonard Jimmie Savage in the United States in mid 20th century.

2.1.1 The Betting Scheme of de Finetti

de Finetti (1937) devised a betting scheme to determine a unique number he qualifies to be the probability of an event of interest. The process is to make the assessor of the probability go through a series of evaluations where he or she is asked to provide a price z on a bet where the gain is z if E occurs and $1 - z$ if E does not occur, $0 \leq z \leq 1$. The person keeps choosing z until becoming indifferent between two possibilities (Berger, 1980). Making a number of assumptions, this intuitive concept is put into a rigorous mathematical framework by de Finetti (Singpurwalla, 2006).

2.1.2 Relative Likelihood - The Savage Axioms

The *Relative Likelihood* axiomatic approach assumes that a person has the ability to compare the likelihood of any two events. Using a series of axioms on rational behavior, it shows that there exist a unique number, in any given situation, that can be considered the probability of the event in question. With the use of an auxiliary experiment that has symmetrical features, the probability of any event can be assessed. It suffices to compare outcomes of the auxiliary experiment with the event considered. By repeated comparisons, a number will be arrived at which will be unique and represent the probability of the event (Degroot, 1970).

2.1.3 Prior Probability

Probability models are used to solve problems that have a relevant amount of uncertainty in them. Bayes' Theorem is a simple probability rule that contributed to the resolution of countless problems in estimation, inference, and prediction. It is a powerful tool, but it requires the specification of prior probabilities. In many problems, one starts with $P(E)$, called a prior probability and computes $P(E|D)$, where D is the set of data. D often comes in pieces over time, $(D_1, D_2, \ldots)$ such as radar measurements in *Target Tracking.* Each time new information D_j arrives, $P(E|D_j, D_{j-1}, \ldots, D_1)$ is obtained from $P(E|D_{j-1}, \ldots, D_1)$ using Bayes' theorem. This mechanism provides for a powerful recursive probabilistic updating that proved successful in many problems. For example, in target tracking, it is the basis of many effective algorithms for tracking targets in all sorts of conditions (Musicki et al., 1994). However, one must start with $P(E)$. This is where the controversy occurs. In a field like target tracking or image analysis, data are abundant and the effect of $P(E)$ is small and does not affect the solution much after a while. But when there is little data, the subjective input weights in significantly on the final answer. Some dictate the use of *Subjective* prior probability. However, while de Finetti, Savage and others did their best to rationalize a subjective $P(E)$, not all statisticians are willing to accept it. The debate is long and is beyond the scope of this article, although we will be making a point along the objection.

3 A Bridge Maintenance Problem

Bridge maintenance optimization has been applied over the past decades due to the large costs associated with the management of networks of ageing structures. Structural deterioration assessment and condition prediction of bridges is a subject of interest that has far reaching consequences, both in terms of public safety and budgeting for asset managers. The Roads and Traffic Authority of the state of New South Wales, Australia, manages more than 5000 bridges. These bridges were built over the last 125 years. They were made out of various materials and technology and built to different loading standards. The structures are exposed to different environments. They are also subjected to various loading patterns and frequencies (Manamperi et al., 2009). All these factors have different ageing effects on the structures. The state is faced with the dual aspects of public safety and maintenance cost in the management of the structures. In a recent study conducted for the Roads and Traffic Authority of the state of New South Wales, a modern approach in modeling structural deterioration in bridges is applied to the maintenance problem. Traditionally, bridge maintenance optimization models optimize based on discounted long-term costs using a Markovian decision model. Aboura et al. (2009) [2] apply the gamma process in modeling deterioration. The gamma process is a stochastic process with independent non-negative increments having a gamma distribution. It was first applied by the Australian scientist Patrick Alfred Pierce Moran [1917-1988] in the 1950's to model water flow into a dam. Abdel-Hameed (1975) was the first to propose the gamma process as a deterioration model. The advantage of the gamma process was recognized and applied in several studies. van Noortwijk (2009) provides a comprehensive overview of the use of the gamma process in the maintenance of structures.

To model deterioration using the gamma process, the power law is incorporated into the stochastic process. The gamma process is defined as follows: Let $v(t)$ be a non-decreasing, right continuous, real-valued function for $t \geq 0$, with $v(0) = 0$. The gamma process with shape function $v(t) > 0$ and scale parameter $u > 0$ is a continuous-time stochastic process $\{Z(t), t \geq 0\}$ with the following properties; (i) $Z(0) = 0$ with probability 1, (ii) $Z(\tau) - Z(t) \sim G(v(\tau) - v(t), u)$, and (iii) $Z(t)$ has independent increments, where $G(z|v, u) = u^v z^{v-1} e^{-uz} / \Gamma(v)$ is the gamma probability density function defined for $z \in (0, \infty)$. Letting $v(t) = \mu^2 t^q / \sigma^2$ and $u = \mu / \sigma^2$, the mean and variance of the deterioration $Z(t)$ are $E(Z(t)) = \mu t^q$ and $V(Z(t)) = \sigma^2 t^q$. Given a set of observations of the deterioration process $Z(t)$, $\{z_i\}_{i=1}^n$ for times $\{t_i\}_{i=1}^n$, the maximization of the likelihood function provides estimates of the three parameters μ, σ and q. For a $Z(t)$ deterioration, a condition $C(t) = 100 - Z(t)$ is defined. At time 0, $C(0) = 100(\%)$ implies a full, 'as new', condition of the bridge element to which the gamma process is applied. The deterioration modeling focuses on elements as the statistical analysis is applied to elements of bridges grouped together (Aboura et al., 2009 [3]). The importance of the structural deterioration estimation is apparent in the execution of the final exercise, the maintenance optimization. It is illustrated in Figure 1, where two different condition paths lead to two very different

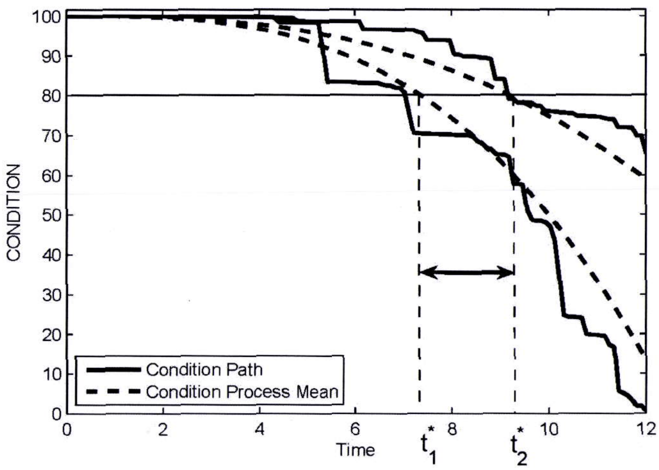

Fig. 1: Time to reach a 20% deterioration target level

times, t_1^* and t_2^*, to reach the 20% deterioration target. There is approximately a 2 years difference between the two times. This can lead to a significant difference in cost if t^* is applied as maintenance time to a large number of elements on different bridges. Estimating properly the condition curve is essential to the problem. The lower on the condition curve are some of the data, the better the estimation (Aboura et al., 2009 [3]). For some bridge elements, low condition data are not available due to maintenance. One way to remedy would be to try to guess the times at which the deterioration reaches some levels. The use of engineering knowledge as an educated guess is commonly referred to as *Expert Opinion.* Significant research was conducted in eliciting expert opinion. We review the concept and offer a solution for the maintenance problem.

4 Expert Opinion Elicitation

The use of expert opinion in the assessment of a prior distribution received much attention in the literature. It allows for the formal incorporation of expert information into a statistical analysis and attempts to provides an answer to the controversial issue of the validity of a prior distribution. Early work to formalize ad-hoc procedures for the use of expert opinion include Dalkey and Helmer (1963). Morris (1974) recognized the importance of treating the expert opinion as data, stating the general principle on which subsequent work was based. The topic was further enlarged by the Bayesian statistical community to the problem of reconciliating prior information from different sources, a topic that dates back to Winkler (1968). Many Bayesian statisticians contributed to the literature on the problem. A lot of the work in Expert Opinion Elicitation remained at the theoretical level, with few notable exceptions, for example Cooke (1991). Cooke's 'classical' method for combining expert probability distributions remains the only method in use in which real data are the basis for evaluating the experts. The name classical model derives from an analogy between calibration measurement and classical statistical hypothesis testing. It is not a Bayesian approach, but it is a method that has been tested in the field.

Garthwaite et al. (2005) present a comprehensive review of the statistical work in the area. The first sentence of their abstract states, in a short but concise manner, that 'elicitation is a key task for subjectivist Bayesians'. It is indeed, as it represents the technology that can validate the theory. However, by the authors' own admission, the statisticians failed to provide an answer to the problem. The paper surveys a wide range of issues, a considerable body of work, but reaches the conclusion that 'too often, ad hoc methods must be used when an expert's opinion is to be quantified'. This is not surprising. Ad hoc methods are used because they are simple and they allow an understanding of the mathematics used to model the expert opinion. In effect, the practitioners that use an ad hoc method are building their own simple likelihood model the best they can, when in fact such expert opinion model should have been derived by the statisticians. No statistician really took time to collect data on expert answers, enough to build a model. All models, at least in the parametric case, were off the shelf models used for mathematical convenience, with possibly absolutely no connection to a model that captures best the expert's behavior. For example, a simplistic model would be that $\xi \sim \mathcal{N}(a\theta + b, \sigma^2)$, for some a, b, σ, where ξ is the expert opinion on θ, a parameter of interest in a decision making problem. While this model is simple, it captures 'bias' and 'inflation' as well as 'variation'. Unfortunately, often, this is the conceptual level of modeling used so far by most Bayesian statisticians in dealing with the expert likelihood. A theoretical model is postulated, supported only by a slight guessing logic in its formulation, and the mechanics are activated. It is done, as if somehow, subjectivity blurs everything, and that common sense will save the day in practice. Well, that is not the case. Often, these models never make it to practice. That is how the day is really saved. More complex models or higher hierarchical levels have been considered. But in principle, it remains that a theoretical model is used on grounds not supported by any statistic about actual expert behavior. The problem is not resolved by adopting a nonparametric approach either (Garthwaite et al., 2005).

4.1 An Expert Opinion Elicitation Solution

In considering the gamma process as a stochastic deterioration model, a typical condition curve is shown in Figure 1. Data on the lower part of the curve is often not available. A way to remedy to the problem is to introduce expert knowledge into the analysis. We consider a maintenance problem where the deterioration model is the gamma process with parameters $\theta = (q, \mu, \sigma)$, as defined in section 3. Given a deterioration level z, we want to determine a probability distribution for the time the deterioration process crosses that level. Given q, μ and σ, we can obtain the exact time the process mean equates that level. It is $t_z^\theta = (z/\mu)^{1/q}$, since $\mu t^q = z$. A good expert opinion would result in a probability distribution around the value t_z^θ. Using this observation in some modeling of the expert, we want to derive the probability distribution $P(\theta|\ t_e, D)$, given the expert input t_e and the data D, a set of observations of the deterioration process $Z(t)$, $\{z_i\}_{i=1}^n$ for times $\{t_i\}_{i=1}^n$. This is done through the application of Bayes theorem,

$$P(\theta|\ t_e, D) \propto P(t_e, D|\ \theta)P(\theta) \propto P(t_e|D, \theta)P(D|\ \theta)P(\theta) \tag{1}$$

The likelihood function $\mathcal{L}(\theta) = P(D|\theta)$ relates to the data and is easily derived from the gamma process (Nicolai et al., 2007). $P(\theta)$ embodies any information the statistician may have about $\theta = (q, \mu, \sigma)$. In most situations, the statistician may not know anything about some bridge element deterioration, as was the case in our study, and consequently a non informative prior may be used. Remains the determination of $\mathcal{L}_e(\theta) = P(t_e|D, \theta)$, the likelihood model at issue. We assume that t_e is independent of the new data $D_{new} \subset D$. The expert opinion is not independent of data in general, and is in fact very much influenced by it. However, we setup the elicitation process such that t_e is provided before the new data D_{new} are observed, hence the independence. The data is collected sequentially over the years. This leaves us with the construction of the likelihood function $\mathcal{L}_e(\theta) = P(t_e|\theta, \mathcal{H})$, where $\mathcal{H}$ is all knowledge the expert has at the elicitation time. At this point, following what has been done so far by subjective Bayesian statisticians, one stipulates, hypothesizes, a model for $P(t_e|\theta, \mathcal{H})$, for example the Normal distribution model $t_e \sim N(at_z^\theta + b, \Sigma^2)$. Given θ, t_z^θ is defined uniquely for a level z, and the expert opinion model is deemed a reasonable one. The triple (a, b, Σ) characterizes the expert; inflation, bias, variability. This is as far as many Bayesians have gone. This is where our criticism lies. What justifies the use of such model and has anyone studied data relevant to the construction of this model.

In the scenario of section 3, we have precisely a situation where we can remedy to the weakness displayed so far by the so called *Supra Bayesian* approach. Instead of speculating on some theoretical model, we can actually build it. There are over 100 major bridge elements in the study of Aboura et al. (2009) [2]. Bridge inspectors are often assigned to the same bridges over the course of many years. These inspectors can provide a pool of experts. The experts can simply provide a guess at a condition before going on site to inspect and assess that condition. This process would ensure enough data are collected to build a statistical model $P(t_e|t_z)$, where t_z is the time at which deterioration z is reached. This model would be expert dependent, as expected, and would be build on actual, real expert information. The likelihood model $\mathcal{L}_e(\theta) = P(t_e|\theta)$ is deducted using the laws of probability

$$P(t_e|\theta) = \int_{t_z} P(t_e|t_z, \theta)P(t_z|\theta)dt_z \tag{2}$$

$P(t_e|t_z, \theta) = P(t_e|t_z)$, the model of the chosen expert, is build through historical data. $P(t_z|\theta)$ is the probability density function of the time for the deterioration to reach level z, given the parameters $\theta = (q, \mu, \sigma)$. $P(t_e|t_z)$ in fact equals $P(t_e|t_z, \theta)$ in the case where the data used to build the model comes from the same bridge or from several bridges with the same θ deterioration characteristics for the bridge element in question. If data from bridges with different element deterioration behavior are used to build the expert model, as it may be the case when there is lack of data, then this aggregation implies an approximation of $P(t_e|t_z, \theta)$ by $P(t_e|t_z)$. But it is still a reasonable approximation that allows the use of an expert data derived model. Or we may simply make the assumption that the expert's answer error is independent of θ, which is most likely an incorrect assumption. All these issues can precisely be answered during the process of building $P(t_e|t_z)$ using actual real expert data and observed target times. The use of information about the expert's answer is the major departure brought about by this solution. The expert model here is not to be theorized, but rather to be built using historical data about the expert. This is a significant conceptual difference from existing solutions.

4.1.1 The Engineering Reality

The reality is that the client refused categorically to consider any subjective input to the solution. The objection was motivated by many reasons, one being that the work was commissioned by a governmental agency that prohibited the idea of having to defend some day a past subjective decision. Another reason is the inspectors being inconsistent in reporting evaluations. The inspectors are needed for the evaluation of the conditions of the bridges as automation is difficult. But these inspectors are prone to error. To use their opinion in guessing a future condition is amplifying the risk of a serious error. We do not speculate as to what other reasons may have contributed to the rejection of the use of a subjective solution.

5 Theoretical Considerations

The rationalization of the use of subjective probability can be made through one of two arguments; (i) de Finetti's betting scheme and (ii) the Relative Likelihood argument. It is the opinion of this author that neither one provides a justification for the use of subjective probability. The betting scheme of de Finetti seems reasonable at first. It is intuitive and along the lines of thinking in gambling situations. The probability of an event $P(E) = 1 - z$ is an equilibrium point the person arrives at after repeatedly choosing a price z. The person keeps choosing z until becoming indifferent between two possibilities. This approach assumes that the thought of money can help a person bring out a personal feeling for an event. This is a strong assumption. Many people may not be able to extract a feeling of likelihood through the consideration of a gain or loss of money. Assuming a person is sensitive to monetary gains and losses, with a linear monetary value (ignoring the circular argument between probability and utility), de Finetti's scheme requires an (imaginary) iterative process, that may take as long as it needs, until it settles on an equilibrium value. No one can guarantee the existence of such an equilibrium point. It is simply postulated. The approach assumes that each time a subjective probability is declared, the gambling scheme has been applied, regardless of how long it took for it to converge. This is a dangerous door. Most people, if allowed to use subjective probabilities, are not going to conduct a long iterative process. They will simply start declaring the first value that comes to their mind.

The Relative Likelihood argument does not fare any better. The development of it is mathematical, rigorous and well thought out. And yet, when it comes to making a jump to the existence and definition of probability, it makes a huge assumption. First, events are organized according to their likelihood. Thanks to the axioms of the theory, they can be ranked by a rational person. By relative likelihood one can compare all events. Finally, a perfect uniform probability distribution is assumed to exist, and its events are compared to the events of interest. It is this last jump, from a real world situation to a completely abstract situation that we object to. Absolutely nothing guarantees that this can be done. The whole approach is an argument that feeds onto itself. It is basically a 'mathematization' of a ranking method that is justified through axiomatization. Then it is saying here is a scale, say between 0 and 1, and where do you think your probability for that event fits here. There is absolutely nothing that says that this can be done. Relative Likelihood simply pushes away the problem, and rephrases it in a mathematical context so that it appears solved. It is a nice axiomatic theory that makes rationality assumptions, then quickly jumps to the declaration of a solution, existence and uniqueness of some number called probability. It does not say where that number comes from. One could almost replace it with "Think hard and give me a number".

Tversky and Kahneman (1986) have shown that people do not conform to the rules of probability. People are not rational and do not have the calculus of probability ingrained in them. All the psychological issues mentioned in Garthwaite et al. (2005) come into play. Without discussing this side of the issue formally, just consider something we are all familiar with; reverse psychology. One could postulate that analytical ability is inversely proportional to the amount of reverse psychology a person displays, if that amount could be measured. The acting in reverse in reaction to an offer is an instinct, residing most likely within the survival tools. This natural caution, along with many other psychological factors we display in our behavior and thinking, would affect any probability assessment. Think of the well known empirical fact that 'the first impression lasts forever'. It hardly ever changes for some, regardless of how much data they are exposed to. For these reasons, one should simply not trust a personal probability. At least not with the current assessment technology.

6 Conclusion

There are many situations that warrant the use of subjective probabilities. Take a gambling situation with people sitting across a table. The situation is so unique and its economic consequences so pressing that only a subjective assessment can be considered. One may not reject outright the concept of subjective probability. However, we are also reporting on a real case of significant economic and safety consequences where subjective probability was strongly objected to. We point to the weakness of attempts by Bayesians at developing expert opinion elicitation methods. We look into the rationalizations of subjective probability and highlight what we think are flaws of the theory. There is no convincing definition of probability. While we admire the likes of de Finetti and Savage, we make the point that probability must be defined properly.

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