Particles of High Bessel Order: a new Candidate to Dark Matter Problem

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Abstract

In previous communications [1,2] we have enhanced quantum theory with the transversal distribution of plane waves associated to quanta, from a more general resolution of the differential wave equation which introduced a new quantum number: the Bessel order. In the communication [2] we have deduced several properties of the presence density of non-dimensionless particles and proposed some applications to Cosmology. In the present paper, we propose a solution to the dark matter problem, an explanation of the presence of antimatter in the Universe and we sketch a new concept of vacuum based on extremely high Bessel orders at the cosmological scale.

Keywords: dark matter, antimatter, hollow mass, ineffective cross-section, mass shell.

1. Introduction

For the reader to understand our development we give an overview of the theory in section 2, we explain the hollow structure of the presence density in section 3, and recall shortly our previous results in Cosmology in section 4. The new concept of vacuum based on extremely high Bessel orders at the cosmological scale is presented in the section 5.

2. Overview of the Theory

We have previously shown [1,2] that particles cannot be dimensionless mass-points (as it is considered in the usual quantum theory) because it implies divergent fields and a divergent self energy.

A general solution of the wave equation introduces mathematically a transversal cylindrical distribution in plane waves of particles and a radial distribution in their spherical waves. The transversal distribution in the wave function of particles uses Bessel functions of first kind $Jn(r/r_0)$ and order n, and thus it introduces a new quantum number with a continuous spectrum: the Bessel order $\beta \geq 0$.

With such distributions in the wave function, mass particles are extended in space with a spatial frequency N which is directly related to its rest mass m_0.

We may think that all particles currently have a Bessel order zero, just to approach the usual concept of point-particles, although strictly positive Bessel orders $\beta > 0$ build hollow structures which introduce the concept of hollow mass or mass shells.

Higher Bessel orders, building particles with an ineffective cross-section, allow antimatter passivation and therefore they have new applications to Cosmology. Mass particles of extremely high Bessel order (very big mass shells) appear to be a new candidate to the dark matter problem in galaxies. More particularly, the observation of a huge quantity of anti-electrons annihilation in the galaxy bulge of the Milky-Way [5,9] suggests that passivated anti-electrons might play an essential role in the life of our galaxy.

Such considerations can lead to a new cosmological scenario: particle-antiparticle pairs might be created at extremely high Bessel orders in the conditions of extremely high temperatures (in the Big-Bang and in some star nucleus) and strong magnetic fields. Furthermore, considering the ineffective cross-section of mass particles at the infinite limit $\beta \rightarrow \infty$ of the Bessel order spectrum suggests a new concept of quantum vacuum: real or virtual mass particles or antiparticles may emerge from vacuum when the Bessel order decreases.

3. Transversal and Radial Distribution of Waves

Quantum theory and the theory of Relativity have inherited the idea of "point-like particle" from the concept of "mass point" in Newton's mechanics and the concept of "point-like electric charge" from classical electrodynamics. In classical models of electrons the concept of a dimensionless electric charge was considered as a solution to the problem of repulsive forces [3] which would occur inside the electric charge if it were expanded in space, although a dimensionless electric charge implies divergent fields and a divergent self energy [1,2].

Our model of particle is not a speculation about the so-called "internal structure" of a boson or a fermion, but it is a study of the distribution of waves in the outer space of the particle, as it can be deduced from the foundations of quantum theory. The wave nature of a fermion which is defined by the well-known frequency ν of its phase wave:

$$h\nu = mc^2 \tag{1}$$

and its minimum frequency ν_0 which is a function of the rest mass m_0 as:

$$h\nu_0 = m_0 c^2 \tag{2}$$

shows that the wave function is to be associated with the rest mass of the fermion.

In a previous communication [1] we have computed the transversal distribution of the plane wave of any free massive particle (fermion or boson) and we have shown that the plane wave associated with the energy-momentum $(E, \boldsymbol{p})$ of any mass particle:

$$\psi_k(\mathbf{x}, t) = \psi_0 \cdot \psi_T(\mathbf{k} \wedge \mathbf{x}) \cdot e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} \tag{3}$$

must have a transversal distribution such as:

$$\psi_T(\mathbf{k} \wedge \mathbf{x}) = a\, J_\beta(r / r_0) \cdot e^{\pm i n \vartheta} \tag{4}$$

where a is the amplitude and r_0 is a scaling factor related to the rest mass m_0 as :

$$r_0 = \frac{\hbar}{m_0 c} \tag{5}$$

where β is a new quantum number, the Bessel order which is a positive real number :

$$\beta \in \Re \quad \beta \geq 0 \tag{6}$$

So the Bessel order β has a continuous spectrum.

We have computed the transversal density of a plane wave and drawn several figures of the transversal distribution of the density of presence of a mass for several values of the Bessel order (see figures 1 to 10 in section 2 of ref. [1]).

The present quantum theory does not predict any value of the Bessel order of each mass particle, as the Bessel order β is neither related to any kinetic variable, nor to any usual quantum number: it is a new quantum number. So a new law will have to be postulated.

In the previous communication [2] we have recalled that the general differential equation of waves:

$$\left(\frac{1}{c^2} \frac{\partial}{\partial t^2} - \nabla^2 \right) \psi = 0 \tag{7}$$

have both solutions of plane waves with a transversal radial distribution and solutions of spherical waves with a spherical radial distribution.

In the case of plane waves, we have computed [2] the transversal radial density of presence d_T :

$$d_\mathrm{T} = \psi_T \cdot \overline{\psi}_T \tag{8}$$

Resulting from a wave distribution function using a Bessel function of order $\beta \geq 0$, the transversal density of presence works as the square of a Bessel function of order β :

$$d_\mathrm{T}(r) = a^2 J_\beta^2 (r / r_0) \tag{9}$$

where r_0 acts as a scaling factor in the argument of the Bessel function.

A spherical wave represents an angular momentum, which may be either an orbital momentum $\boldsymbol{l}$ with a projection $\boldsymbol{l}_z$ or a spin momentum $\boldsymbol{s}$ or their sum $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$.
Spherical waves propagate with a spherical expansion in space, which has to match the law in $1/r$ and the homogeneous spherical density has to match the law in $1/r^2$ because it is related to a solid angle. So we have postulated [2] the following spherical distribution $\Delta(r)$:

$$\Delta(r) = a' \frac{1}{r} J_\beta (r / r_0) \tag{10}$$

which is formally related to the cylindrical distribution by:

$$\psi_T(r) = \frac{a}{a'} r \, \Delta(r) \cdot e^{\pm i n \vartheta} \tag{11}$$

Consequently the presence density of the rest mass in 3 dimensions can be expressed as:

$$D(r) = \left(a'\right)^2 \frac{1}{r^2} J_\beta^2\left(r / r_0\right) \tag{12}$$

We have shown [2] that the transversal distribution $\psi_T(r)$ of the phase wave has a spatial period Λ which is identical to the Compton length and depends only on the rest mass of the particle, as:

$$\Lambda = \lambda_c = 2\pi r_0 \tag{13}$$

and we have deduced that the spatial frequency Ω is equal to the wave number:

$$\Omega = 2\pi N = k_0 \tag{14}$$

4. A Hollow Structure of Presence Density

The radius r_1 is defined as the radius of the primary maximum of presence density. The ratio r_1/r_0 is a function of the Bessel order β. At Bessel orders $\beta > 0$ radial distributions have a central hole which is described by Bessel functions.

The radius r_2 of the central hole is defined as the shorter radius of half the primary maximum of presence density. The ratio r_2/r_0 is a function of the Bessel order β, and we have shown [2] that we have, with an accuracy better than 1% :

$$\frac{r_2(\beta)}{r_0} \approx \beta \tag{15}$$

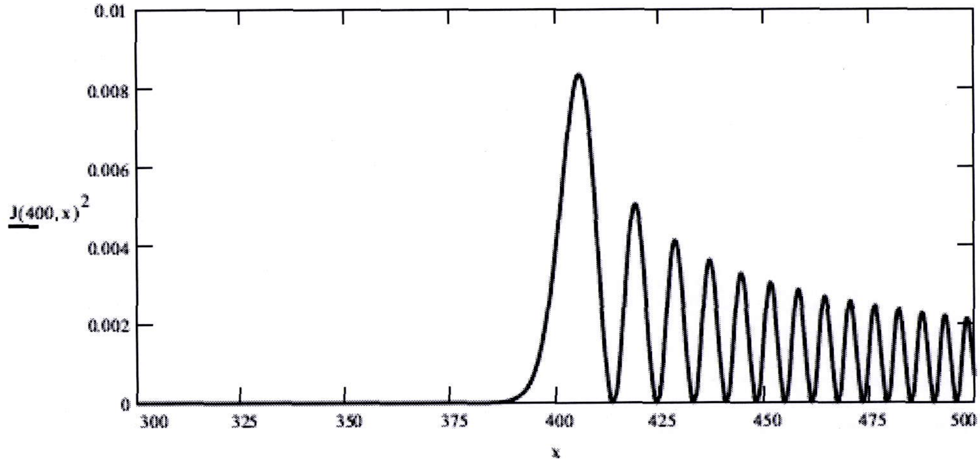

Figure 1. Transversal radial density of presence at Bessel order 400.

At higher Bessel orders the presence density of a particle is quite zero from the centre of the particle until very near the radius r_2, and this results into a lower probability of interaction of a particle P_1 with a particle P_2.

The figure 1 shows the presence density of a particle at Bessel order 400. The x coordinate represents the ratio r/r_0. The figure 2 with a zoom at a very low density scale, shows that the presence density is quite zero from the centre of the particle until very near the radius r_2.

Let us consider two particles P_1 and P_2 (e.g. an electron and an anti-electron), they can interact only at a close enough distance which is experimentally defined by their efficient cross-section. If the particle P_1 has a high Bessel order, the radius r_2 of its hole is much greater than the efficient cross-section of the particle P_2 so they cannot interact, whatever their usual properties: therefore P_2 comes out of the scope of P_1. We then say that the particle P_1 has an "inefficient cross-section".

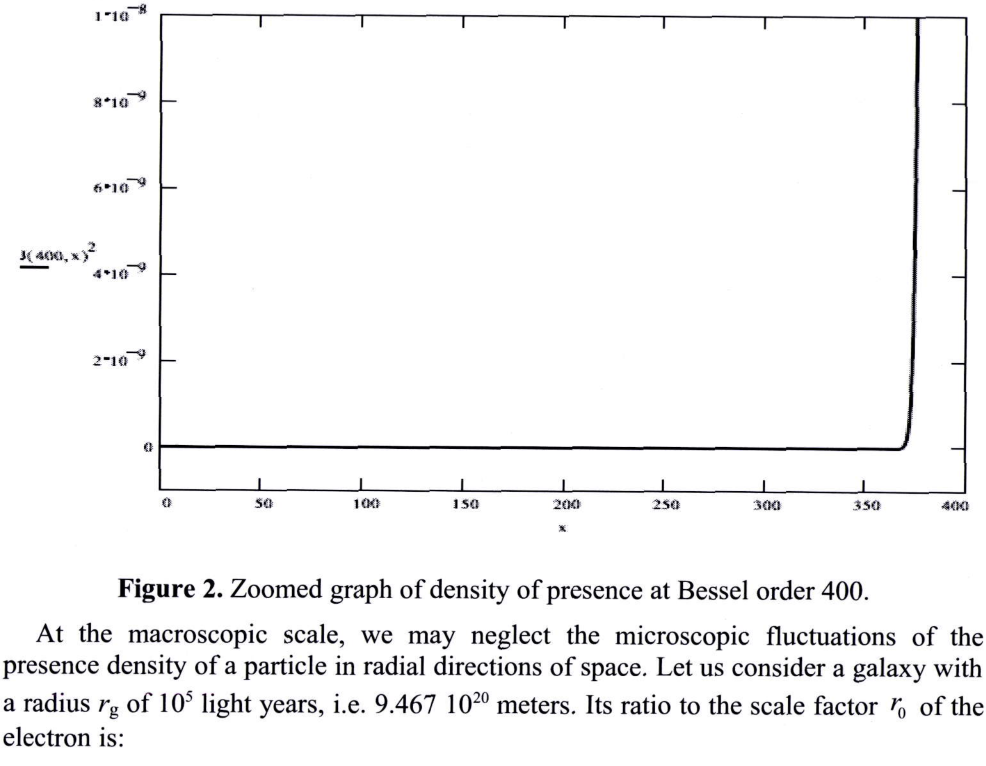

Figure 2. Zoomed graph of density of presence at Bessel order 400.

At the macroscopic scale, we may neglect the microscopic fluctuations of the presence density of a particle in radial directions of space. Let us consider a galaxy with a radius r_g of 10^5 light years, i.e. $9.467\ 10^{20}$ meters. Its ratio to the scale factor r_0 of the electron is:

$$\frac{r_g}{r_0}=2.45\cdot 10^{33} \tag{16}$$

so the equation (12) of the 3-dimensional presence density at the radius r_g :

$$D(r_g)=(a')^2\frac{1}{r_g^2}J_\beta^2\left(\frac{r_g}{r_0}\right) \tag{17}$$

evaluates to:

$$D(r_g) \approx 10^{-42} (a')^2 J_\beta^2 (2.4\,10^{33}) \tag{18}$$

and we see that the fluctuating 3-dimensional presence density is nearly zero:

$$D(r_g) \approx 0 \tag{19}$$

Considering the equation (9) of the 2-dimensional transversal distribution of plane waves, the average transversal density $<d_T(r)>$ at a macroscopic scale is given by the equation (65) of reference [2] which is:

$$\langle d_T(r) \rangle \approx a^2 \frac{1}{\pi} \frac{r_0}{r} \tag{20}$$

so for the galaxy:

$$\langle d_T(r_g) \rangle \approx a^2 \frac{1}{\pi} \frac{r_0}{r_g} \tag{21}$$

evaluates to:

$$\langle d_T(r_g) \rangle \approx 1.3 \cdot 10^{-34} a^2 \tag{22}$$

Therefore microscopic fluctuations are not to be considered at the cosmological scale. In the same way we can show that spatial wavelength can be neglected.

5. Applications to Cosmology

In our previous communication [2] the properties of hollow mass and inefficient cross-section have been applied to the domain of Cosmology, so we have proposed a solution to the problems of antimatter and dark matter. For a better understanding of our cosmological concepts we recall shortly these results in the next sections 5.1. and 5.2..

5.1. A Passive Antimatter in the Universe

In the standard Big-Bang scenario, it is usualy assumed that half of the mass of the Universe (antimatter) has been annihilated while the other half (matter) has been prevented from annihilation. This should have occured during the inflation time of 10^{-33} second, although the 3 Sakharov conditions [4] are not met, as we have explained it [2].

As it is known from the baryon asymmetry parameter: only one antiparticle is cancelled when 10 billions pairs of particle-antiparticle are produced. So it is hardly difficult to obtain a baryogenesis at the large scale of the cosmos, which does not include a great quantity of antimatter [2].

The European astronomical satellite "Integral" [5] has detected gamma photons of 511 KeV from the centre of our galaxy. The Dapnia laboratory of Saclay's CEA has called it a fountain of anti-electrons, because this 511 KeV photons flow has been evaluated to 10^{+43} positron annihilations per second. It is 400.10^{+9} times less than from Sirius. The 511 keV positron annihilation line emission has been measured [6], then its sky distribution has been analyzed [7,8] and drawn into pictures [9].

The figure 3 shows the all-sky distribution of the 611 keV γ-ray emission. The contour levels indicate intensity levels of 10−2, 10−3, and 10−4 ph cm−2 s−1 sr−1 from the centre outwards.

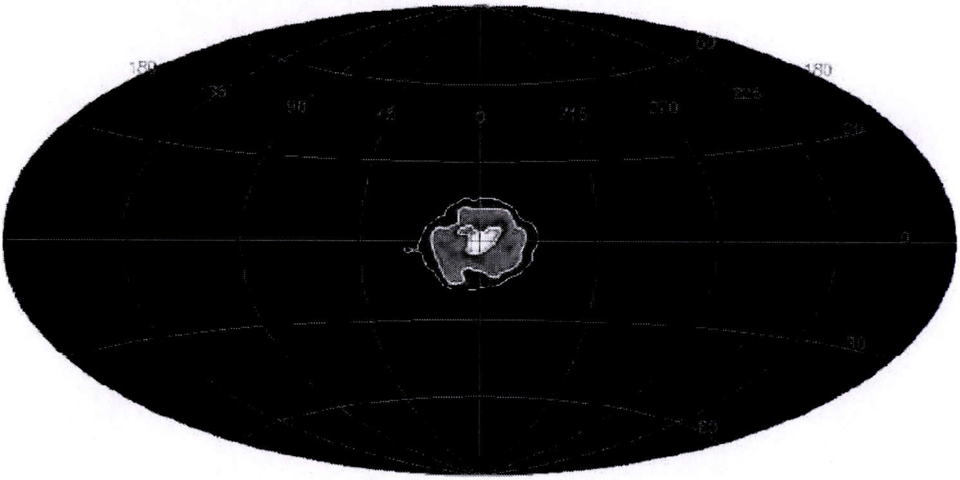

Figure 3. Richardson-Lucy image of 511 keV gamma-ray line emission.
Credit: J. Knödlseder et al., A&A, vol. 441, p. 513 (2005), fig. 4
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Several scenarios have been proposed and postulate a source of positrons [10,11,12] within the conventional framework.

The observation of a continuous annihilation of positrons indicate the presence of huge quantities of anti-electrons inside the galaxy bulge of the Milky-Way.

Antiparticles at high Bessel orders have a passive state, which forbid any possible interaction with the corresponding particles, because of their inefficient cross-section.

5.2. A Solution to the Dark Matter Problem

To explain dark matter, many hypothetical particles have been proposed [13], with several different cosmological implications.

A fundamental question has been asked [14]: "Does the missing mass problem signal the breakdown of Newtonian gravity ?" and an other fundamental question should also be asked: Does the missing mass problem signal the breakdown of point-like particle gravity ?

At extremely high Bessel orders, the hole radius r_2 of particles in a galaxy bulge can reach the radial distance of peripheral stars, and therefore the mass of such particles is mainly distributed far away from the centre of the galaxy. This is the expected property of dark matter. It means that a galaxy bulge may contain such a specific matter, which produces a gravitational field only at large distances.

In our previous communication [2] we have done a simple simulation, by computing the angular velocity $\omega(r)$ for an orbit of radius r around an equivalent central mass M(r):

$$\mathrm{M}(r) = 4\pi \int_0^r D(\rho)\rho^2 \, d\rho \tag{23}$$

which depends on the mass distribution $D(r)$ of a set of mass particles of extremely high Bessel orders which are stored in the galaxy bulge. We have considered the galaxy bulge as a set of particles of one kind, e.g. only electrons (or anti-electrons), i.e. all particles have the same scale factor r_0. And for convenience we have set it to $r_0 = 1$. The particles have several different Bessel orders. To simplify, only integer orders are considered and spherical densities are added as equation (33) in reference [2] which is:

$$D(r) = (a')^2 \frac{1}{r^2} \sum_{\beta=0}^{N} A_\beta J_\beta^2(r/r_0) \tag{24}$$

and then integrated in equation (23) with the software Mathcad 6.0 SE. The resulting figures of our simulations are not to be scaled as we have set the gravitation constant to $G = 1$ and the coefficient a' has been set to $a' = 1$ for simplicity.

The coefficients A_β define the abundance of particles of Bessel orders β. We have adjusted the coefficients A_β to have star velocities which decrease slightly with the radius r as it is observed in the arms of spiral galaxies.

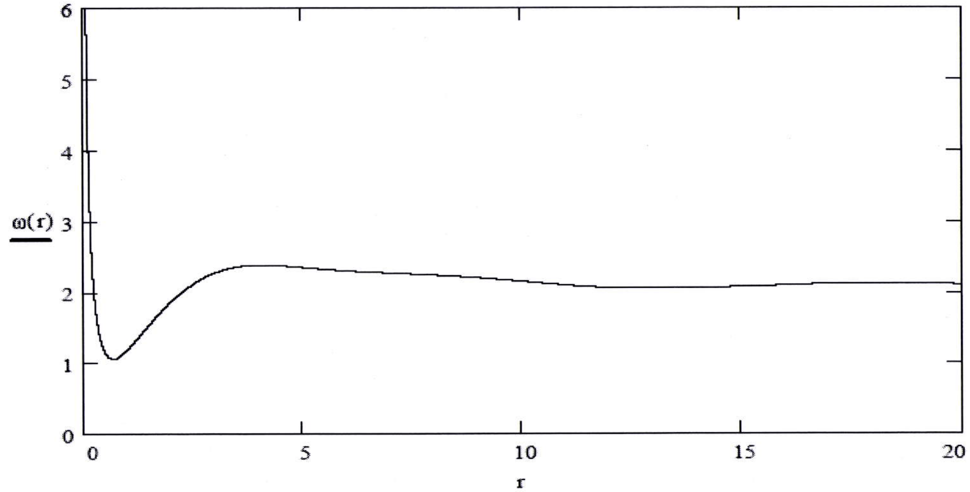

Figure 4. Distribution of velocities around a given set of hollow masses.

The chosen coefficients are the following:

$$\begin{aligned} A = (&0.02;0.1;30;50;80;120;150;250;300;100;500;\\ &300;600;800;900;1200;1500;1000;1200;1500;1500) \end{aligned} \tag{25}$$

where the line matrix A starts with the index $\beta = 0$.

The figure 4 shows the resulting distribution of star velocities. To obtain a nearly flat curve in a wide interval of radii, it has been necessary to give a very low value to the coefficient A_0. This shows that particles of Bessel order zero must be in a very limited quantity in the galactic bulge.

In the Milky-Way, particles of high bessel order may be electrons but they are mainly anti-electrons which annihilate when their Bessel order β decreases to zero and gives the observed γ-bursts of 511 KeV.

Consequently particles or antiparticles of high Bessel order can be regarded as the central engine of the galaxy.

6. A new Concept of Quantum Vacuum

6.1. Functions of the Bessel Order

We have computed the primary maximum of the presence density at the radius r_1, the ratio r_1/r_0 and the ratio r_2/r_0 as functions of the Bessel order β, numerically with the software MathCAD 6.0 SE, from the order zero to the order 400 (see the results in the Data Array 2 of reference [2]). We have shown [2] that the radius r_2 of the hole in a mass particle of high Bessel order β is defined by:

$$\frac{r_2(\beta)}{r_0} \approx \beta \qquad (15)$$

Now let's consider the values of the primary maximum of the presence density as a function of the Bessel order β. We store the previously computed values in a line matrix **M** :

$$M = (0.092;0.059;0.046;0.038;0.033;0.029;\\ 0.026;0.024;0.022;0.021;0.013;0.010;0.008) \qquad (26)$$

and the associated Bessel orders in a line matrix **B** :

$$B = (10;20;30;40;50;60;70;80;90;100;200;300;400) \qquad (27)$$

where the maximum M_j correspond to the Bessel order B_j of same indice *j*. From these matrix we can draw the graph of this function with the software Mathcad 6.0 SE as shown in figure 5, where the rhombuses represent the points in the graph of the matrix function $M_j(B_j)$. The dashed line represents the maximum d_1 of the presence density as a function of Bessel order β; it is the graph drawn through these points by the software.
As the function $d_1(\beta)$ has no known mathematical expression, we have approximated it with a linear function which is represented by the continuous line and has been numerically computed from the matrix function $H_j(B_j)$ defined by:

$$H_j = \frac{1}{k(B_j)^m} \qquad (28)$$

Then we have determined the value of the coefficient k and the exponent m with a accuracy better than 1%, and we have found the following values:

$$k = 2 \qquad m = 0.695 \qquad (29)$$

so we can deduce that the primary maximum d_1 of the presence density at the radius r_1, is the function of the Bessel order β defined by:

$$d_1(\beta)=\frac{1}{k\cdot(\beta)^m} \tag{30}$$

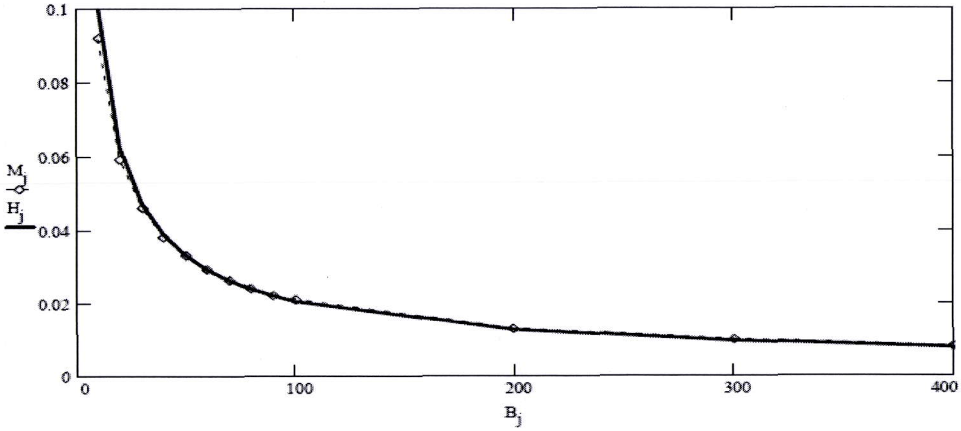

Figure 5. Primary maximum of the presence density, function of the Bessel order.

6.2. Particles at the Limit of Infinite Bessel Order

We see from equation (15) at the infinite limit of the Bessel order $\beta \to \infty$, that the radius r_2 of the hole is infinite:

$$\lim_{\beta\to\infty} r_2(\beta)=\infty \tag{31}$$

and we see from equation (30) that the maximum d_1 of the presence density is zero:

$$\lim_{\beta\to\infty} d_1(\beta)=0 \tag{32}$$

Therefore at the limit of infinite Bessel order, any mass particle has a null presence density everywhere and its ineffective cross-section has an infinite radius. Such a particle seems to be "diluted" in the whole space-time of the Universe, and so it vanishes in vacuum.

These considerations lead us to a new concept of quantum vacuum which is made of mass particles of extremely high Bessel orders, with hole radii at the scale of cosmological distances, such as galaxies, galaxy clusters or the whole Universe itself.

So we can define a new model of quantum vacuum which is made of mass particles at the infinite Bessel order. This new concept of vacuum is different from Dirac's vacuum, but infinite Bessel order particles do not replace negative energy particles.

Let's consider a given particle in such a quantum vacuum. When the Bessel order decreases, the radius of the hole decreases, the ineffective cross-section decreases, while the presence density increases (and the amplitude of spatial frequencies also increases). Therefore any particle with a decreasing Bessel order seems to emerge from vacuum, but the particle is not created in the sense of a particle-pair creation in usual quantum theory. We then rather talk of "particle emergence" from vacuum.

It leads to a new scenario of nucleosynthesis, where dark matter is first created before the ordinary matter of Bessel order zero.

In such a scenario real or virtual particles may emerge from quantum vacuum when their Bessel order decreases. A particle may emerge without its anti-particle, but the creation of particle pairs remains possible from photons.

In the case of real particles the amplitude of the presence density will follow the law of equation (30). As virtual particles have a short life time, they require a short time in decreasing their Bessel order to be able to emerge from such a vacuum.

Consequently all the possible emerged particles depend on the "initial populations" defined in quantum vacuum with several different types of particles. A possible relation between the emerged mass-energy and the decreasing Bessel order will have to be studied, while some relations between magnetic field, gravitational field and the Bessel order have to be postulated.

7. Conclusions

We think that the Bessel order is always zero in most terrestrial experiments, but it may take extremely high values for some particles or antiparticles, under some physical conditions: extremely strong gravitation fields or extremely strong magnetic fields in galaxy bulges and in star nucleus.

We have shown that a population of particles which have a hollow mass with a hole radius as large as the galaxy radius, behaves as the required dark matter. In other words, dark matter is hidden in the galactic bulge, and it has a gravitational mass at very long distances.

The presence of antimatter in the Universe is actually proven by the detection of γ–rays of 511 keV from the centre of our galaxy. Antimatter (mainly composed of anti-electrons in the Milky-Way) is made passive by extremely high Bessel orders and stored in the galaxy bulge of the Milky-Way, and also in some star nucleus.

The assumed inflation period has not destroyed antimatter but another mechanism involving hollow mass particles has made the antimatter passive. Moreover the Universe is using hollow matter or hollow antimatter of extremely high Bessel orders to build central engine of galaxies.

Acknowledgements

We are grateful to J. Knödlseder for his kind communication of his publication [9] and his help to obtain the reprint permission from Astronomy & Astrophysics.

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