Towards a Computational Derivation of a Dual Relativity with Forward-Backward Space-Time Shifts

Daniel M. Dubois
Centre for Hyperincursion and Anticipation in Ordered Systems,
CHAOS asbl, Institute of Mathematics, University of Liège
Grande Traverse 12, B-4000 LIEGE 1, Belgium
Fax: + 32 4 366 94 89 - Email: Daniel.Dubois@ulg.ac.be
http://www.ulg.ac.be/mathgen/CHAOS/CASYS.html
HEC, rue Louvrex 14, B-4000 LIEGE, Belgium

Gilles Nibart
Laboratoire de Physique Théorique Fondamentale de Paris
31, rue de l'Evèque
F-06140 Coursegoules, France
Phone/Fax 33 (0)4 93 59 14 46

Abstract

A computational derivation of the Klein-Gordon quantum relativist equation and the Schrödinger quantum equation with forward and backward space-time shifts was developed in Dubois (1999, 2000). The forward-backward space λ and time τ shifts are related to a phase velocity $\mathbf{u} = \lambda/\tau$. The ratio $\mathbf{v}/\mathbf{u}$, where $\mathbf{v}$ is a group velocity, is related to the mass of particles: for $\mathbf{v} < \mathbf{u}$, particles have a real mass and for $\mathbf{v} = \mathbf{u}$, there is no mass, as for photons.
In this paper, it is shown that this formalism gives rise to a quantum interpretation of the mass in relation with plane waves. Moreover there is a third case for mass of particle: when for $\mathbf{v} > \mathbf{u}$, particles have an imaginary mass, as for tachyons.
From these considerations, we look at the possibility to develop a dual relativity including these three types of mass.

Keywords: Quantum Schrödinger equation, Relativist Klein-Gordon equation, forward-backward space-time shifts, mass, dual relativity.

1 Introduction

A computational derivation of the Klein-Gordon quantum relativist equation and the Schrödinger quantum equation with forward and backward one-dimension space and time shifts was developed in Dubois (1998, 1999).
With three-dimension space and time shifts, Klein-Gordon equation for electromagnetic field was derived (Dubois. 2000).

This paper introduces firstly the forward and backward derivatives for discrete and continuous systems. Generalized complex discrete and continuous derivatives are deduced. The Klein-Gordon equation is deduced from the space-time complex continuous derivatives. These derivatives take into account forward-backward space $\boldsymbol{\lambda}$ and time τ shifts related to a phase velocity $\mathbf{u} = \lambda/\tau$. The ratio $\mathbf{v}/\mathbf{u}$, where $\mathbf{v}$ is a group velocity, is related to the mass of particles: for $\mathbf{v} < \mathbf{u}$, particles have a real mass and for $\mathbf{v} = \mathbf{u}$, there is no mass, as for photons.
From this formalism, new interpretations are obtained which deal with a dual Relativity: three fundamental particles velocities are obtained: $v < c$, particle with real mass; $v = c$, photons; and, $v > c$, particle with imaginary mass for tachyons.

1.1. A Generalized Forward and Backward Discrete Derivative

Let us consider a function F(t) of the time t. Two discrete time derivatives can be defined: a forward and a backward derivatives

$$\Delta_f F / \Delta t = (F(t + \Delta t) - F(t))/\Delta t \qquad (1a)$$

$$\Delta_b F / \Delta t = (F(t) - F(t - \Delta t))/\Delta t \qquad (1b)$$

where Δt is the discrete time interval. The successive application of the forward derivative to the backward derivative, or the inverse, gives the second order derivative:

$$\Delta^2 F / \Delta t^2 = [\, F(t + \Delta t) - 2F(t) + F(t - \Delta t)\,] / \Delta t^2 \qquad (2)$$

A generalized discrete derivative by a weighted sum of these derivatives was defined (Dubois, 1998) as follows:

$$\Delta_w F/\Delta t = w.\Delta_f F /\Delta t+(1-w).\Delta_b F/\Delta t = [w.F(t+\Delta t)+(1-2.w).F(t)-(1-w).F(t-\Delta t)]/\Delta t \quad (3)$$

where the weight w is defined in the interval [0,1]. For w = 1, the forward derivative 1a is obtained and for w = 0, the backward derivative 1b. For w = 1/2, derivative 3 becomes

$$\Delta_{1/2} F/\Delta t = (F(t+\Delta t) - F(t-\Delta t))/2\Delta t = [\Delta_f F/\Delta t + \Delta_b F/\Delta t]/2 \qquad (4)$$

which is an average derivative. From eq. 3 of the generalized discrete derivative, the second order derivative is given by the successive application of eq. 3 for w and (1–w), or the inverse:

$$\Delta_w\Delta_{1-w}F/\Delta t^2 =$$

$$[F(t+\Delta t)-2F(t)+F(t-\Delta t)]/\Delta t^2 + w(1-w)[F(t+2\Delta t)-4F(t+\Delta t)+6F(t)-4F(t-\Delta t)+F(t-2\Delta t)]/\Delta t^2$$

$$= \Delta_{1-w}\Delta_w F/\Delta t^2 \qquad (5)$$

which is the sum of the classical discrete second order derivative and a factor, weighted by $w(1 - w)$, which is similar to a fourth order discrete derivative (multiplied by Δt^2). For $w = 0$ and $w = 1$, the classical second order derivative is obtained: $w(1 - w) = 0$.
For $w = 1/2$, $w(1 - w) = 1/4$, the second order derivative is also obtained but with a double time interval $2\,\Delta t$.

1.2. A Generalized Complex Discrete Derivative

In choosing the value of the $w(1 - w)$ equal to $1/2$, we obtain weights w, solution of

$$w^2 - w + 1/2 = 0 \qquad (6)$$

which are given by the complex numbers

$$w = (1 \pm i) / 2 \qquad (7)$$

and $1-w = (1 \pm (- i)) / 2 = (1 \pm i^*) / 2 = w^*$, where w^* is the complex conjugate of w.

So eq.3 of the generalized discrete derivative can be rewritten as (Dubois, 1999):

$$\Delta_w \Phi /\Delta t = w.\Delta_f \Phi/\Delta t + w{*}.\Delta_b \Phi /\Delta t = [w.\Phi(t+\Delta t) + (w^* - w).\Phi(t) - w^*.\Phi(t-\Delta t)] /\Delta t$$

$$= [\Phi(t+\Delta t) - \Phi(t-\Delta t)] /2\Delta t \pm i\, [\Phi(t+\Delta t) - 2\Phi(t) + \Phi(t-\Delta t)] /2\Delta t \qquad (8)$$

where the generalized complex derivative is applied to a complex function $\Phi = F + i\,G$.

The second order derivative is given by the successive applications of eq.8 for w and w^*, or the inverse:

$$\Delta_w\Delta_{w*}\Phi/\Delta t^2= [\Phi(t+\Delta t)-2\Phi(t)+\Phi(t-\Delta t)]/\Delta t^2$$

$$+(1/2)[\Phi(t+2\Delta t)-4\Phi(t+\Delta t)+6\Phi(t)-4\Phi(t-\Delta t)+\Phi(t-2\Delta t)]/\Delta t^2 = \Delta_{w*}\Delta_w\Phi/\Delta t^2 \qquad (9)$$

which is the sum of the classical discrete second order derivative and a factor, weighted by the real number $1/2$, which is similar to a fourth order discrete derivative (multiplied by Δt^2).

1.3. Generalized Complex Continuous Derivatives

We will consider successively generalized continuous time and space derivatives deduced for the generalized discrete derivative and applied to complex space-time functions.

1.3.1. A Generalized Complex Continuous Time Derivative (Dubois, 1999)

From the generalized complex time derivative applied to the complex space-time function $\Phi(\mathbf{r},t)$, let us write

$$\Delta\Phi(\mathbf{r},t)/\Delta t = (\partial^+\Phi/\partial t + \partial^-\Phi/\partial t)/2 \pm i(\partial^+\Phi/\partial t - \partial^-\Phi/\partial t)/2 \tag{10}$$

where

$$\partial^+\Phi(\mathbf{r},t)/\partial t \approx [\Phi(\mathbf{r},t + \Delta t) - \Phi(\mathbf{r},t)]/\Delta t \tag{11a}$$

$$\partial^-\Phi(\mathbf{r},t)/\partial t \approx [\Phi(\mathbf{r},t) - \Phi(\mathbf{r},t - \Delta t)]/\Delta t \tag{11b}$$

which give the discrete forward and backward derivatives for $\Delta t > 0$. Then, the continuous version of the forward and backward discrete derivatives are:

$$\partial^+\Phi(\mathbf{r},t)/\partial t = \partial\Phi(\mathbf{r},t + \tau/2)/\partial t = \partial\Phi(\mathbf{r},t)/\partial t + (\tau/2)\partial^2\Phi(\mathbf{r},t)/\partial t^2 \tag{12a}$$

$$\partial^-\Phi(\mathbf{r},t)/\partial t = \partial\Phi(\mathbf{r},t - \tau/2)/\partial t = \partial\Phi(\mathbf{r},t)/\partial t - (\tau/2)\partial^2\Phi(\mathbf{r},t)/\partial t^2 \tag{12b}$$

where $\tau > 0$, in using the development in Taylor's series until the first order. These continuous forward and backward time derivatives mean that the derivatives are computed in a future and past times $t \pm \tau/2$, respectively, which correspond to anticipatory and memory effects. The total time duration is τ around the current time t, which can be interpreted as a temporal non-locality. If the function Φ is related to a particle moving in one direction, the forward and backward derivatives mean that the particle anticipates the time in the direction of moving and has a memory of the time in the opposite direction. This can be also interpreted as a time extension of the particle around the current time.

These continuous forward and backward derivatives give the discrete ones in taking $\tau = \Delta t > 0$, with the following discrete versions of the continuous derivatives:

$$\partial\Phi(\mathbf{r},t)/\partial t \approx [\Phi(\mathbf{r},t + \Delta t) - \Phi(\mathbf{r},t - \Delta t)]/2\Delta t \tag{13}$$

$$\partial^2\Phi(\mathbf{r},t)/\partial t^2 \approx [\Phi(\mathbf{r},t + \Delta t) - 2\Phi(\mathbf{r},t) + \Phi(\mathbf{r},t - \Delta t)]/\Delta t^2 \tag{14}$$

Thus

$$\Delta\Phi(\mathbf{r},t)/\Delta t = \partial\Phi(\mathbf{r},t)/\partial t \pm i(\tau/2)\partial^2\Phi(\mathbf{r},t)/\partial t^2 \tag{15a}$$

which can be written as

$$\Delta\Phi(\mathbf{r},t)/\Delta t = \partial\Phi(\mathbf{r},t \pm i\tau/2)/\partial t = \partial\Phi(\mathbf{r},t)/\partial t \pm i(\tau/2)\partial^2\Phi(\mathbf{r},t)/\partial t^2 \tag{15b}$$

With the forward and backward time derivatives applied to continuous functions, the one-dimensional time domain is transformed into a two-dimensional time domain with one real time t variable and one complex time $\pm i\tau/2$.

If

$$\Phi(\mathbf{r},t) = \phi(\mathbf{r},t) \exp(\pm it/\tau) \tag{16}$$

the first order derivative disappears

$$[\partial\Phi(\mathbf{r},t)/\partial t \pm i(\tau/2)\partial^2\Phi(\mathbf{r},t)/\partial t^2]/\Phi(\mathbf{r},t) = \pm i[(\tau/2)\partial^2\phi(\mathbf{r},t)/\partial t^2 + (1/2\tau)\phi(\mathbf{r},t)]/\phi(\mathbf{r},t) \tag{17}$$

Proof: With

$\Phi(\mathbf{r},t) = \phi(\mathbf{r},t)\exp(i\alpha t)$ (18)

we obtain

$\partial\Phi(\mathbf{r},t)/\partial t \pm i(\tau/2)\partial^2\Phi(\mathbf{r},t)/\partial t^2 =$

$[\partial\phi(\mathbf{r},t)/\partial t + i\alpha\phi(\mathbf{r},t)]\exp(i\alpha t) \pm i(\tau/2)[\partial^2\phi(\mathbf{r},t)/\partial t^2 + 2i\alpha\partial\Phi(\mathbf{r},t)/\partial t - \alpha^2\phi(\mathbf{r},t)]\exp(i\alpha t)$ (19)

and the first order derivative $\partial\phi(\mathbf{r},t)/\partial t$ disappears for

$1 \pm i(\tau/2)2i\alpha = 0$ (20)

or $\alpha = \pm 1/\tau$ so that the coefficient of $\phi(\mathbf{r},t)$ becomes $i\alpha \pm i(\tau/2)(-\alpha^2) = \pm i/2\tau$ ■

1.3.2. A Generalized Complex Continuous Space Derivative (Dubois, 1999, 2000)

In three space dimensions, partial space derivative is given by a gradient ∇, and second order derivative by the Laplacian written as $\nabla.\nabla$ or ∇^2 instead of Δ to avoid confusion with the discrete operator Δ.

From a similar reasoning as for the time derivative, we obtain successively:

$\Delta\Phi(\mathbf{r},t)/\Delta\mathbf{r} = (\nabla^+\Phi + \nabla^-\Phi)/2 \pm i(\nabla^+\Phi - \nabla^-\Phi)/2$ (21)

in considering the forward and backward derivatives, with **a space shift vector λ** :

$\nabla^+\Phi(\mathbf{r},t) = \nabla\Phi(\mathbf{r} + \lambda/2,t) = \nabla\Phi(\mathbf{r},t) + (\lambda/2)\nabla^2\Phi(\mathbf{r},t)$ (22a)

$\nabla^-\Phi(\mathbf{r},t) = \nabla\Phi(\mathbf{r} - \lambda/2,t) = \nabla\Phi(\mathbf{r},t) - (\lambda/2)\nabla^2\Phi(\mathbf{r},t)$ (22b)

These continuous forward and backward space derivatives mean that the derivatives are computed in two opposite directions $\mathbf{r} \pm \lambda/2$, respectively. The total space length of computation is λ around the current position. This can be interpreted as a spatial non-locality. If the function Φ is related to a particle moving in one direction, the forward and backward derivatives mean that the particle anticipates the space in the direction of moving and has a memory of the space in the opposite direction. This can be also interpreted as a spatial extension of the particle which is no more represented by a point but a ball (a wave packet).

$\Delta\Phi(\mathbf{r},t)/\Delta\mathbf{r} = \nabla\Phi(\mathbf{r},t) \pm i(\lambda/2)\nabla^2\Phi(\mathbf{r},t)$ (23a)

which can be written as

$\Delta\Phi(\mathbf{r},t)/\Delta\mathbf{r} = \nabla\Phi(\mathbf{r} \pm i\lambda/2,t) = \nabla\Phi(\mathbf{r},t) \pm i(\lambda/2)\nabla^2\Phi(\mathbf{r},t)$ (23b)

With forward and backward space derivatives applied to continuous functions, the three-dimensional spatial domain is transformed into a six-dimensional space domain with one real space $\mathbf{r}$ variable and one complex space volume $\pm i\lambda/2$.

If

$$\Phi(\mathbf{r},t) = \phi(\mathbf{r},t)\exp(\pm i\mathbf{r}/\lambda) \quad (24)$$

the first order derivative disappears:

$$[\nabla\Phi(\mathbf{r},t) \pm i(\lambda/2)\nabla^2\Phi(\mathbf{r},t)]/\Phi(\mathbf{r},t) = \pm i[(\lambda/2)\nabla^2\phi(\mathbf{r},t) + (1/2\lambda)\phi(\mathbf{r},t)]/\phi(\mathbf{r},t) \quad (25)$$

Proof: With

$$\Phi(\mathbf{r},t) = \phi(\mathbf{r},t)\exp(i\beta.\mathbf{r}) \quad (26)$$

we deduce

$$\nabla\Phi(\mathbf{r},t) \pm i(\lambda/2)\,\nabla^2\Phi(\mathbf{r},t) =$$

$$[\nabla\phi(\mathbf{r},t)+i\beta\phi(\mathbf{r},t)]\exp(i\beta.\mathbf{r}) \pm i(\lambda/2)[\nabla^2\phi(\mathbf{r},t)+2i\beta.\nabla\Phi(\mathbf{r},t)-\beta^2\phi(\mathbf{r},t)]\exp(i\beta.\mathbf{r}) \quad (27)$$

and the first order derivative $\nabla\phi(\mathbf{r},t)$ disappears for

$$\beta = \pm 1/\lambda \quad (28)$$

so that the coefficient of $\phi(\mathbf{r},t)$ becomes $i\beta \pm i(\lambda/2)(-\beta^2) = \pm i/2\lambda$ ■

2 Deduction of Quantum Relativity Equation Systems

2.1. A Generalized Continuous Space-Time Derivative (Dubois, 1999, 2000)

Let us consider the equation

$$\Delta\Phi(\mathbf{r},t)/\Delta t = \mathbf{v}.\,\Delta\Phi(\mathbf{r},t)/\Delta\mathbf{r} \quad (29)$$

where $\mathbf{v} = \Delta\mathbf{r}/\Delta t$ is a velocity. In using the forward and backward continuous derivatives 15a and 23a , we obtain

$$\partial\Phi(\mathbf{r},t)/\partial t \pm i(\tau/2)\partial^2\Phi(\mathbf{r},t)/\partial t^2 = \mathbf{v}.\,[\nabla\Phi(\mathbf{r},t) \pm i(\lambda/2)\,\nabla^2\Phi(\mathbf{r},t)] \quad (30)$$

With

$$\Phi(\mathbf{r},t) = \phi(\mathbf{r},t)\exp(\pm it/\tau \pm i\mathbf{r}/\lambda) \quad (31)$$

we obtain

$$\pm i[(\tau/2)\partial^2\phi(\mathbf{r},t)/\partial t^2 + (1/2\tau)\phi(\mathbf{r},t)] = \pm i\,\mathbf{v}.\,[(\lambda/2)\nabla^2\phi(\mathbf{r},t) + (1/2\lambda)\phi(\mathbf{r},t)]$$

or

$$\partial^2\phi(\mathbf{r},t)/\partial t^2 = \mathbf{v}.\mathbf{u}\nabla^2\phi(\mathbf{r},t) - (1/\tau^2)[1-(\mathbf{v}/\mathbf{u})]\phi(\mathbf{r},t) \quad (32)$$

with $\mathbf{u} = \lambda/\tau$.

Let us give the phase and group velocities in introducing a plane wave solution

$$\Phi = \exp(i\omega t + i\mathbf{k}.\mathbf{r}) \quad (33)$$

in this equation. We obtain

$$\omega^2 = \mathbf{v.u}\ \mathbf{k}^2 + (1/\tau^2)[1 - (\mathbf{v}/\mathbf{u})] \quad (34)$$

and

$$\omega = \pm \sqrt{\{\mathbf{v.u}\ \mathbf{k}^2 + (1/\tau^2)[1 - (\mathbf{v}/\mathbf{u})]\}} \quad (35)$$

so the phase velocity is

$$\mathbf{v}_p = \omega/\mathbf{k} = \pm\, \mathbf{e}_k \sqrt{\{\mathbf{v.u} + (1/\tau^2)[1 - (\mathbf{v}/\mathbf{u})]/\mathbf{k}^2\}} \quad (36)$$

where $\mathbf{e}_k$ is the unit vector $\mathbf{k}/|\mathbf{k}|$, and the group velocity

$$\mathbf{v}_g = d\omega/d\mathbf{k} = \pm\, \mathbf{e}_k\, \mathbf{v.u}/\sqrt{\{\mathbf{v.u} + (1/\tau^2)[1 - (\mathbf{v}/\mathbf{u})]/\mathbf{k}^2\}} \quad (37)$$

so the product $\mathbf{v}_p.\mathbf{v}_g$ is

$$\mathbf{v}_p.\mathbf{v}_g = \mathbf{v.u} = c^2 \quad (38)$$

where c is a speed, and when

$$\mathbf{v} = \mathbf{u} \quad (39)$$

so

$$\mathbf{v}_p = \mathbf{v}_g \quad (40)$$

and we can write the relation

$$\mathbf{u}.d\mathbf{v} = -\,\mathbf{v}.d\mathbf{u} \quad (41)$$

If

$$\mathbf{u} \geq \mathbf{v} \quad (42)$$

then

$$c \geq \mathbf{e}_k.\mathbf{v}g \text{ and } \mathbf{e}_k.\mathbf{v}_p \geq c \quad (43)$$

Thus we may interpret **v** as a group-related velocity and **u** a phase-related velocity. Dubois (2000) called **v** and **u**, the internal (endo) and external (exo) group and phase velocities, respectively. This remarkable property is that the product $\mathbf{v}_p.\mathbf{v}_g$ is equal to the product **v.u** which is equal to c^2, with the same inequalities.

Let us now show that the Klein-Gordon quantum relativist equation and the Schrödinger quantum equation can be derived in our formalism.

2.2. Deduction of the Klein-Gordon Quantum Relativity Equation

In taking $\mathbf{v.u} = c^2$, where c is the light speed, and multiplying both members of eq. 32 by $\hbar^2$, where $\hbar$ is the Planck constant, so that the equation dimension is a square energy, one obtains (Dubois, 1999, 2000):

$$\hbar^2\, \partial^2\phi(\mathbf{r},t)/\partial t^2 = \hbar^2 c^2\, \nabla^2\phi(\mathbf{r},t) - (\hbar^2/\tau^2)(1 - \mathbf{v}/\mathbf{u})\phi(\mathbf{r},t) \quad (44)$$

This equation is the Klein-Gordon equation in taking

$$(\hbar^2/\tau^2)(1 - \mathbf{v}/\mathbf{u}) = m_0^2c^4 \tag{45}$$

or

$$\hbar/\tau = \pm\, m_0\, c^2 /\sqrt{(1 - \mathbf{v}/\mathbf{u})} \tag{46}$$

where m_0 is a rest mass so that the relativist mass m is

$$m = m_0 /\sqrt{(1 - \mathbf{v}/\mathbf{u})} \tag{47}$$

When the mass is negative, τ is negative. Eq. 45 could be written as

$$(\hbar^2/\tau^2)(1 - \mathbf{v}/\mathbf{u}) = \hbar^2\, \omega_0^2 \tag{48}$$

or

$$\hbar/\tau = \pm\, \hbar\, \omega_0 /\sqrt{(1 - \mathbf{v}/\mathbf{u})} \tag{48a}$$

so the forward-backward time shift τ satisfies the relativist equation

$$1/\tau = \omega = \omega_0 /\sqrt{(1 - \mathbf{v}/\mathbf{u})} \tag{49}$$

From the relation $\mathbf{u} = \lambda/\tau$, we deduce that the forward-backward space shift λ satisfies also a relativist equation.
Thus, equation 44, with equation 45, gives

$$-\,\hbar^2\, \partial^2\phi(\mathbf{r},t)/\partial t^2 = -\,\hbar^2\, c^2\, \partial^2\phi(\mathbf{r},t)/\partial \mathbf{r}^2 + m_0^2c^4\phi(\mathbf{r},t) \tag{50}$$

which is the Quantum Relativity Klein-Gordon equation for bosons.

Let us remark that $\mathbf{v}/\mathbf{u} = \mathbf{v}.\mathbf{u}/\mathbf{u}.\mathbf{u} = c^2/\mathbf{u}^2$. If $\mathbf{u}$ is parallel to $\mathbf{v}$, similarly to $\mathbf{v}_p$ was parallel to $\mathbf{v}_g$ for a plane wave, we have also $\mathbf{v}/\mathbf{u} = \mathbf{v}^2/c^2$.
In this case, eq. 48 can be written as

$$(\hbar^2/\tau^2)(1 - \mathbf{v}^2/c^2) = \hbar^2\, \omega_0^2 \tag{51a}$$

or

$$(\pm\, \hbar/\tau)(1 + v/c)\, (\pm\, \hbar/\tau)(1 - v/c) = (\pm\, \hbar\omega_+)\, (\pm\, \hbar\omega_-) = \hbar^2\, \omega_0^2 \tag{51b}$$

so

$$\hbar^2\, \omega_0^2 = (\pm\, \hbar\omega_+)\, (\pm\, \hbar\omega_-) \tag{51c}$$

which define two frequencies, which could be related to forward (positive) and backward (negative) shifts, similar to the Doppler effect.
Similarly, the rest mass could be interpreted as the product of two masses

$$(\hbar^2/\tau^2)(1 - \mathbf{v}/\mathbf{u}) = m_0^2c^4 = (\pm\, m_+ c^2)\, (\pm\, m_- c^2) \tag{52}$$

where m_+ and m_- could be defined as masses related to internal group and phase velocities.

Phase and group momentum can be also defined as

$\mathbf{p}_u = m_u \mathbf{u}$ (53a)

$\mathbf{p}_v = m_v \mathbf{v}$ (53b)

in introducing phase and group masses

$m_+ = m_v$ and $m_- = m_u$ (53c)

See the interpretation of Schrödinger equation as a fluid with a complex momentum, a phase and a group momentum (Dubois, 1999).

2.3. Quantum Interpretation of the Mass in Relation to Plane Waves

In usual presentations of quantum mechanics, most authors introduce wave functions having imaginary exponents without any explanation but just relating them to the classical case of plane wave propagation.

This is not a pertinent justification to use plane wave functions, and some authors seem to choose such functions to simplify the calculus.

Moreover, the usual quantum theory has never proposed a quantum definition of mass.

The above demonstration has introduced a plane wave function, such as eq. 31:

$\Phi(\mathbf{r},t) = \phi(\mathbf{r},t) \exp(\pm it/\tau \pm i\mathbf{r}/\lambda)$

which has allowed to eliminate first order derivatives from eq. 30 and has led to give a quantum definition of mass by eq. 45 in relation to quantum time shift:

$m_0^2 = (1/c^4)(\hbar^2/\tau^2)(1 - v^2/c^2)$

Obviously, plane wave propagation is required to allow energy to exist as being mass bodies, because without it, energy would dilute with the propagation in several directions.

This new concept in physics is extremely important, and we think that :
The plane wave propagation is the cause of mass, in relation to time shift.

2.4. Wave Equation for Photons

If equation 67 is satisfied, $\mathbf{v} = \mathbf{u}$, so the term $(1/\tau^2)(1 - \mathbf{v}/\mathbf{u}) = 0$ disappears from equation 32 which becomes

$$\partial^2\phi(\mathbf{r},t)/\partial t^2 = c^2 \nabla^2\phi(\mathbf{r},t) \quad (54)$$

which is the wave equation for photons. The condition **v** = **u** implies that the rest mass is null, $m_0 = 0$, from eq. 45.

This result is remarkable in the sense that we do not deduce that the phase velocity **u** is equal to the group velocity **v** from the fact that the mass is null, but the inverse: when the phase and group velocities are equal, then the mass of the particle is null.
Can we conclude that the creation of rest mass in particles is due to a difference from the phase velocity **u** to the group velocity **v**, the phase velocity being related to forward and backward space-time shifts λ and τ.

Another remarkable property is the fact that the sign of the mass of the particle is related to the sign of the time shift τ. A positive time shift creates particles with a positive mass and negative time shift creates particles with a negative mass.

Last but not least, the time shift τ is also at the basis of the finite light speed. When τ is null, the particles can propagate with an unlimited group velocity, as in the non-relativist Schrödinger equation.

2.5. Deduction of the Schrödinger Quantum Equation (Dubois, 1999, 2000)

In taking τ = 0 in the equation 30, we obtain

$$\partial\Phi(\mathbf{r},t)/\partial t = \mathbf{v}.\,[\nabla\Phi(\mathbf{r},t) \pm i(\lambda/2)\,\nabla^2\Phi(\mathbf{r},t)] \qquad \textbf{(55)}$$

With

$$\Phi(\mathbf{r},t) = \phi(\mathbf{r},t)\exp(\pm\, i\mathbf{r}/\lambda) \qquad \textbf{(56)}$$

we obtain

$$\partial\phi(\mathbf{r},t)/\partial t = \pm\, i\,\mathbf{v}.\,[(\lambda/2)\,\nabla^2\phi(\mathbf{r},t) + (1/2\lambda)\phi(\mathbf{r},t)] \qquad (57a)$$

In taking $\mathbf{v}.\lambda = \hbar/m$, where $\hbar$ is the Planck constant and m the rest mass, we obtain after multiplication by $i\hbar$

$$i\hbar\,\partial\phi(\mathbf{r},t)/\partial t = \pm\,[(-\hbar^2/2m)\,\nabla^2\phi(\mathbf{r},t) - (\hbar^2/2m\lambda^2)\phi(\mathbf{r},t)] \qquad (57b)$$

which is the Schrödinger equation for a free particle in a constant negative potential $V = -\hbar^2/2m\lambda^2 = -\hbar\mathbf{v}/2\lambda$.
This supplementary term is particularly intriguing: is it related to quantum void?.
Its study will be made in a next paper.

Remark 1: The second term in the second member of eq. 57b disappears in introducing

$$\phi = \phi_1\exp(\pm\, i\,\mathbf{v}\,t/2\lambda) \qquad (58)$$

so we obtain

$$i\hbar\, \partial\phi_1(\mathbf{r},t)/\partial t = (-\hbar^2/2m)\, \nabla^2\phi_1(\mathbf{r},t) \tag{59}$$

the classical Schrödinger equation for a free particle of positive mass.

Remark 2: from eq. 52 $(\hbar^2/\tau^2)(1 - \mathbf{v}/\mathbf{u}) = (\hbar^2/\tau^2)(1 - \mathbf{v}\tau/\lambda) = m_0^2c^4 = m_0^2\mathbf{v}^2\lambda^2/\tau^2$, so, with $\tau = 0$, we can write $\hbar = m_0\mathbf{v}.\lambda$.

Remark 3: We write m instead of m_0, because when $\tau = 0$, they are equal, which corresponds to a non-relativist case.

3 Towards a Dual Relativity

From eq. 45

$$(\hbar^2/\tau^2).(1 - \mathbf{v}/\mathbf{u}) = m_0^2c^4$$

the rest mass can be defined as a function of the phase and group velocities:

$$m_0 = (\hbar/c^2\tau)\, \sqrt{(1 - \mathbf{v}/\mathbf{u})} \tag{60}$$

where $c^2 = \mathbf{u}.\mathbf{v}$ and $\mathbf{u} = \lambda/\tau$.

The square root may be imaginary and three cases can be defined.

If $\mathbf{v} < \mathbf{u}$ the rest mass is real:

$$m_0 = (\hbar/c^2\tau)\, \sqrt{(1 - \mathbf{v}/\mathbf{u})} \tag{60a}$$

If $\mathbf{u} = \mathbf{v}$ the rest mass is null (photons):

$$m_0 = 0 \tag{60b}$$

If $\mathbf{v} > \mathbf{u}$ the mass is imaginary (tachyons):

$$m_0 = (i\,\hbar/c^2\tau)\, \sqrt{(\mathbf{v}/\mathbf{u} - 1)} \tag{60c}$$

Einstein's relativity deals only with the first two cases, eqs. 60ab.

So, a dual relativity can be developed in including the third case, eq. 60c.

Several avenues may be considered.

In previous papers R. DUTHEIL and G. NIBART using the tensor formalism, have shown that particles named *tachyons* (Bilaniuk et al, 1962) have a superluminal velocity in Tachyonic Referential Frames, may exist (Dutheil et Nibart, 1986) and do not violate the Causality Principle (Nibart et Dutheil, 1986). According to their *reinterpretation principle*, (the TBI Principle) (Nibart et Dutheil, 1986) *tachyons* would always be perceived by any natural observer, using Ordinary Referential Frames, as being antiparticles having a subluminal velocity. See also G. Nibart (2000).

Another way is to consider a 6-dimensional universe with a 3-dimensional time $\mathbf{t} = (t_1,t_2,t_3)$ similarly to the 3-dimensional space $\mathbf{r} = (x,y,z)$, so $\mathbf{ds} = \mathbf{dr} + i\, c\, \mathbf{dt}$
and the dual relativity would be defined as $\mathbf{d\mathfrak{s}} = i\, \mathbf{ds} = i\, \mathbf{dr} - c\, \mathbf{dt}$
In the dual universe, real space - imaginary time would be replaced by real time - imaginary space. It has already been suggested to formulate physics in a six-dimensional space by Marchildon and Antippa (1983) to generalize the Lorentz transformations to superluminal velocities.

In looking at eq. 60c, if τ is transformed to $i\,\tau$, the mass remains real, but for conserving $\mathbf{u}$ real, we must also transform λ to $i\,\lambda$, because $\mathbf{u} = \lambda/\tau$.
This case will be developed in a forthcoming paper.

Let us finally point out some questions which will be answered in a next paper.

The sign of a scalar variation has not the same physical meaning for time or space.

What type of coordinate variation can be known as forward or backward ?
In the discrete model, developed above, can Δt be related to the time arrow, to define a forward derivative and a backward derivative. Moreover, $\Delta \mathbf{r}$ depends on the choice of the referential frame, i.e. of an arbitrary convention of the observer.

In the space derivative $\Delta\Phi(\mathbf{r},t)/\Delta\mathbf{r}$ of eq. 21, the variation of the function Φ is divided by the vector $\Delta\mathbf{r}$. So, to introduce a generalized space derivative in a three dimensional space, we may have to define the division by a vector. But here we have just defined the space derivative by using the space shift λ. Because λ is here a vector of the three dimensional space, the space shift vector λ is to be considered as having a particular direction. So, **a privileged direction** has to be granted for the space shift of the generalized discrete space derivatives in the three dimensional space. In the above demonstrations the privileged directions is the normal direction of the plane waves.
In a next work we will show that a vector derivative can always be defined, in relation with the scalar product, and thus the vector derivative expression depends on the metrics of the space-time manifold.

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