Anticipatory Networks and Superanticipatory Systems

Andrzej M.J. Skulimowski,

Department of Decision Science,
Chair of Automatic Control and Biomedical Engineering,
AGH University of Science and Technology
al. Mickiewicza 30, 30-050 Kraków, Poland
and
International Centre for Decision Sciences and Forecasting
Progress & Business Foundation, Kraków, Poland
Email: ams@agh.edu.pl, URL:www.foresight.pl

Abstract: We will present the theory of anticipatory networks and show that it generalizes earlier models of consequence anticipation in multicriteria decision problem solving. The theory bases on an assumption that the decision-maker takes into account the anticipated outcomes of future decision problems linked in a prescribed manner by the causal relations with the present one. So arises a multigraph of decision problems linked causally (first relation) and representing additionally one or more anticipation relations. Such multigraphs will be termed anticipatory networks. Then we will introduce the notion of superanticipatory systems, which are anticipatory systems that contain a future model of at least one more anticipatory system beyond itself. It will be shown that non-trivial anticipatory networks are superanticipatory systems. Finally, we will discuss some real-life applications of anticipatory networks.

Keywords: anticipatory networks, superanticipatory systems, multicriteria optimizers, multicriteria decision making, foresight

1. Introduction

This paper presents the theory of anticipatory networks that generalizes the ideas related to anticipatory models of consequences in multicriteria optimization problems presented in [14], [15], and [19]. We assume that when making a decision, the decision-maker takes into account the anticipated outcomes of each future decision problem linked by the causal relations with the present one. In a network of linked decision problems the causal relations are defined between the time-ordered nodes. The future scenarios of the causal consequences of each decision are modeled by multiple edges starting from an appropriate node. The network is supplemented by one or more relations of anticipation, or anticipatory feedback, that describes a situation where decision-makers take into account the anticipated results of some future optimization problems while making their choice. Then they use the causal dependences of future constraints and preferences on the choice just made to influence future outcomes in such a way that they fulfill the conditions contained in the definition of the anticipatory feedback relations.

Both types of relations as well as forecasts and scenarios regarding the future model parameters form an information model, called *anticipatory network* [19]. In Secs. 2 we will show the basic properties of anticipatory networks and idea of the method of their reduction and computing.

Following [14] and [19], in Sec. 3 we will present an application of anticipatory networks to select compromise solutions to multicriteria planning problems for the anticipatory trees and general networks. Then, motivated by the properties of the anticipatory networks, in Sec. 4 we will introduce the notion of superanticipatory systems. By definition, a *superanticipatory system* is a system that is anticipatory in the sense of Rosen [13], or weakly anticipatory in the sense of Dubois [2], and contains a future model of at least one other anticipatory system which outcomes may influence its current decisions by so-called anticipatory feedback relation. This notion is idempotent, i.e. the inclusion of other superanticipatory systems into the model of the future does not yield an extended class of systems, but we can classify them according to the grade that counts the number of nested superanticipation. We will observe that most anticipatory networks can be regarded as superanticipatory systems if we assume that future decisions can be based on similar anticipatory principles as the current one.

The motivation for the above outlined theory came from a need to create an alternative approach to selecting a solution to multicriteria optimization problems that takes into account direct multi-stage models of the future consequences of the decision made which was presented in [14]. The anticipatory behavior of decision-makers correspond to the definition of anticipatory system proposed by Rosen [13] and developed further in a series of publications by Dubois and other researchers [2,4,8,12]. A bibliographic survey of these ideas can be found in [8]. An ability of creating a model of the future of the outer environment and of itself that characterizes an anticipatory system is also a prerequisite for an anticipatory network, where each node models an anticipatory system and they are able to influence each other according to the causal order. In this paper we restrict the anticipatory networks to model decisions made in so-called networks of optimizers, where each node models an optimization problem [19]. This class of information processing systems has been introduced in [17].

We will point out that most anticipatory networks are model-based so that their nodes correspond to the weak anticipatipatory systems [2]. The networks of anticipatory agents can be constructed applying scenarios of anticipated consequences provided by a foresight project. In the final Sec.5 we will outline an example of such construction applied in IT foresight. Similarly to the anticipatory networks of optimizers, one can construct networks with nodes modeling Nash equilibria, set choice problems, random or irrational decision-makers, or hybrid networks containing nodes of all types[18].

2. Anticipatory networks as generic causal models

The original motivation ideas behind introducing anticipatory networks as models of consequences was formulated in [14,17,19].The basic principle is to use the forecasts and foresight scenarios to estimate the parameters (cf. below) of future decision making agents and to build the network of them. The anticipated future consequences of a decision made are modeled as changes of constraints and/or preference structures of future decision problems. The nature of these changes is assumed a priori known. It may result from model-based forecasts or foresight as well. Then the anticipated outcomes of future decision making problems that – of course – depend on constraints and preference structures, serve as a source of additional information that can be used to solve the current problem. In addition, the future decision-making agents may use the same principle to make their decisions and this fact must be taken into account at the preceding decision stages.

The constructive algorithms of computing the solutions to the current multicriteria decision making problem taking into account the above anticipatory preference information feedback may be applied if we know that:

A) All agents whose decisions are modeled in the network are rational, i.e. they make their decisions complying to their preference structures.
B) An agent can assess outcomes of some or all future decision problems that are causally dependent on the present one as more or less wanted. This dependence is described as relations (usually multifunctions) between the decisions to be made now and the constraints and/or preference structures of future problems.
C) The above assessments are transformed into decision rules for the current solution choice problem that affect the outcomes of future problems in such a way that they comply with the agent's assessments in this or another way. The decision rules so derived form an additional preference structure for the decision problem just considered.
D) There exist a relevance hierarchy in the network; usually the more distant in the future from the present is an agent, the less relevant is the choice of solution of its problem, but this rule is not a paradigm.

Anticipatory networks, which contain only decision-making agents solving optimization problems are termed *optimizer networks.* According to [19], an *optimizer* O is a function that acts on a set of feasible decisions U and on the preference structure P and selects a subset $X \subset U$ according to P and to the fixed set of optimization criteria F with values in an ordered space E that define this optimizer. Throughout this paper we will assume that the optimization problems solved by the optimizers have the form

$$(F: U \rightarrow E) \rightarrow min(\theta), \tag{1}$$

where E is a vector space with a partial order $\leq_\theta$ defined by a convex cone θ, i.e. iff

$$x \leq_\theta y \Leftrightarrow y - x \in \theta \text{ for each } x, y \in E.$$

The solution to (1) is the set of nondominated points defined as

$$\Pi(U,F,\theta) := \{u \in U: [\forall v \in U: F(v) \leq_\theta F(u) \Rightarrow v = u]\}.$$

Most often the decision maker's aim is to select and apply just one nondominated solution to (1). Thus the role of the preference structure P that occurs in the definition of an optimizer is to restrict the set of nondominated points in the solution process. Without a loss of generality we can assume that P is defined explicitly by pointing out for each $u \in U$ which elements of U dominate u. These are termed *dominating sets* and form a *domination structure* [1] which models the way the decision maker takes into account additional information about preferences when making the decision. Thus P can be defined as a family of subsets of U in the following way

$$P:=\{\pi(u)\subset U:\ u\in\pi(u)\ and\ if\ v\in\pi(u)\ and\ w\in\pi(v)\ then\ w\in\pi(u)\}_{u\in U},$$

i.e. for each $u\in U$ $\pi(u)$ is the set of elements preferred to u.

Similarly as in case of spaces ordered by convex cones, an element $u\in U$ is *nondominated* with respect to P iff $\pi(u)\cap U=\{u\}$, which means that no element of U is preferred to u. The set of nondominated points with respect to P will be denoted by $\Pi(U,F,P)$. In a most common case where an additional preference structure P is defined by a convex cone ζ,

$$\pi(u):=\pi(u,\zeta)=\{v\in U:\ F(v)\leq_{\zeta} F(u)\} \tag{2}$$

and $\Pi(U,F,P)=\Pi(U,F,\zeta)$. Conversely, in problem (1) $\Pi(U,F,\theta)=\Pi(U,F,P_{\theta})$ with P_{θ} defined by (3). Now we can formulate the following

Definition 1. A *free multicriteria optimizer* O is a mapping that selects a solution u_0 from U that is nondominated with respect to θ in (1) and P, i.e. if u_0 is an element of

$$\Pi(U,F,\theta,P):=\{u\in U:\ [\forall v\in U:\ F(v)\leq_{\theta} F(u)\Rightarrow v=u]\}\cap\Pi(U,F,P). \tag{3}$$

O is characterized by U, F, θ, and P and will be denoted by a 4-tuple $O:=(U,F,\theta,P)$. ■

If, beyond the criteria F, the ordering θ, and the preference structure P an optimizer O realizes certain decision rules R, such as a heuristics or random choice from $\Pi(U,F,\theta,P)$ then the admissible solution set returned by this optimizer may be different from $\Pi(U,F,\theta,P)$ and equal to $X\subset\Pi(U,F,\theta,P)$. In such cases we will use the notation $O:=X(U,F,\theta,P)$, where X is interpreted as the set of actually selected solutions to the optimization problem (1). For brevity's sake, whenever it does not cause an ambiguity, free multicriteria optimizers will be referred to as *optimizers*.

Observe that if for a convex cone $\zeta\subset E$ $P:=P_{\zeta}$ and $\theta\subset\zeta$ then, of course,

$$\Pi(U,F,\theta,P)=\{u\in U:[\forall v\in U:F(v)\leq_{\theta}F(u)\Rightarrow v=u]\}\cap\{u\in U:[\forall v\in U:F(v)\leq_{\zeta}F(u)\Rightarrow v=u]\}=\Pi(U,F,\zeta).$$

However, in such cases the resulting preference structure represented by the cone ζ is result usually from an iterative process of restricting gradually the set of nondominated points to (1). This technique is referred to as *contracting cone method* (cf. [5]) since the dual cones is to an increasing sequence of ordering cones $\theta\subset\zeta_1\subset\zeta_2\ldots\subset\zeta$ are contracting as well as do the sets $\Pi(U,F,\theta)$, $\Pi(U,F,\zeta_1)$, ..., $\Pi(U,F,\zeta)$. Here, we refer to this methodology to show its similarity to the anticipatory network technique described in the Anticipatory Decision-Making Problem (ADMP). Indeed, one can see [19] that the more anticipatory feedbacks are taken into account in an anticipatory network with the starting node , the more opportunities exist to confine the choice in a problem (1) to a smaller subset of the set $\Pi(U,F,\theta,P)$.

If for all $u,v \in U$

$$F(v) \leq_{\theta} F(u) \Rightarrow v \in \pi(u) \quad (4)$$

that we will say that P conforms to the criteria F and the order θ, in brief P is *conforming*. Observe that this is the case if $P:=P_{\zeta}$ and $\theta \subset \zeta$. If P is conforming than to select an $X \subset \Pi(U,F,\theta)$ the action of the optimizer can be stretched on all the set U, without computing $\Pi(U,F,\theta)$, otherwise it must be restricted to $\Pi(U,F,\theta)$. However, the computation or even an approximation of $\Pi(U,F,\theta)$ can be a hard task.

As we have already mentioned, besides of their optimizing capabilities, the optimizers may form networks with some new properties compared to the theory of sequential decision problems. In particular, in feed-forward networks of optimizers constraints and preference structures in some optimizers are causally linked to the results of solving other problems and may depend on their preference structures. Thus, in a network of optimizers the parameters of the actual instances of optimization problems to be solved vary as results of solving other problems in the network.

Definition 2. If $O_1:=X_1(U_1,F_1,\theta_1,P_1)$ and $O_2:=X_2(U_2,F_2,\theta_2,P_2)$ are multicriteria optimizers then a constraint influence relation r between is defined as

$$O_1\, r\, O_2 \Leftrightarrow \exists \varphi: X_1 \rightarrow 2^{U_2}: X_2=\varphi(X_1). \quad (4)$$

Acyclic r are termed *causal constraint influence relations*, in short, *causal relations*. ∎

Causal relations are represented by the (causal) *network of optimizers*. The Def. 2 models the situation where the decision maker anticipating a decision output at a future optimizer can react by creating or forbidding decision alternatives, which is described by influencing the constraints by multifunctions φ depending on the outputs from the preceding problems. Similarly as in [19] and [18], from this point on the term *causal network* will refer to the graph of a causal constraint influence relation.

To complete the definition of anticipatory networks, we need first to define the anticipatory feedback relation.

Definition 3. Suppose that G is a causal network consisting of free optimizers and that an optimizer O_i in G precedes another one, O_j, in the causal order r. Then the *anticipatory feedback* between O_j and O_i in G is an information concerning the model-based anticipated output from O_j which serves as an input influencing the choice of decision at the optimizer O_i. Such relation will be denoted by $f_{j,i}$. ∎

By the above definition, the existence of an anticipatory information feedback between the optimizers O_n and O_m means that

- the decision maker at O_m is able to anticipate the decisions to be made at O_n, and
- the results of this anticipation are to be taken into account when selecting the decision at O_m.

This relation does not need to be transitive. Similarly as in the case of causal relations, there may also exist multiple types of anticipatory information feedback in a network, each one related to the different way the anticipated future optimization results are considered at an optimizer O_m. The multigraph of r and one or more anticipatory feedbacks define an anticipatory network of optimizers:

Definition 4. A causal network of optimizers with the starting node O_0 and at least one additional anticipatory feedback relation linking O_0 with another node in the network will be termed an *anticipatory network* (of optimizers). ■

In [19] the anticipatory information feedback in causal networks of optimizers has been applied to selecting a solution to the optimization problem modelled by the starting element in an anticipatory optimizer network G. Specifically, while making the decision, the decision maker takes into account the following information contained in G:

- forecasts concerning the parameters of future decision problems represented by the decision sets U, criteria F, and the ordering structure of the criteria values θ,
- the anticipation concerning the behaviour of future decision makers acting at optimizers, represented by the preference structures P,
- the forecasted causal dependence relations r linking the parameters of optimizers in the network, and
- the anticipatory relations pointing out which future outcomes are relevant when making decisions at specified optimizers and the anticipatory feedback conditions.

Now we will present a few key definitions referring to solving the multicriteria decision problems using an anticipatory network of optimizers as a source of additional preference information.

Definition 5. An anticipatory network (of optimizers) is said to be *solvable* if the process of considering all anticipatory information feedbacks results in selecting a non-empty solution set at the starting problem. ■

Definition 6. A causal graph of optimizers G that can be embedded in a straight line will be called a *chain* of optimizers. If it contains at least one an anticipatory feedback $f_{i,0}$ then G will be termed an *anticipatory chain (of optimizers)* ■

Example. The Fig.1 contains an example of an anticipatory chain of optimizers, where a decision made at the optimizer $O_0=(U_1,F_1,IR_+^2)$ will take into account the anticipated outcomes at $O_{j-1}=(U_1,F_1,IR_+^2)$ and $O_j, =(U_1,F_1,IR_+^2)$. The causal constraint influence relations $\varphi_{i,j}$ (cf. Def.2) are defined as $\varphi_{i,j}:=Y_j \circ F_i$, where the multifunctions $Y_j:F_i(U_i)\rightarrow\rightarrow U_j$ model the dependence of the scope of decisions available at O_j on the optimization outcomes of the problem O_i. Following [14], the total restriction of the decision scope at O_j generated by Y_j is denoted by R_j, i.e. $R_j:=Y_i(F_i(U_i))$. The resulting restriction of the set of nondominated outcomes at O_j is denoted by S_j (in a chain, as exemplified in Fig.1, i can be replaced by $j-1$). By definition, the causal relation represented by $\varphi_{i,j}$ is *non restrictive* iff $S_j= \Pi(U_j,F_j,\theta_j)$. We will say that $\varphi_{i,j}$ *complies with* O_j iff $S_j\subset \Pi(U_j,F_j,\theta_j)$ - this is the case shown in Fig.1.

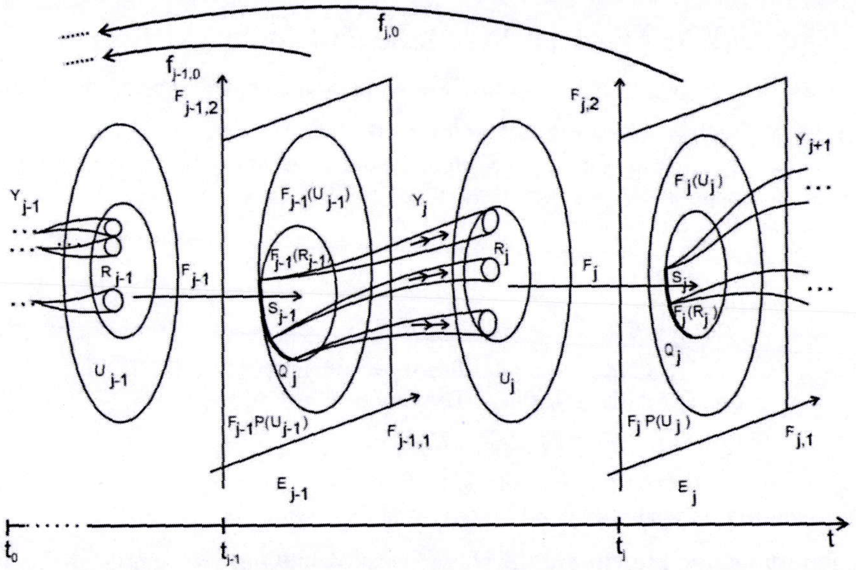

Fig. 1. A chain of optimizers with two anticipatory feedbacks $f_{j-1,0}$ and $f_{j,0}$ linking O_{j-1} and O_j, with the starting node O_0, respectively. The temporal order complies with the causal relations defined by multifunctions $\varphi_{j-1,j} := Y_j \circ F_{j-1}$.

Definition 7. A causal graph of optimizers G that is a tree and contains at least one an anticipatory feedback $f_{i,0}$ will be termed an *anticipatory tree (of optimizers)* ■

A simple tree of optimizers is shown in Fig. 2.

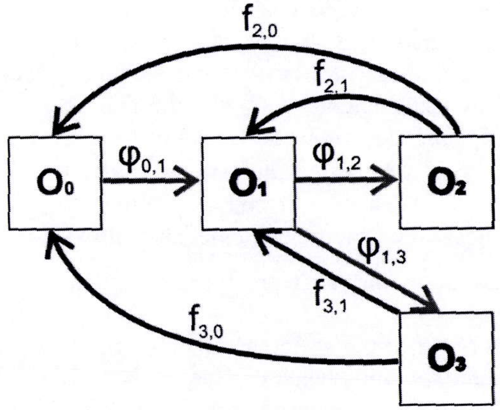

Fig. 2. An example of a simple tree of optimizers, where O_1 is the bifurcation optimizer [19] for O_2, and O_3. Causal relations are defined by the multifunctions $\varphi_{i,j} := Y_i \circ F_{i-1}$. Four anticipatory feedback relations are denoted by $f_{k,m}$, $k=0,1$, $m=2,3$.

3. Decision making problems in general anticipatory networks

In a non-trivial anticipatory network the following decision problem can be formulated:

Anticipatory Decision-Making Problem (ADMP). For an anticipatory network G with finite decision sets, for all chains of optimizers find the set of all admissible sequences of decisions $(u_0, \ldots, u_n)$ that maximize the function

$$g(u_0, \ldots, u_n) := \sum_{i \in J(0)} h(u_i, q(0,i)) w_{0,i} \tag{5}$$

and such that for all i, $1 \leq i < n$, the truncated decision chain $(u_i, \ldots, u_n)$ maximizes

$$g(u_i, \ldots, u_n) := \sum_{j \in J(i)} h(u_j, q(i,j)) w_{i,j}, \tag{6}$$

where $J(i)$, $i=0,1,\ldots,n$, denote the sets indices of decision units in G, which are in the anticipatory feedback relations with O_i. The function h is defined as

$$h(u_i, q(i,j)) := \| F_{i-1}(u_i) - q(i,j) \|, \tag{7}$$

and $w_{i,j}$ are positive coefficients corresponding to the relevance of each anticipatory feedback relation between the optimizers O_i and O_j. ■

From the formulation principle at the above decision making problem it follows that the decision maker at O_0, while selecting a decision $u_0 \in U$ that is the first element of an admissible decision sequence, uses the anticipatory network G and the function g as an auxiliary preference structure to solve the problem (1).

The key notion in this section can now be defined as follows:

Definition 8. A solution to the ADMP, a family of decision sequences $u_{0,m(0)}, \ldots, u_{N,m(N)}$ minimizing *(4)-(6), will be called anticipatory chains.* ■

Constructive solution algorithms to solve the ADMP taking into account the information contained in an anticipatory network G have been proposed in [19] (Algs.1 and 2) for a class of anticipatory networks with discrete decision sets U_i, when the graph of causal relation r is either a chain or a tree. The anticipatory feedback conditions have been there defined as the requirement of O_i that the decisions at O_j, for j from certain index set $J(i)$ such as O_i precedes O_j in the causal order r are selected from the subsets $\{V_{ij}\}_{j \in J(i)}$, $V_{ij} \subset U_j$. Usually, it means that the criteria values on V_{ij}, $F_j(V_{ij})$ are of special importance to the decision makers and can be defined as reference sets [16]. The general ideas of these algorithms can be presented as follows:

1. Decompose the anticipatory network into causal chains of optimizers linked by causal relations,
2. Identify in each chain of the anticipatory network elementary cycles, i.e. cycles, which do not contain other such cycles, consisting of causal relations along chains and anticipatory feedback relations,
3. Solve the decision problem for each chain, by eliminating the elementary cycles,
4. Use the logical conditions that defined the anticipatory requirements to bind the solutions sequences on the common parts of the anticipatory chains.

Thus it is possible to reduce the analysis of anticipatory trees to the recursive analysis of anticipatory chains in the tree. Moreover, a general network can be decomposed into trees or chains, which makes it possible to apply solution rules for chains iteratively, gradually eliminating solved trees and chains. However, the solution procedures for anticipatory trees cannot be directly adopted to the solution of the problems where in a network of optimizers there may exist units that are influenced causally by two or more predecessors without taking into account the synchronisation problems.

Such networks can model the problems where multiple resources, provided as outcomes of multiple different and independent decision processes, determine the scope of a later-stage decision, for example to optimize the decisions in a potential future joint venture created to develop a new product (NPD) one has to consider the outputs provided by the potential future partners of this joint venture. It can be shown that taking into account a possibility to create a future production alliances and representing such relations in an anticipatory network gives a competitive advantage over agents optimizing their own future outputs only. An example of a general anticipatory network is shown in Fig.3.

To analyze general networked optimizers, we will have to admit an assumption that if an optimizer O_p is directly influenced by more than one predecessor then the aggregation rules are defined for each subset of influencing factors generated by the preceding optimizer (e.g. as intersection or a union of the sets of feasible alternatives, each one imposed by a different preceding optimizer).

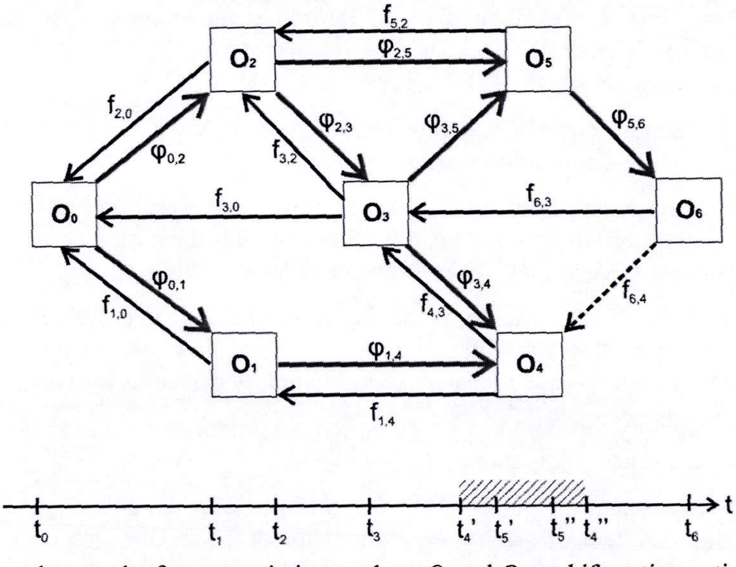

Fig.3. A causal network of seven optimizers, where O_2 and O_3 are bifurcation optimizers, while O_4 and O_5 are each influenced by two predecessors. The shadowed area between t_4' and t_4'' on the time axis denotes the synchronization interval for the simultaneous influence of O_1 and O_3 on the decisions of O_4. An analogous interval for O_5 is contained in $[t_4', t_4'']$. The dotted arrow between O_6 and O_4 denotes an irrelevant anticipatory feedback, because there is no causal relation between these optimizers.

In addition, these rules must take into account the synchronization of influence that was not necessary in case of anticipatory trees. Specifically, simultaneous action of predecessors on O_p may be restricted to the prescribed time intervals. This is depicted in the above Fig. 3, where t_i' and t_i'' denote the start and end of a synchronization time intervals for the *i-th* optimizer.

In a most common situation, where the preceding optimizers influence imposes a logical products of individual influences, the synchronization problem reduces to analysing the time conditions when the intersection of constraints resulting from multiple influencing multifunctions can still yield a feasible solution. However, in general, all combinations of logical conditions binding independent influences should be considered, including the situation where one agents influence consists in removing another agent's constraints. The analysis of such cases requires further studies, which however can be based on the solution scheme presented above and in [19].

4. Anticipatory networks as superanticipatory systems

Let us observe that in the above presented approach to solving anticipatory networks we have assumed that the anticipation is a universal principle governing the solution of optimization problems at all stages. In particular, future decision makers modelled at the starting decision node O_0 can in the same way take into account the network of their relative future optimizers when making their decisions. Thus, the model of the future of the decision-maker at O_0 contains models of future agents including their respective future models. This has motivated us to introduce the notion of superanticipatory systems, that directly generalize the anticipatory systems in the sense of Rosen [13] and weak anticipation in the sense of Dubois [2]:

Definition 9. *A superanticipatory system is an anticipatory system that contains at least one model of another future anticipatory system.* ■

Observe that by the requirement that the model of *another* system must be contained in a superanticipatory system, the above definition excludes the case when an anticipatory system models recursively itself. This is discussed later in this section.

By definition, this notion is idempotent, i.e. the inclusion of other superanticipatory systems into the model of the future of a superanticipatory system does not yield an extended class of systems since, every superanticipatory system is also anticipatory.

The superanticipatory systems can be classified according to the grade that counts the number of nested superanticipation.

Definition 10. A superanticipatory system *S* is of grade *n* if it contains the model of a superanticipatory system of grade *n-1*. An anticipatory system which does not contain any model of another anticipatory system is defined as superanticipatory of grade *0*.

Let us observe that the actual grade *n* of a superanticipatory system *S* depends on the accuracy of the model of other systems used by *S*. If, according to When constructing its model of the environment *S* may not be able to estimate the actual content of the other systems' models. It may be conjectured that if a superanticipatory system uses an empirical and rational modeling approach then it is more likely that the other systems will have models of a higher grade than S has estimated based on experiments. Thus the grade of the rational system S, when determined based on the information coming solely from the same system, can be regarded as a lower bound of an actual grade. The perfect knowledge of the grade can be attributed to a hypothetical ideal external observer only. When referring to an anticipatory network, which is always a result of certain modeling compromise, one can see that the following statement can be formulated

Theorem 1. Let $G=(O,r,f)$ be an anticipatory network, where O is the (finite) family of optimizers, r is the causal influence relation, and f is the anticipatory feedback relation. If G contains an anticipatory chain C such that there exist exactly n optimizers in C, $\{O_{C,1},...,O_{C,n}\}\subset C=(O_1,...,O_N,r,f)$, $N\geq n$, with the following property:

$$\forall i\in\{1,...,n\}\ J_C(i)\neq\emptyset \text{ and } (\exists j\neq i\text{: } O_{C,j}\ r\ O_{C,i} \text{ and } i\in J_C(j)), \qquad (8)$$

where $J_C(i)$ is the set of indices of optimizers in G, which are in the anticipatory feedback relation with O_i. and no other chain in G has the property (8) with $m>n$ then G is a superanticipatory system of a grade at least n.

The proof of the above Theorem 1 follows directly from the definitions of anticipatory networks (Def.4) and superanticipatory systems (Def. 9,10). ■

It is easy to see that an anticipatory network containing a chain on n optimizers, each one linked with O_0 and with all its causal predecessors by an anticipatory feedback is an example of a superanticipatory system of grade n.

The notion of superanticipation is obviously related to the general recursive properties of anticipation. By its definition, superanticipation makes sense only when the anticipation of the future is based on a predictive model. Problems to be solved that arise in a natural way are related to the accuracy of such models, to the grade of superanticipation, and to the relation between the *internal* (system's) time, when it builds the model and analyzes it, and the *external* real-life time, when the modeled objects evolve. Other recursive approaches related to anticipation include meta anticipation defined by Dubois [2] and information set models in multi-step games.

In [9] Nadin supposes that for every anticipatory system in the sense of Rosen, from the fact that it contains a model of itself (and the environment) it follows that the "itself" portion of this model is nested, i.e. the model of itself contained in the system must contain the model of this model etc. This would lead to a kind of 'self-superanticipation' of infinite grade, something like a view into a mirror having another one behind. However, as the Rosen's expression 'contains' was strongly motivated by biology, where the agents are usually not able to store the information outside of their brains. In the computational theory of anticipation attributed to humans and artificial systems, the 'model of itself' can be actually an external one, stored in a computer or elsewhere, and the term '*contains*' should be better replaced by '*is made available*'. This lets us avoid the paradox of unlimited memory needs to cope with the nested models. Indeed, every anticipatory or superanticipatory system manages only limited memory and computational resources and every real-life anticipatory network can be constructed as a finite model with a limited anticipation horizon.

Another idea related to the recursion and anticipation is due to Dubois [2,3]. It bases on the observation that some coupled chaotic systems with delays manifest a synchronization behavior similar to that one which occurs in periodic systems [11]. Namely, if to the chaotic time-delay system $x=f(x-\tau,x)$, $x\in I\!R^p$, $\tau>0$, there is associated another system with the state vector y_1 described by the same equation with a dissipative coupling term

$$y_1=f(y_1(t-\tau),y_1) - \sigma D[x(t)- y_1(t)], \tag{9}$$

where $\sigma>0$, $D=[d_{ij}]_{pxp}$, $d_{ij}\in\{0,1\}$, then, under additional assumptions concerning the Lyapunov exponents of (8), $lim[|y_1(t)-x(t+\tau)|]$ vanishes when $t\to\infty$ [cf. e.g. 11]. This allows to construct an approximate functional relation between the present state of y_1 and future states of x, called by Dubois incursion [4]. Moreover, when another state vector y_2 is defined by substituting in the same eq. (9) y_1 to x and y_2 to y_1, we conclude that $y_2(t)$ is approximately equal to $x(t+2\tau)$. This operation can be repeated recursively, yielding the approximate functional dependence between y_n and $x(t+n\tau)$ after n iterations. It means that this phenomenon is closely related to strong anticipation in the sense of Dubois [4]. Such n-level synchronization, where the *k-th* state vector is the feedback cascade defined by (9) depends on $x(t+k\tau)$ for $k=1,...,n$ is termed *meta anticipation* [2,3]. Both notions, superanticipation and meta anticipation are disjoint as they refer to different types of anticipation: weak (model based) in case of superanticipation, and strong (functional dependence based) in case of meta anticipation.

However, an intermediate recursive notion can be defined for n-stage games, when the player anticipate the behavior of other ones. From the point of view of the player G_1 the anticipation is defined here for $k(G_1)$ steps forward and includes the anticipatory models of other players $G_2,\dots G_N$. Each one of them can also possess a model of ourselves (G_1) and of some or all remaining game participants with the anticipation horizon of $k(G_i)$ moves, $i=2,..,N$. The player G_1 fulfills thus the definition of superanticipatory system and the game can be represented as an evolving anticipatory network. However, when the future moves of the other players results from a deterministic algorithm rather than from a decision making model, the anticipation may be based on the knowledge of the (deterministic) function identified with that algorithm's operation. This may happen when a human player plays a deterministic game with a computer, or when machine-machine interaction is modeled. Such games would refer to the Dubois' strong and meta anticipation, where the master-slave (or driver-response) structure of the coupled system is an analogue of the leader-follower relation in multi stage Stackelberg games [6,10].

5. Concluding Remarks

This paper surveyed the main ideas concerning the anticipatory networks, the basic methods to solve them, and presented their extension, termed superanticipatory systems. Anticipatory networks may be applied to model and solve a broad range of problems. Apart from the above-mentioned inspirations coming from potential uses in foresight, roadmapping, and socio-econometric modelling, there are further potential fields of application, such as

- Anticipatory modelling of sustainable development, where the anticipatory network assumption that the present-time decision maker wants to assure a possibly best opportunities to make a satisfactory decisions to future agents modelled by other nodes in the network corresponds to the 'future generation' paradigm
- Anticipatory planning based on results of foresight studies, such as development trends, scenarios, and relevance rankings of key technologies, strategic goals etc. The planning can use deterministic as well as stochastic planning techniques and include multi-step game models.

Further applications are discussed in [19].

The anticipatory networks can also contribute to implement the knowledge contained in foresight scenarios in a clear, formal way. Specifically, the development of theory outlined above has been motivated by the problem of modelling finding feasible foresight scenarios based on the identification of future decision-making processes and on anticipating their outcomes. Scenarios, such as those defined and used in foresight and strategic planning [5], can depend on the choice of a decision in one of the networked optimization problems as well as be external-event driven. When included in a causal network of optimizers, the anticipation of future decisions and alternative external events would allow us to generate alternative structures of optimizers in the network.

Anticipatory networks, those that contain the optimizers as well as the hybrid ones [18] extend the plethora of modeling tools that can be used to formulate and solve decision making problems taking into account new future-dependent preference structures. When regarded as a class of world models for robotic systems, the anticipatory networks provide a flexible representation of the outer environment, while the superanticipation allows us to model collective decision phenomena in autonomous robot swarms. Further studies on this class of models can also contribute to the general theory of causality and may lead to discovering surprising links to physics and neural decision mechanisms.

Acknowledgement. The author is grateful for the support of research project No. WND-POIG.01.01.01-00-021/09: "Scenarios and development trends of selected information society technologies until 2025" funded by the ERDF within the Innovative Economy Operational Programme, 2006-2013.

References

1. Bergstresser, K., Charnes, A., Yu, P. L. (1976). Generalization of domination structures and nondominated solutions in multicriteria decision making. *Journal of Optimization Theory and Applications*, 18(1), p. 3-13
2. Dubois D.M. (2001), Theory of Incursive Synchronization and Application to the Anticipation of a Delayed Pearl-Verhulst Chaotic System, *Systems Science*, 27(1), p.71-86.
3. Dubois D.M. (2002). Theory of Incursive Synchronization and Application to the Anticipation of Delayed Linear and Nonlinear Systems. *Computing Anticipatory Systems: CASYS 2001 - Fifth International Conference*. American Institute of Physics, AIP Conference Proceedings 627, p. 182-195.

4. Dubois D.M. (2003). Mathematical Foundations of Discrete and Functional Systems with Strong and Weak Anticipations. In: Butz, M.V., Sigaud, O., Gérard, P. (eds.): *Anticipatory Behavior in Adaptive Learning Systems, State-of-the-Art Survey. Lecture Notes in Artificial Intelligence* 2684, Springer, p. 110-132.
5. Godet M. (2001). *Creating Futures – Scenario Planning as a Strategic Management Tool.* Economica, London.
6. Kaliszewski I. (1994). *Quantitative Pareto analysis by cone separation technique.* Kluwer Academic Publishers, Boston.
7. Kijima, K., Xu, Ch. (1994). Incentive Strategies Dealing With Uncertainty About The Followers MCDM Behavior. *International Journal of Systems Science* 25(9), p. 1427-1436.
8. Nadin M. (2010). Annotated Bibliography. Anticipation. *Int. J. General Sys.* 39(1), 35-133.
9. NadinM. (2012): The anticipatory profile. An attempt to describe anticipation as process, *International Journal of General Systems* 41(1), p.43-75.
10. Nishizaki I., Sakawa M. (2009).*Cooperative and Noncooperative Multi-Level Programming.* Operations Research/Computer Science Interfaces Series Vol. 48, Springer, Dordrecht, Heidelberg, London, New York.
11. Pikovsky, A., Rosenblum, M., and Kurths, J. (2001). *Synchronization: A universal concept in nonlinear sciences.* Cambridge University Press, Cambridge, UK
12. Poli, R.: An introduction to the ontology of anticipation. *Futures* 42, p.769-776 (2010)
13. Rosen R. (1985).*Anticipatory Systems - Philosophical, Mathematical and Methodological Foundations.* Pergamon Press, London, 2nd Ed. Springer, 2012.
14. Skulimowski A.M.J. (1985). Solving Vector Optimization Problems via Multilevel Analysis of Foreseen Consequences. *Found. Control Engrg.* 10(1), p.25-38.
15. Skulimowski A.M.J. (1986). Foreseen Utility in Multi-Stage Multicriteria Optimization. In: J. Kacprzyk (ed.), *Seminar on Nonconventional Problems of Optimization*, Warsaw, May 9-11, 1984. Proceedings, Part III, *Prace IBS PAN* 136, p. 365-386.
16. Skulimowski A.M.J. (1997). Methods of Multicriteria Decision Support Based on Reference Sets, In: Caballero R., Ruiz F., Steuer R.E. (eds.) *Advances in Multiple Objective and Goal Programming, Lecture Notes in Economics and Mathematical Systems* 455, Springer, Berlin-Heidelberg-New York, p. 282-290.
17. Skulimowski A.M.J. (2008). The Networks of Optimizers. Paper presented at the 18th International Conference on Multicriteria Decision Making, Auckland, Jan.5-9, 2008.
18. Skulimowski A.M.J. (2012). Hybrid anticipatory networks. In: L. Rutkowski et al. (eds.), Proc. ICAISC 2012, *Lecture Notes in Artificial Intelligence* 7268, Springer, Berlin-Heidelberg, p.706-715.
19. Skulimowski A.M.J. (2013). Anticipatory Network Models of Multicriteria Decision-Making Processes. *International Journal of Systems Science* 44 (iFirst), http://www.tandfonline.com/doi/pdf/10.1080/00207721.2012.670308.