Dimension Calculus and Anticipatory Systems

Arturo G. Grappone
International Review "Metalogicon",
Piazza Albania, 6 - 00153 Roma – ITALY
a.grappone@mclink.it - www.grappone.it

Abstract

Dimensional analysis permits us the rigorous comprehension of physical quantities by its reduction in terms of mass M, length L, time T, electric charge Q, and temperature Θ. E.g., speed is LT^{-1}, force is MLT^{-2} and so on. However, Saumont has observed that it is no rational to give dimensions to constant quantity and not to give dimensions to variable quantity. Also, standard dimension analysis implies an evident hyper-cubic topology $M^n L^p T^p Q^r \Theta^s$ that isn't incompatible with (hyper)incursive systems that have hypersphere or torus topology. Finally, Grappone has proven that anticipatory systems, in terms of set inclusive networks, are equivalent to first order theories in mathematical logic, i.e. polyadic or cilindric algebras that haven't a simple hypercubic structure. This paper is an attempt to start the solving of these problems.

Keywords: Computative Topology, Dimension Analysis, Incursivity

1 Introduction

Rémi Saumont has put an important problem.[1] He has observed that similitude theory has got marginal attention from fundamental sciences also if great scientist as Galilee, Newton and Fourier has been interested in them.

Saumont affirms that such a deficiency is provoked by difficulty to attribute a scale that isn't built by fundamental relation among electrical forces.

He observe that, in fact, Newton's gravitation law permit to utilize scales that starting phenomena defines; instead, Coulomb's law on electric interaction that involves punctual masses doesn't permit this one.

But electromagnetic forces that Newton calls "mechanic causes" are preponderant in human scale in the greatest part of physical phenomena (various motor processes, thermodynamics, material resistance, viscosity, friction, and so on).

Thus a method that can be applied to gravitational phenomena has hystorical value only.

However, to find models in various sciences electric forces need of a proper scale. This one is empirically built because it is found on electric laws that have be done till now.

[1] See Saumont, R., (1988), *Présentation de l'ouvrage*.

The limit of such an empirical scale appears clearly in physiology if we consider the difficulty to put biological laws without that the force increase in proportion to the square of linear dimension, as well as more how much the more the living beings are small (surface law, Lambert and Tessier's law).

Beyond all, Newton's statement that mechanic causes increase proportionally to surfaces is verified in materials resistance, crystals increasing, energy or heat changes in thermo machines and in living beings.

All these experimental data induce Saumont to review the studies on condictions of physical likeness because it is perhaps too hurled to affirm their inadequacy to reality.

But scale definition for electromagnetic forces is very difficult and its result has got developments for which there is discomfort to preview the width.

To solve the problem Saumont reaches some interesting conclusion.

The fundamental relations of Newton's mechanics haven't an immediate relativistic character if dimensioned constants appear in them; otherwise their relativistic character appears.

Dimensioned constants introduction needs because the used space-time used representation has got a lack of generality.

So-called universal constants permit a mathematical representation of the phenomena in a common dimension system in the tally of a three dimensions Euclidean space but a rough knowledge of phenomena mechanisms. Physics progresses instead with the knowledge progress of these constants.

Actual knowledges indicate that three dimension space usual representation isn't adequate to represent all the reality and that it can be described only by a really unitary system.

Inertia is linked to time by definition. Thus it can locally be identified either to gravitation (general relativity) or to electromagnetism by the choice of the same opportune representation in the space.

But gravity and electromagnetism can be identified never because the dimensional formula of the former is the square of the dimensional formula of the latter when such formulas are defined in the same space domain.

Unity only can be equal to its square: if we don't define largenesses as regard an extreme then it is an obstacle.

It is usual and reasonable to put all phenomena in a same space-time context and some theories haven't the elegance and the internal coherence of general relativity, but this last one is elegant and coherent only in representation of gravitation-inertia and manages electromagnetism only phenomenologically.

Primary relations permit in effect to define the dimensional base of general relativity by the two-dimensional formulas on gravity and electromagnetism from which the question is born. So Saumont concludes that such a dimensional base develops a dimensional fundamental indetermination that is little compatible with quantum electromagnetism.

Thus Saumont suggests a space-time structure where gravity could have four dimensions instead of three as electromagnetism.

If we accept a no usual space-time representation as regard the dimensions then gravitation, inertia and elettromagnetism becomes interdependent phenomena of a same emtity.

To solve the problem, particularly for inertia and time definition, Saumont suppose the work hypothesis of a vacuum-like potential energetic hyperspace.

He suggests the geometric character of the energy and so an equivalence between energy and space.

This space energetic property isn't necessary linked to a given curve. It could justify physical phenomena beyond the action ray of nuclear forces by a poly-dimensional Euclidean representation in a five-dimension tally that is perfectly compatible with the restricted relativity.

Thus Saumont supposes gravity as a five-dimension electromagnetism that is represented by linear equations and one function in the five-dimension space will give the gravity charge quantity.

Similarity theory has permitted this work hypothesis and so itreveals itself as a guide to dimension analysis ansd physical system conception.

Saumont supposes that electromagnetism becomes starting process of inertia and gravitation phenomena by its own interaction law that is deduced by experience.

In fact, virial theorem proves that the balance of a set of interacting charge particles can be dynamic only, i.e. electromagnetism is continous movement at corpuscular scale.

Gravitation and inertia should be the reaction of the five- and four-dimension potential energetic mediums to a perturbation by movement that destroys the isotropy because it happens in three dimensions.

This Saumont statement explains because the gravitational interaction cannot be repulsive differently from electromagnetism interaction.

It suggests that space topologic properties are essential in physics building.

Thus time nature is a problem of combinatory topology: as spaces can be oriented only if they are odd-dimensioned, time has orientation properties that aren't equal to those ones of length because it is defined by an inertial law in an even-dimension space and its orientation properties play a role in an odd-dimension space.

Saumont observe that the electromagnetical genesis of inertial and gravitational phenomena is subject to a law of all or nothing for elementary levels. E.g. inertial and gravitational mass is independent from its temperature till to absolute zero where it is null.

This fact makes perhaps impossible to obtain gravitational and inertial effects from the electromagnetic usual processes of our technical apparatuses.

Saumont supposes that the management of gravity and inertia effects by electromagnetic phenomena has to pass through the structure topology of the field generator devices.

E. g., such a structure toplogy should permit to produce fields of more than three dimensions with alteration of five and four-dimension isotropy.

It isn't important to accord with all Saumont's conclusions. He opens a door. To make easier the introduction of (hyper)incursivity in physics this paper propose some developments of his approach.

2 Proposal to Develop Physical Dimension Analysis

2.1 Definition of Computative topology

2.1.1 Premises of Computative Topology

Ancient Greek mathematician Menæchmus approached conic section studies and derived properties by using concepts that are very near to coordinates of analytical geometry.

But we cannot attribute an analytical geometry in modern sense to him because he was unaware that any equation in two unknown quantities determines a curve: the general concept of an equation in unknown quantities was alien to ancient Greek thought. Imperfections of the algebric notation have prevented above all sciluppo of a mature geometry analytics in ancient Greece.[2]

Ancient Greek mathematician Apollonius gave an important contribution too. Apollonian tratise on Section Determination could be considered one dimension analytic geometry.

He has considered the following general problem by using algebraic analysis of its age in geometric form: given four points A, B, C, D on a straight line, find such a fiveth point P that the rectangle on AP and CP have got a given ratio with the rectangle on BP and DP. Here he has transormed the problem in a quadratic expression whose he analyzes all the possibility and the solution number as in other cases.[3] Apollonius' method on conics is so similar to modern analytic geometry that someones juge that his work anticipate Cartesius of 1800 years.

The general application of reference lines and of a diameter with a tangent at its extremity in particular is very similar to use a coordinate frame either rectangular or, more generally, oblique. Distances from tangency point along the diameter can be considered abscissas and segments between axis and curve that are parallel to the tangent can be considered ordinates. Thus Apollonian relationships between these abscissas and the corresponding ordinates can be considered rethorical forms of curve equations.

But ancient Greek mathematic didn't previde negative magnitudes and coordinate system was superimposed *a posteriori* upon a given curve in order to study its properties. In fact, cases where a reference coordinate frame has be put *a priori* to represent a relation or equation don't appear.

We can affirm on ancient Greek geometry that equations, naturally in rethoric form, are determined by curves, but not *vice versa*. Coordinates, variables and equations were subsidiary notions due to specific geometric situations. Too little number of available curves prevented to develop of analytical geometry to Apollonius, the greatest geometer of antiquity, rather than his thought. Problems that concern always a limited number of particular cases don't require general methods.[4]

---

[2] See Boyer, C. B., (1991), *The Age of Plato and Aristotle*, pp. 94-95.
[3] See Boyer, C. B., (1991), *Apollonius of Perga*, pp. 142.
[4] See Boyer, C. B., (1991), *Apollonius of Perga*, pp. 156.

Omar Khayyam (ca. 1050-1123) has written an *Algebra* that exceeded that of al-Khwarizmi because it includes third degree equations. He has provided either arithmentic or geometric solutions for quadratic equations as his Ariab predecessors; he believed arithmetic solution of general cubic equations impossible mistakely and he gave only geometric solutions for them.

To solve cubics Menaechmus, Archimedes, and Alhazan used intersecting conics already, but Omar Khayyam has had the merit of generalizing this method to every third degree equation with positive roots.

As ordinary space has three dimensions, Omar Khayyam didn't previde similar geometric methods for equations of higher degree than three.

The tendency to close the gap between numeric and geometric algebra was a very fruitful contribution of Arabic medieval eclectism. Also if the decisive step in this direction came much later with Descartes, Omar Khayyan saw: "*Whoever thinks algebra is a trick in obtaining unknowns has thought it in vain. No attention should be paid to the fact that algebra and geometry are different in appearance. Algebras are geometric facts which are proved.*". [5]

The previous achievements have influenced Fermat and, principally, Descartes in analytic geometry foundation.[6] This last one has obtained important achievements that are reported in the appendix *Geometry* of his book "*Discourse on the Method of Rightly Conducting the Reason in the Search for Truth in the Sciences*", briefly *Discourse on Method.* This paper, written in French tongue, has provided to foundation of infinitesimal calculus in Europe.

Abraham de Moivre also pioneered analytic geometry.[7]

Cantor-Dedekind axiom permits us to transport every theorem of Euclidean geometry in Analytic geometry and vice versa.[8] This statement has permitted to Alfred Tarski's to identify the proof of the decidability of the ordered real field to the proof that Euclidean geometry is consistent and decidable.[9]

Topology is mathematic area that studies space property at less of object continuous deformations as stretching but no tearing or gluing. It is born from geometry and set theory by concepts of space, dimension an transformations.[10] Its preliminaries was *geometria situs* (latin: geometry of place) or *analysis situs* (latin: picking apart of place).[11]

Robert Rosen define an anticipatory system in 1985 as follows: "*A system containing a predictive model of itself and/or its environment, which allows it to change state at an instant in accord with the model's predictions pertaining to a latter instant.*"

This definition can be appled to any system that includes machine learning, i.e. the issue is how much of a system behaviour is determined by reasoning on dedicated situations, how much by on-line planning and how-much by system designer providing.[12]

---

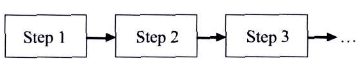
[5] See Boyer, C. B., (1991), *The Arabic Hegemony*, pp. 241-242.
[6] See Stillwell, J., (2004), *Analytic Geometry*, pp. 105.
[7] Swokowski, E. W., Cole, J. A., (2006), pp. 506-507
[8] Erlich, P., (1994), *General introduction*, pp. vi–xxxii.
[9] See Henkin, L., Suppes, P., Tarsky, A., (1959), *What is elementar geometry?* (Tarski, A.), pp. 16-29.
[10] See Basener, W., (2006).
[11] See Euler, L., (1736), *Solutio problematis ad geometriam situs pertinentis*.
[12] See Rosen, R., (1985).

In 1997, Daniel M. Dubois defined incursion and hyperincursion as follows: "*The computation is incursive, for inclusive recursion, in the sense that an automaton is computed at future time t+1 as a function of its neighboring automata at the present and/or past time steps but also at future time t+1. The hyperincursion is an incursion when several values can be generated for each time step. External incursive inputs cannot be transformed to recursion. This is really a practical example of the final cause of Aristotle. Internal incursive inputs defined at the future time can be transformed to recursive inputs by self-reference defining then a self-referential system. A particular case of self-reference with the fractal machine shows a non-deterministic hyperincursive field. The concepts of incursion and hyperincursion can be related to the theory of hyper-sets where a set includes itself. Secondly, the incursion is applied to generate fractals with different scaling symmetries. This is used to generate the same fractal at different scales like the box counting method for computing a fractal dimension. The simulation of fractals with an initial condition given by pictures is shown to be a process similar to a hologram. Interference of the pictures with some symmetry gives rise to complex patterns. This method is also used to generate fractal interlacing. Thirdly, it is shown that fractals can also be generated from digital diffusion and wave equations, that is to say from the modulo N of their finite difference equations with integer coefficients.*"[13]

2.1.2 Definition of Computative Topology

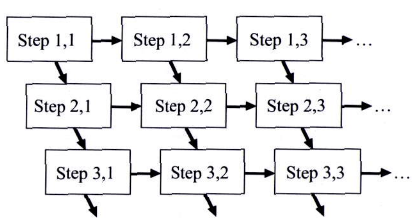
We can see analytic geometry as the spreading of mathematical analysis on a geometric space: every analytical function corresponds to a geometrical entity. Now, consider computation theory: is it possible that every algorithm can be spread on an opportune topological structure?

E.g., observe this generic serial algorithm:



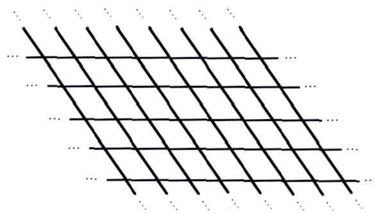
It can be considered as spread on a straight line:

Now, observe this generic parallel algorithm:

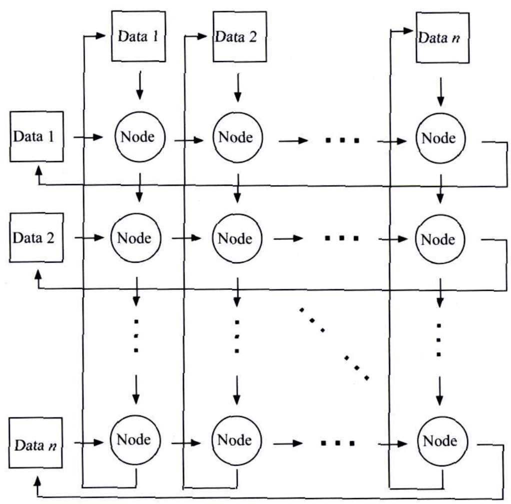


[13] See Dubois, D. M., (1997).

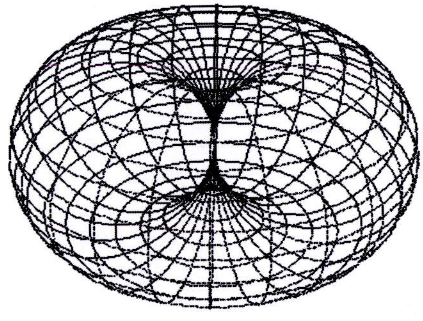
It can be considered as spread on a Euclidean plane:



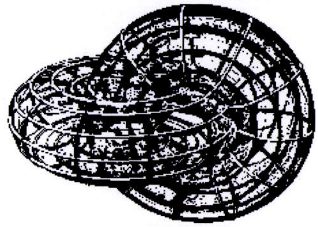
Finally, observe Dubois' generic incursive algorithm:



The periodic copy of the last row on the first row and of the last column on the first column impose us to consider the incursive algorithm spread on a torus:



Dubois' definition of hyperincursivity make very easy to image the spreading of a hyperincursive algorithm; e.g., in a simple case we have:



We have considered three algorithms. The serial and the parallel algorithms aren't anticipatory. The incursive algorithm yes. What is the difference? Single step or node contents can be identical in the three algorithms. Thus it is connection topology that makes the difference. The non-anticipatory algorithms are spread in an evident Euclidean hyper-cubic topology (line, square, cube and so on) exactly as standard dimension analysis expressions; instead the simplest incursive algorithm has a torus topology. This last one permits anticipation.

Is torus topology only that permits anticipation? It is true that Dubois has proven that a very efficient anticipation is obtained by spreading an algorithm on a torus but other opportune topological structure can be used too. E.g., we think that we can obtain anticipation by spread algorithms on the greatest part of Escher's picture.

In general, an opportune step or node connection topology can give important properties to algorithms that they cannot have with Euclidean hypercubic topologies. E.g., we have seen that Saumont affirms that gravity and electromagnetism are different because they haven't the same number of dimension and he puts hypotheses on an important topology role in the future antigravitary generators. If Saumont has right then the mathematical relation between gravity and electromagnetism cannot be compatible with the standard hypercubic dimension analysis. New ways should be covered.

2.2 Physical Dimension Analysis: but of what dimensions?

2.2.1 Definition of Fractal

Let the *topological dimension* of a set be the number of independent parameters needed to describe a point in the set.

Let X be a metric space. If $S \subset X$ and $d \in [0,\infty)$, the d-dimensional Hausdorff content of S is defined by $C_H^d(S) = \inf\left\{\sum_i r_i^d \text{ where } \text{“}\sum_i r_i^d\text{”} \text{ is a cover of } S \text{ by balls with radii } r_i > 0\right.$. Then *Hausdorff dimension* of X is $\dim_H(X) := \inf\left\{d \geq 0 : C_H^d(X) = 0\right\}$.

Let a *fractal* be "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole, a property called self-similarity. Often its Hausdorff dimension strictly exceeds its topological dimension.

2.2.2 Sets of Inclusive Networks

We suppose Dubois-Resconi statements on this matter.[14] "*Consider a quantity* y *and more quantities* $x_1,\ldots,x_n$. *If we must affirm that "* y *depends upon* $x_1,\ldots,x_n$ *", then we can write: (1.1)* $y = y(x_1,\ldots,x_n)$ *in usual mathematical language,*

$$\text{(1.2)}\quad \left.\begin{matrix} x_1 \xrightarrow{\Delta t_1} \\ \vdots \quad\quad \vdots \\ x_n \xrightarrow{\Delta t_n} \end{matrix}\right\} \longrightarrow y(t)$$

as an algorithm (with some negative Δt_i *in the case of hyperincursive relations), (1.3)*

$$\begin{cases} (x_t)A^2_=\left(\varphi^1_{S_y}(x_t),\varphi^n_p\left(\varphi^1_{S_{x_1}}\left(\varphi^1_{\Delta t_1}(x_t)\right),\ldots,\varphi^1_{S_{x_n}}\left(\varphi^1_{\Delta t_n}(x_t)\right)\right)\right) \\ (x_q)A^2_=(x_q,x_q) \\ (x_q)(x_r)\left(A^2_=(x_q,x_r)\supset\left(\mathsf{A}(x_q,x_q)\supset\mathsf{A}(x_q,x_r)\right)\right) \end{cases}$$

in a well known logical mathematical language, where $\begin{cases} p_1 \\ \vdots \\ p_n \end{cases}$ *is equivalent to* $p_1 \wedge \ldots \wedge p_n$ *and where "*$(x_q)A^2_=(x_q,x_q)$*" and "*$(x_q)(x_r)\left(A^2_=(x_q,x_r)\supset\left(\mathsf{A}(x_q,x_q)\supset\mathsf{A}(x_q,x_r)\right)\right)$*" are axiom schemes whose set can define the predicate "...is equal to ...", and, finally, we can write (1.4)* $S_y(t)=\{S_{x_1}(\Delta t_1),\ldots,S_{x_n}(\Delta t_n)\}$ *in the language of the theory of a set inclusive network, a theory which is used by D. Dubois to describe the incursive and the hyperincursive processes. Particularly,* $S_y(t)=\{S_{x_1}(\Delta t_1),\ldots,S_{x_n}(\Delta t_n)\}$ *and similar equations are structural equations in the theory of a set inclusive network.*

...

Consider the brace brackets "$\{\ldots\}$*". If such brace brackets are not contained in other brace brackets, then we call them first level brace brackets. Also, we call second level brace brackets those that are contained in first level brace brackets only. In general, call n-th level brace those that are contained in brace brackets of (n-1)-th level at most.*

Let a structural equation be of n-th degree if and only if it contains brace brackets n-th degree at most.

E. g., $S_y(t)=\{S_{x_1}(\Delta t_1),\ldots,S_{x_n}(\Delta t_n)\}$ *is a structural equantion of the first degree because it contains a couple of brace brackets only, instead* $S_{11}(t)=\{\{\{S_{01}(\Delta t_0),S_{10}(\Delta t_0),S_{11}(\Delta t_1)\},S_{02}(\Delta t_0),S_{12}(\Delta t_1)\},S_{03},(\Delta t_0)S_{13}(\Delta t_1)\}$ *is a structural equation of the third degree because it has a couple of brace brackets which*

[14] See Dubois, D. M., and Resconi, G., (1992), *STRUCTURAL EQUATION OF HYPERINCURSIVE PROCESSES*, pp. 85-122.

are contained in a couple of brace brackets which are contained in a couple of brace brackets …

Theorem 3.1: *Every structural equation system (with a sole equation also) in the theory of a set inclusive network is equivalent to a wff of standard sentence logic where its atomic sentences are structural equations whose degree is equal to two at most.*"[15]

This last achievement shows that the sets of inclusive networks force their structural equations always in a two topological dimension context.

2.2.3 Are Sufficient the Topological Dimensions in Physical Dimension Analysis?

Grappone has proven that every recursive algorithm is equivalent to an opportune (hyper)incursive algorithm.[16] But this last one corresponds always to an inclusive network set. This fact implies that any computable mathematical system can be forced in a two topological dimension description by an inclusive network set that can represent it. Thus topological dimension number becomes less important. Also, we have before seen that fractals have Hausdorff dimension often strictly greater than topological dimension. So, the introduction of fractals and anticipatory mathematics in physics makes necessary to consider in physical dimension analysis not only topological dimension, but also Hausdorff dimension at less. However it may also be useful to use Minkowski dimension,[17] a Hausdorff dimension variant, Hamel dimension[18] by which Hilley has build a quantum mechanics without Hilbert space,[19] manifold theory dimension,[20] Lebesgue covering dimension[21] and so on.

The use of computative topology that we have before considered makes this fact still more obvious.

2.3 Other Ambiguities in Usual Dimension Analysis: Space and Universal Constants

2.3.1 Space

To represent a physical largeness the usual dimension formula is $l^n t^p m^q i^r$. Now, t, m, I are effectively scalar largeness. l, instead, measures an interval in a very three space dimension. Call them x, y, z. Thus, if r, s, t, u, v, w are real numbers, we can write $\mathrm{r}l = \sqrt{(\mathrm{s}x)^2 + (\mathrm{t}y)^2 + (\mathrm{u}z)^2}$. What is the l dimension formula as regard x, y, z? It cannot evidently have a hypercubic structure $x^n y^p z^q$: the use of l as physical dimension excludes the same use of x, y, z automatically. Thus we have: $\mathrm{s}x = \mathrm{s}l$, $\mathrm{t}y = \mathrm{t}l$, $\mathrm{u}z = \mathrm{u}l$, $\mathrm{v}xy^{-1} = \mathrm{v}$, $\mathrm{w}xz^{-1} = \mathrm{w}$ and so on: three distinct dimension x, y, z are confused in an one dimension l and variable largeness as xy^{-1}, xz^{-1} and so on are without dimensions. This paradox is

[15] See Grappone, A. G., (1999).
[16] See Grappone, A. G., (1997).
[17] See Schroeder, M., (1991), pp. 41-45.
[18] See Gannon, T., (2006).
[19] See Hilley, B. J. (2002).
[20] See Spivak, M., (1965).
[21] See V.V. Fedorchuk, (1993), *The Fundamentals of Dimension Theory*, in: Arkhangel'skii A. V., Pontryagin, L. S., (Eds.), (1993), *Encyclopaedia of Mathematical Sciences, Vol. 17, General Topology I*

more evident in Minkowski space that has spacelike dimensions but a timelike dimension too.[22] Its interval definition is $r\sigma = \sqrt{(sx)^2 + (ty)^2 + (uz)^2 - (vct)^2}$: what is the σ dimension formula in terms of $l^n t^p m^q i^r$? Also, speed has dimensions $v = lt^{-1}$ in a mechanic that supposes its increasing to infinite, but, as there is a greatest speed c, a more realistic speed representation in term of c fraction, i.e. $v = lct^{-1}$ is without dimensions exactly as xy^{-1}, xz^{-1} and so on. Is it acceptable?

Consider this proposal. Put $\sqrt{r^2 + s^2 + u^2 - c^2v^2}\left[xyz[t]\right] = \sqrt{r^2x^2 + s^2y^2 + u^2z^2 - v^2c^2t^2}$. Put $\sqrt{r^2 + s^2 + u^2}\left[xyz[0]\right] = \sqrt{r^2x^2 + s^2y^2 + u^2z^2}$. Thus $l = \left[xyz[0]\right]$. We can use as standard dimension formula $\left[xyz[0]\right]^n t^p m^q i^r$. The distributive property of multiplication as regard the operator $\left[abc[d]\right]$ makes the dimension calculus very easy.

2.3.2 Universal Constants

Is it acceptable that universal constants have physical dimension? Saumont observe that not and shows that very important developments may happen in physics.[23] We accord with him. E.g., consider gravity law in dimension terms: $ma = G\frac{mm}{l^2}$. If we eliminate the dimension of the universal constant G then we obtain $ma = \frac{mm}{l^2}$ that can be transformed in this way: $ma = \frac{mm}{l^2} \Leftrightarrow m\frac{l}{t^2} = \frac{mm}{l^2} \Leftrightarrow m\frac{l^3}{t^2} = mm \Leftrightarrow m = \frac{l^3}{t^2}$. m becomes an acceleration of volume change. This interpretation of the mass is not absurd. If Universe volume V is in continuous expansion and gravity is equivalent to the curvature of V then we may interpret this curvature as a non-homogeneous expansion of V and m in the place ΔV as the negative acceleration of ΔV expansion as regard the remaining expansion. It is a work hypothesis only, but shows as the dimensioned universal constants can obstacle physics development.

Conclusions

Three work proposals:

- To replace the dimension expression $l^n t^p m^q i^r$ with $\left[xyz[0]\right]^n t^p m^q i^r$.
- To introduce at less Hausdorff dimension beside the topological dimension, i.e. replace $\left[xyz[0]\right]^n t^p m^q i^r$ with $\left[xyz[0]\right]_s^n t\ m^q i^r$.
- To eliminate the dimension from physic universal constants.

Ulterior studies are necessary, but it can continue the debate opened from Saumont.

[22] See Catoni F., Boccaletti D., Cannata R., (2008), *Mathematics of Minkowski Space*, Birkhäuser, Basel.
[23] See Saumont, R., (1988).

References

Basener, W., (2006), *Topology and Its Applications* (1st ed.), Wiley, Hoboken, NJ, U.S.A.

Boyer, C. B., (1991), *A History of Mathematics*, (2nd Edition ed.), John Wiley & Sons Inc., Hoboken, NJ, U.S.A.

Catoni F., Boccaletti D., Cannata R., (2008), *Mathematics of Minkowski Space*, Birkhäuser, Basel, CH.

Dubois, D. M., and Resconi, G., (1992), *Hyperincursivity – A new mathematical theory*, Presses Universitaires de Liège, Liège, B.

Dubois, D. M., (1997), *BioSystems*, 43, 97-114.

Dubois, D. M., (2008), *On the Quantum Potential and Pulsating Wawe Packet in the Harmonic Oscillator*, COMPUTING ANTICIPATORY SYSTEMS: CASYS'07 - Eighth International Conference. AIP Conf. Proceedings, Volume 1051, pp. 100-113.

Arkhangel'skii A. V., Pontryagin, L. S., (Eds.), (1993), *Encyclopaedia of Mathematical Sciences,* Springer-Verlag, Berlin, Germany.

Erlich, P., (1994), *Real Numbers, Generalizations of the Reals, and Theories of Continua*, P. Erlich, Kluwer Academic Publishers, Dordrecht, NL

Gannon, T., (2006), *Moonshine beyond the Monster: The Bridge Connecting Algebra, Modular Forms and Physics*, Cambridge University Press, Cambridge.U.K.Grappone, A. G., (1997), *Genesis of the Rationality from 'Chaos*, International Journal of Computing Anticipatory Systems (1997) 1, pp 272-281

Grappone, A. G., (1999), *Structural Equations in the Theory of a Set Inclusive Network: Highest Degree Theorem* in G. E. Lasker, ADVANCES in ARTIFICIAL INTELLIGENCE and …, vol. V, I.I.A.S., University of Windsor, Canada, pp 78-82.

Henkin, L., Suppes, P., Alfred Tarski, A., (1959), *The axiomatic method. With special reference to geometry and physics. Proceedings of an International Symposium held at the Univ. of Calif.,* Berkeley, Dec. 26, 1957-Jan. 4, 1958, *Studies in Logic and the Foundations of Mathematics*, North-Holland, Amsterdam, rep 2007 of Brouwer Press.

Hiley, B. J., (2002), *Algebraic Quantum Mechanics, Algebraic Spinors and Hilbert Space,* http://www.bbk.ac.uk/tpru/BasilHiley/Algebraic%20Quantum%20Mechanic%205.pdf

Rosen, R., (1985), *Anticipatory Systems*, Pergamon Press, New York, NY, U.S.A.

Saumont, R. (1988) *Analyse dimensionelle et similitude en physique fondamentale*, Edition Européennes, Antony, FR

Schroeder, M., (1991), *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise,* W. H. Freeman, New York, NY, U.S.A.

Spivak, M., (1965), *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, *The standard graduate text*, HarperCollins Publishers, New York, NY, U.S.A.

Stillwell, J., (2004), *Mathematics and its History* (Second Edition ed.), Springer Science + Business Media Inc., New York, NY, U.S.A.

Swokowski, E. W., Cole, J. A., (2006), *Algebra and Trigonometry with Analytic Geometry*, *Classic*, (Eleventh Edition), Thomas Learning Inc., Belmont, CA, U.S.A.pp. 506-507