For a Naturalist Approach to Anticipation: from Catastrophe Theory to Hyperincursive Modelling

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Abstract
Living systems, in certain circumstances, try to predict future situations, and by that, begin to adapt in advance. The behaviour of adaptation of the sportsman, or of the predator, that begins before the properly told effort, show us remarkable anticipatory characteristics. It is impossible to understand the anticipatory behaviour, and the autonomous actions, of the individuals without having recourse to a dual control. We must distinguished a direct control, each element of the action is felt, and a dual control, only the aim of the target is consciously present (attended to), the other elements of the action are relegated to the periphery of the attention. The living being deals with its external middle to establish its internal coherence, but it takes this into account only to distinguish itself by the action. The relationship/separation between the living being and its environment leans both on an internal action (to adapt it), and on an external action (to adapt its middle). The importance of the dual action for the living being, holds in the fact that it exists by the means of its self-constituent activity, "connected to" and "distinct of" a no-self. We use a dynamic structure involving catastrophe theory, to model anticipative process. The dynamics of the predation, a good example of anticipating system, can be described by an attraction of the predator with regard to the prey. René Thom showed how to use the cusp catastrophe to model predation. The predation activity can be defined by a potential. This structure takes in account the duality of the living being, the substance which is a material organisation, and the goal, which is a relational abstraction.

In this paper, a new interpretation of the catastrophe theory is given in the framework of hyperincursion: a hyperincursive system is an extension of recursive systems in which the state of the system is computed from a function of itself. A Hyperincursive Cusp Function can be modelled by a Heaviside Cusp Function. A Hyperincursive Boolean Table can be built and a hyperincursive algebraic linear function can model a cusp which represents an elementary flip-flop one bit memory.

A recursive process defines the successive states from its initial conditions and a hyperincursive process defines the successive states from the path chosen in the control parameters space. The recursive process is related to an internal observer and the hyperincursive process is related to an external observer.

Keywords: Anticipation, catastrophe, hyperincursion, cusp, internal/external observers.

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Introduction

The study of living system show us, in certain circumstances, behaviours which sign anticipatory capacities (Rosen, 1985). From given situations, the living being tries to predict future situations, and by that, begins to adapt in advance (Dubois, 1998). Anticipation seemed to be an implicit component of adaptive capabilities, that can be pointed out in different biological functions, along the phylogenic scale (Goodwin, 1983). We find this in micro-biological, as well as in social, behaviours. Economic agents are founded on past and current events in view of representing the future: anticipation is articulated around the memory and the project.

In opening the "Pandora Box" of anticipation, temptation is great to reduce the study of the living system, to the analysis of inflections due to the future. So the analysis of anticipatory behaviour leads to the question of the living behaviour. Do anticipatory behaviours appear during the development as made structures, or as partial and incomplete schemas which evolve ? Are they attached to physical-chemical laws of the nature, or laws of cybernetic behaviours ? Can we explicit them completely and/or can we formally construct a model of them ?

Some of these questions may appear difficult, and our project may appear ambitious, but as the title indicates, this paper deals with a tentative program. Moreover, the elaboration of this program benefices of researches realised these last years. We integrate some work in the field of theoretical biology at one hand, and in the field of cognitive sciences on another hand. More precisely, our study belongs to a naturalist approach of cognition. Anticipation is linked to an elementary form of know-how implicated in the self-constitution of the living being defined by Aristotle. "Among the natural bodies (i.e. not built by man), certain have life and certain do not have life. We understand by life, the fact to eat, grow, die" (Aristote, De l'âme, II, 1) (Collectif, 1978).

1 Biological Examples

The living being maintains, with its material environment relationships, but it distinguishes itself by the conquest of a biological autonomy. Anticipation seems to be an implicit component of adaptive capabilities, which can be pointed out in different biological functions: nutrition, locomotion, reproduction, etc.

1.1 Anticipative Stress

When the hour of the meal arrives, we secrete saliva and gastric juices. These stimulations of our secretions, sometimes associated to cramps of stomach, are linked to an anticipation of the arrival of foods in the stomach (Goldstein, 1983). To the contact of the gastric wall, rich in nervous plexus connected to the vague nerve, foods provoke the secretions necessary for the digestion. These secretions are associated to hormonal or nervous reflexes that one finds in other situations. The behaviour of adaptation of the sportsman, that begins before the properly told effort, shows us also remarkable characteristics. The preparation of the action is translated into an acceleration of the cardiac rhythm controlled by the adrenaline. This tachychardia is furthermore connected to the stimulation of the receptors of vena cava and to the elevation of the temperature. The process is limited by an antagonistic action of the cardio-moderator system, staked by the nerves of Hering and Cyon. The initial acceleration, released by the cerebral bark is linked to a stress of anticipation.

These adaptations prepare the arrival of a conditional stimulus. For the hungry predator, the form of the prey, carries a biological significance (pregnance). The recognition of such a form gives rise to a very ample reaction in the subject: freeing of hormones, emotive excitement, and behaviour designed to attract or repulse the inductive form. In higher animals, the pregnances encountered are rather few: hunger, fear, sexual desire. By opposite, the forms which have no significance for the individuals are salient. In the beginning of the Pavlov's classic experiment, the meat is a pregnant form, and the ring of the bell is a salient form. But when the association "meat ⇔ bell ringing" has been repeated often enough (and simultaneously reinforced by the satisfaction of hunger), the sound of the bell alone will appear to the dog as a salient form rich in alimentary pregnance. The salient form that is the sound of the bell is invested by the alimentary pregnance of the meat. The bell is in itself sufficient to bring in the dog a behaviour related to an appetite for food (it will salivate). A pregnance effect can be triggered in a subject by a sensory stimulus with very little "figuration" (here, an auditory stimulus).

Pavlovian conditioning is often considered as a pure manifestation of neurophysiological (or psychic) automatism, initiated by an external perturbation. This point of view does not take into account the profoundly purposive character of this phenomenon (Piaget, 1967). It does not trackle the initial questions: Why does a pregnant form invest its neighbours whilst a simple salient form does not ? Why a salient form placed before reinforcement is more rapid and thoroughly invested than if it comes "after" ? How a pregnant form can act as an inductive source to invest a first salient form, and so on to build up fairly complicated sequences of actions ? But we can also look on a pregnance as an invasive fluid spreading through the field of perceived salient form. In this case, the individual has to deal with an action released in a uncertain situation. And during the processing of this action, the individual gathers informations on the environment to reduce the uncertainty. This is the framework of a very frequent anticipative process.

1.2 Dual Control of Actions

It is impossible to understand the autonomous actions of the individuals without having recourse to a aim (Searle, 1983). A classic example allows to illustrate this observation: the archery. During the first archery, the archer feels the weight and vibrations of the arm. In revenge, it does not feel the junction of the arrow and the target. But little by little, the arrow becomes the continuation of his hand, and he feels the focal contact of the target. The junction of the arrow to the target becomes then intelligible. Between the first clumsy archery and the mastery of the archery, the control of the action changes. There is a switch from a direct control, where each element of the action is felt, to a dual control, where only the aim of the target is consciously present (attended to), the other elements of the action being relegated to the periphery of the attention (Polanyi, 1969). These two levels are not qualitatively merged together, and the schematisation of their reciprocal reports is more difficult when a part of the action becomes tacit (non transparent).

The objective aim (of the object) organises into two levels of conscience: one focal, relative to the sought-after purpose (focal awareness), and the other, peripheral, relative to the movement (subsidiary awareness). In the autonomous action, the awareness is focused on the objective to reach, by hiding the movement to realise in the background (Polanyi, 1969). Components of the action are indeed not obtained neither by the exterior observation, nor by the self-analysis. Thus an archer that would displaces his attention from the target, to place it on the movement of its hands, would disturb his archery and would lack his target. If we could succeed to list the totality of elementary components of the action, then we would be not able to describe the global finality. The anticipation is therefore linked to a dualistic organisation of the intentional control of the action: the awareness is awareness of something.

1.3 Differentiation of a Self and a No-Self

The global finality of the living being can be described in term of a self-determinism (Bertalanffy, 1961, Varela, 1989). If one considers a predator P ingesting a prey p, the digestion can be seen as a return $(P + p) \Rightarrow (P)$. In the same way, the injured predator is susceptible to regenerate a missing part m by a process $(P - m \Rightarrow (P)$. In this framework, p or m are determined by P itself. The predator P possesses, by integrating elementary causalities of the action, its self-determinism. However, to admit a biological self-determinism of the predator P, it is necessary to consider that despite a lesion m, it is P that determines the material regeneration. But materially P no longer exists, because it transforms in $(P - m)$. There is no physical coexistence of $(P - m)$ and m. Therefore if the ideal determinism is circular, the biological determinism is temporal. Any regulation based on a physical stabilisation supposes a temporal determinism in order to exert it (Pichot, 1980). Confusing the two levels leads to consider that the regulations are instantaneous, and to accept that living beings are immutable. It is ignoring biological processes such as the development, the adaptation, the accommodation, the reproduction, the sickness, the ageing, the death. In taking into account the two levels, it is accepted that the living being (for example the predator P) could act in an anticipated manner, in front of the movement of its environment (for example the prey p).

Its identity as a living being is not defined in an absolute manner but in connection with an external middle, in seeking furthermore to distinguish itself from its environment. The living being maintains, with its material environment, relationships that follow physico-chemical laws, but it distinguishes itself by the conquest of a biological autonomy. To connect totally the living being to its physical environment, it is necessary to ascend until the origin of its biological evolution (phylogeny, ontogeny). The living being has indeed evolved from a primitive continuity with its environment, in the prebiotic middle, but in a different way (Pichot, 1993). The constitution of the living being cannot be defined without considering the movement by which its "self" and its "no-self" are permanently separated and connected. The individual deals with its external middle to establish its internal coherence, but it takes it in account only to distinguish itself by the action. The relationship/separation between the living being and its environment leans both on an internal action (to adapt it), and on an external action (to adapt its middle). The importance of the action for the living being, holds in the fact that it exists by the means of its self-constituent activity, "connected to" and "distinct of" a no-self (Pichot, 1980).

2 Self-Constitution of the Living Being

The recognition of the natural environment, and the constitution of the living being, make more use of an actual experience, than a logic, of the difference. The experience of the difference is here the experience that the individual possesses on the natural laws. The importance of experience, in the biological knowledge, makes Canguilhem to say: "We suspect that, to make mathematics, it would suffice to us to be angels. But to make the biology, even with the assistance of intelligence, we need to feel us beasts" (Canguilhem, 1989).

2.1 Physical Experience

The research of the autonomy, that guides the living being, stakes an experience of construction of an external middle, structured within its geographical environment. Uexküll has very well described this construction of an external middle (umwelt) from a geographical environment (umgebung), by the biological being (Uexküll, 1972).Thus the tick has built an umwelt constituted by three signals: the odour of the butyric acid; the heat; and the texture of the skin. This external middle denotes a umgebung relative to a mammal possessing hair, and hairless zones. This representation allows the female tick to choose a host, on which to leave to fall, when it is suspended to a branch of tree, after a coupling. Elements of the environment that act on the subject are indeed endowed of a certain organisation, but this organisation has undergone a complex analysis to be known by the subject.

Objects of the environment are thus constructed (reconstructed) by the subject. A cube, for example, shows one, two or three faces to the observer. The global perception of the cube necessitating a movement of rotation of the subject around the object. It is necessary to be able to integrate a multitude of points of views, to reach the universal form (Husserl, 1950). For the subject, the constitution of the world (umwelt), and simultaneously his self-constitution, stakes a cognitive, sensory and motive experience. The living being "knows" his external middle, "knows" himself (in the face of the former), and "knows" consequently the physical reality, by his physical action (Pichot, 1991). However the knowledge of the physical world (external middle), and therefore the living being himself, are incomplete and unfinished. The living being lives in the incompletion and not in the perfection. The unfinished constitution of the objective world by the subject, cannot be dissociated of the unfinished constitution of the subject himself.

2.2 Spatial Experience

The unfinished experience of the physical world allows the subject to access to a mental experience of the spatiality (Pichot, 1991). The movement gives access to an internal spatiality (perception), and to a external spatiality (motility). The internal spatiality of the subject concerns the form of his body, his size, his volume, etc. The external spatiality concerns his co-ordination, his manner to act on the objective world. The spatiality of the body is a frame that allows objects to be present to the subject, identical to them, even in the movement (time and space). This spatiality relates to a universal structure, but at the same time, it leans on the motive co-ordination of the subject. The representation of the movement of the object (and the subject) and the modification of the image of the object in the movement, imply a nervous computation. But, it is necessary to distinguish the nervous representations which are computed from the mental presence which is immediate. The space, as a continuous extent, is an immediate intuition. And the action to which this intuition is linked, is computed in its sensitive and motive parameters.

At one hand, the space is immediately present to the subject, and on the other hand, it is not immediately present to the nervous system. There is no contradiction, if we consider that the space is not the result of an "a posteriori" computation for the movement, but of a preliminary to the former. Indeed, the appearance of the image of the object in the conscience of the subject appears before the former has realised the totality of the necessary movement for the prehension of the object. Then the data with a sense are directly integrated, when the object is perceived, by the motive co-ordination. The action, in following the "fissures" of the physical world, plays the role of a developer of the spatial structure and allows a spatial presence of this structure. This structure is not the result of a computation, it is inherent to the space, and to the physical action. The subject who walks has conscience of the balance/unbalance, but it is incapable to explain how he reaches this.

2.3 Symbolic Experience

The perceptive action does not suffice to constitute a world (and correlatively a subject) flawless objective with which it would be in continuity (Pichot, 1991). Absent objects, that would allow to perfect the perceived world, are constituted in desired objects. Meaningful values, marked by the characteristic of the desire, are attributed to these objects (first meaning). The distribution of these values, puts some objects in the forefront of the conscience, relegates others to the rear plan. Some objects are positively connoted: they are parts of the external middle that the subject tries to constitute. Other objects are connoted negatively, because they constitute only a part of this external middle. This distribution creates a residue, irreducible by the action, which is responsible of the incompletion of the world (and of the subject). This incompletion is linked in the fact that a part of the lack can not be reduced and objectivated.

The primary meanings of these desired objects can be refined, by adding different connotations that remain connected directly to the subject. These intuitive meanings notice an hermeneutic thought directly anchored in the experience of the subject. Values can be transferred from an object to the other: for example, from the meat to the bell (Pavlov). But they can also be transferred in words when the language appeared. The logical-linguistic thought intervenes then to refine primitive meanings directly elaborated by the contact to the world. These intuitive meanings play then a role of roots, and allow the blooming of a system of discursive meanings, like a tree. The language serves thus to establish the finer meanings, constituent of semantic branches. These meanings are secondary not only because they appeared after, but also because they are irrigated by primary meanings (Petitot, 1992). The language, more than other movements of the body, cannot be reduced to a pure combinatory action. Underlying to the syntactic combination there is an hermeneutic thought (intuitive roots). The language, and laws of the grammar, channel intuitive meanings, but did not suffice to make sensible sentences. This is due to the fact that, at the differences of some qualities, meanings do not qualify the objects but qualify the existence of the object for the subject.

3 Modelling of Anticipation and Catastrophe Theory

We will use a dynamic structure involving the catastrophe theory, to model an anticipatory process. This structure must take into account the duality of the living being, the substance which is a material organisation, and the goal, which is a relational abstraction (Thom, 1977).

3.1 General Modelling Specifications

The general specifications of the system S suppose that:

(a) The activity is represented by a biophysical flow X of a functional space χ. This process remains implicit because the total activity is too complicated.

(b) The internal process X defines globally a set of internal states of S: (A, B, C, ...). These states are competing, and only one of them can become the actual state. The other states remain virtual.

(c) The system S is controlled by many control parameters that make up a space of production factors. We call them the external space W in contrast to the internal space χ of process X. We use a continuous field $\sigma: W \rightarrow \chi$ that combines $w = (u_1,\dots,u_r) \in W$ of the control space, with the process X_w of the internal space.

A system $S = (W, \chi, \sigma)$, occupying an internal state A., is perceived by the sensory qualities $q^1,\dots, q^k$ associated to A. In other words, the internal process X_w, appears through physical qualities q_w^i. When the control W varies continuously, the internal state (and so the qualities) varies continuously. This variation is not always significant, and can remain qualitatively unvarying. This situation corresponds to variations of the production factors that do not modify the expression of a given state. However a little variation of these control parameters could also cause a sudden change of one sensory quality q_w^i, that expresses a change of an internal state. This set of critical points, that causes a switch of system perception, is called catastrophe set K_w

Let A_w be an internal state of the system, and let I be a selector agent that brings it up to date. This state A_w is in contact, by reciprocal determining, with other states B_w, C_w, *etc.* The agent I decides, according to the hypothesis (b), the states that remain virtual. Let us suppose that the control w covers a path γ in W, X_w - and consequently the structure A_w - are subject to a deformation along γ. It can happen that A_w does not fulfil any more the condition of selection imposed by I, after a crossing of a critical value w_i. According the hypothesis (c), the system bifurcates then spontaneously from A_w towards a new actual state (until then virtual) B_w. The destabilisation of the internal states (due to the agent I), comes with the change of the control w, and induces a set of qualitative discontinuities K_w in the external space W

According to this general model, there is index ($w = (u_1,\dots,u_r)$) that controls the object (or phenomenon) qualities ($q^1,\dots, q^k$) (Thom, 1988; Petitot, 1992). So the notion of distinctive features of an object (or a phenomenon) must be reviewed. It is necessary to distinguish the external features linked to the index, and the internal features that are qualitatively unvarying. The first one changes continuously (for example acidity, or temperature), as the second one changes suddenly (for example the elasticity). This supposes the use of two phenomenological types of oppositions:

(i) competition of two unvarying states, which is a qualitative opposition corresponding to a conflict catastrophe;

(ii) appearance/disappearance of an unvarying state, which is a privative opposition corresponding to a bifurcation catastrophe.

3.2. Modelling of the Predation

The capture of a prey by the predator is a good example of biological activity implementing an anticipatory system. The dynamics of the predation can be described by an attraction with regard to the prey (Thom, 1990). After the capture of the prey, the activity of the predator fall down (Figure 1). His satiety rises a threshold value K. The research of a prey starts again when the satiety of the predator rises another threshold value J. Satiety controls the activity level by the means of an hysteresis cycle (capture cycle). Satiety v is a normal factor influencing the predation activity. When the predator is satiated, it falls asleep, its alertness falls down. After a sleep, the predator P is waking up hungry again. The alertness u is named the splitting bias factor.

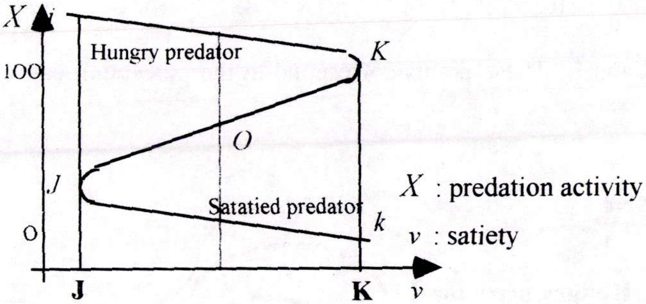

Figure 1 : The capture cycle. Satiety controls the predation activity by means of an hysteresis cycle.

René Thom (1977) showed how to use the cusp catastrophe to model predation (Figure 2). The predation activity can be defined by a potential

(1) $f_{u,v}(x) = x^4\ 4 + ux^2\ 2 + vx$

To represent the graph V of this potential $f_{u,v}(x)$, given by the equation

(2) $f_{u,v}(x) - y = 0$

we need a R^4 space (x, u, v, y).

But we can study the projection Σ in the space control W of the critical locus of V, defined by the catastrophe application $\chi : \Sigma \to W$. Σ is defined as the set of critical points of $f_{u,v}$, and therefore the surface of equation

(3) $f'_{u,v}(x) = x^3 + ux + v = 0$

We can remark that this equation becomes the one of the total graph of the fold singularity x^3 if we change v in $-y$. Σ is a cusp surface, which became the name of the graph of x^4. The circle G around O of equation

(4) $u^2 + v^2 = 1$

intersects the bifurcation locus, the semi-cubic parabola of equation

(5) $4u^2 + 27v^2 = 0$

in two points J and K. These points correspond to the perception catastrophe, and the capture catastrophe.

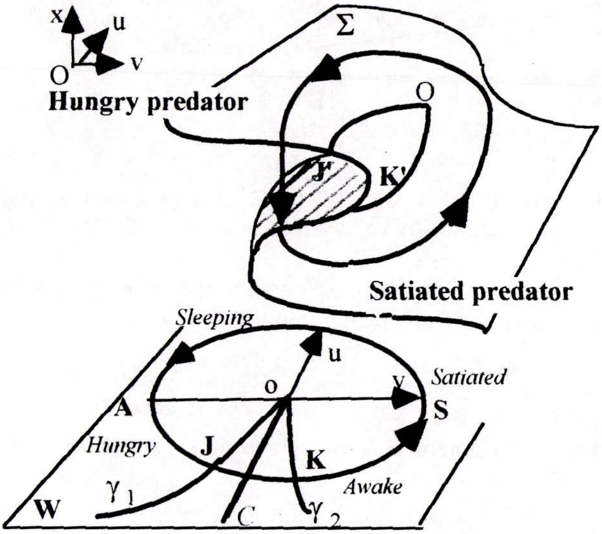

Figure 2 : Predation cusp.

If we interpret the semi-circle SA (v > 0), as the sleep period, the other semi-circle is the awake period (Figure 3) (Thom, 1990). After a sleep, to the left of J, the predator is alienated by the image $p°$ of its prey $(P + p°)$. If some external prey p arrives on the scene, it is recognised as such and located by P, then alienation ceased. The predator P jumps to a new equilibrium point which had appeared in J. Then the prey p will occupy the bottom of the minimum basin. Then during the passage from J to K, the values of the two minima will equalise, and in K the basin P, situated at the lowest point captures the vanishing basin of the prey p. The motor activity of pursuit, ending in capture, is triggered off. The predator jumps in K to a new equilibrium point.

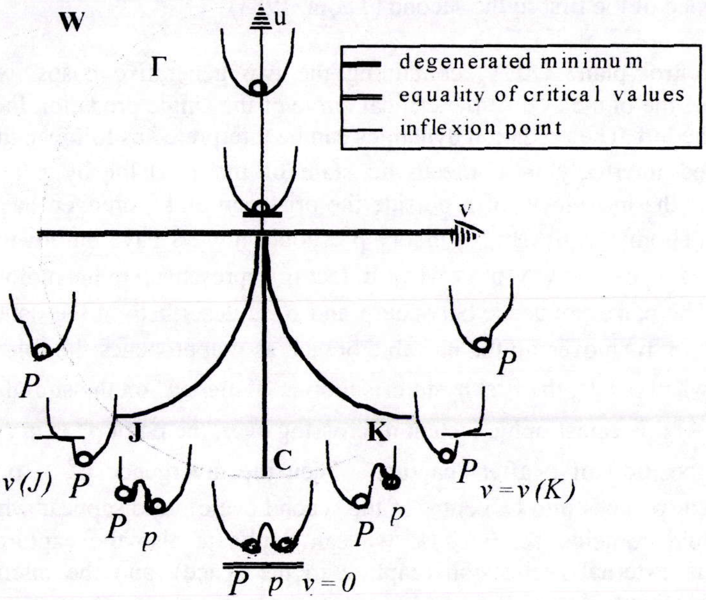

Figure 3 : Analysis of the potential minima in the predation loop.

3.3. Duplication of the Hysteresis Loop

There is no difficulty in interpreting the capture catastrophe in K, but it is more difficult to resolve the problem of the perception in J. During the bimodal period corresponding to the segment JK, the predator is not really in some point of the stable branch of the critical curve

$$(6)\qquad v + x^3 - x = 0$$

corresponding to $u = -1$, but rather in the centre o (Figure 1).

We can interpret this as follows: the activity level of the predator decreases to $x(o)$ after the recognition, and before the capture, of the prey (Thom, 1990). The capture of prey in space has to be preceded by an indispensable stage: the recognition and location of external prey p. So we will need to place a preliminary cycle denoted RrJ_1j (perception or recognition cycle) on the left of JjKk which is properly speaking the hysteresis loop of capture (motor or capture cycle) (Figure 4). The point R represents the predation "awakening". When the predator *P* wakes up, he is famished and can think of nothing else but "breakfasting" on an external prey. But the prey must first be found, and so it has to look for this. The period of searching is represented by the arc RJ, organised by a centre r (awakening), and the motor period by the segment JK. These two cycles are concatenated according to the rule of co-fold coincidence, this means that there is a subservience of the first to the second (Thom, 1988).

In the control plane Ou v, *containing* the two generative cusps, we shall led to "duplicate" the branch OJ of the critical curve of the single predation loop, by adding a cusp on the left. The predation dynamics can be interpreted as follows: the real prey p is represented internally in a metabolic state of the predator by p°; In a sense p° anticipates the movement of p outside the organism and moreover - as we shall see - inside it (Thom, 1990). But in theory p° should always have an advance on p in the direction of a positive v: $v(p^\circ) > v(p)$. In fact p° represents p in his motor and metabolic activity. The correspondence between p and p° is described on the concatenated cycle in the figure: p° moves up the unstable branch as p approaches the fold point J_1 of the fall. But when $p = J_1$, the first hysteresis loop is "flattened" on the straight line jJ_1 of the co-folds. At this catastrophic instant of crossing over, the point p° is in J_1 and coincides with the position of p after "capture". Then the divergence $p^\circ > p$ is at once re-established (p° goes into O, centre of the second cycle), to disappear when it meets the next co-fold coincidence. Because we can duplicate also the capture cycle in two cycles: the external prehension (capture in the space), and the internal prehension (digestion) (Figure 4).

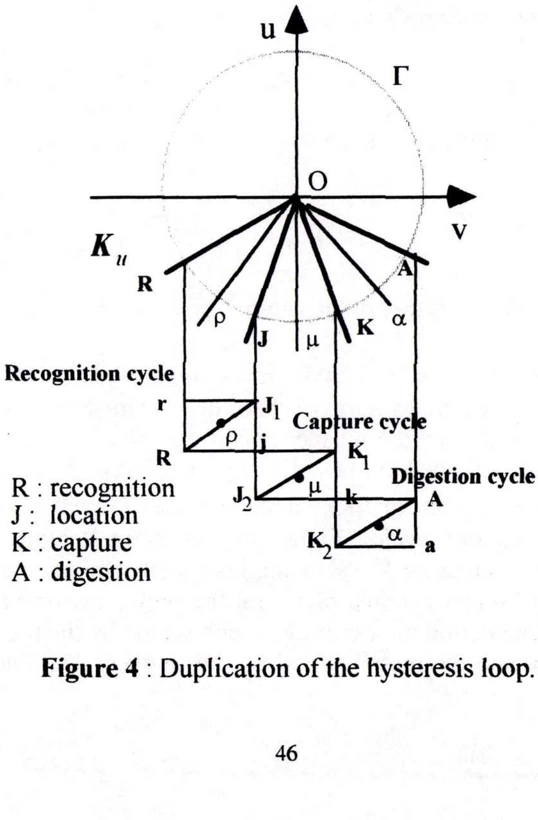

Figure 4 : Duplication of the hysteresis loop.

Thom (1988) said that this geometry could be paraphrased in control terms: it is those catastrophic moments when the state of the prey in relation to the predator changes abruptly that command the sudden jumps of p° anticipating the movement p. In other words, the passage of the prey p through these catastrophic instants (recognition, location, capture in space, digestion in the stomach) acts as an opening pre-program for the outflow of the predator system p°. Anatomically, this is controlled in vertebrates by the nervous system (central for the external part of the prey's trajectory, vegetative for the internal part, its digestion). The quasi-synchronous evolution of p and p° throughout the physiological blastula will constitute the global "field" of alimentation (the "chreod" of capture and assimilation of prey). The projection $p \rightarrow p^\circ$ plays, with respect to the prey, the same role as the parameterised segment in the Van der Pol theory (Thom, 1977).

4 Hyperincursive Theory of Cusp Catastrophes

Let us consider the following hyperincursive relation

$$(7) \qquad y(t+\Delta t) = (r-s)/2 + (r-s)^2/2 + [1-(r-s)^2]\,[3-2y(t+\Delta t)]\,y(t+\Delta t)^2$$

This relation is hyperincursive (Dubois, 1998) because the value of $y(t+\Delta t)$ at time $t+\Delta t$ is defined as a function of itself $y(t+\Delta t)$ at the same time $t+\Delta t$. For defined values of the control parameters r and s, there can exist several values of $y(t+\Delta t)$.

The following Table I gives the values of $y(t+\Delta t)$ for a few integer values of $r(t)$ and $s(t)$:

TABLE I

r(t)	s(t)	y(t+Δt)
0	0	0
0	0	1/2
0	0	1
0	1	0
1	0	1
1	1	0
1	1	1/2
1	1	1

For the same values of $r = s = 0$ and $r = s = 1$, there are three values for $y(t+\Delta t)=0$, $y(t+\Delta t) = 1/2$, $y(t+\Delta t) = 1$, and for $r = 0$, $s= 1$, there is one value of $y(t+\Delta t)=0$, and for $r = 1$, $s = 0$, there is also one value for $y(t+\Delta t) = 1$. Thus, the relation (7) represents a cusp in the framework of the catastrophe theory.

Let us remark that the relation (7) is a true catastrophe equation for $\Delta t=0$, as follows

$$(8)\qquad C(y(t), r, s) = -y(t) + (r-s)/2 + (r-s)^2/2 + [1-(r-s)^2]\,[3-2y]\,y(t)^2 = 0$$

that is a logical time-independent system. This relation is not algorithmic, that is to say not recursive: this is a non-computable relational function.

But it is possible to transform such an hyperincursive relation to a hyper recursive equation in introducing an internal time τ in the following way

$$(9)\qquad y(t+\Delta t, \tau+\Delta\tau) = (r-s)/2 + (r-s)^2/2 + [1-(r-s)^2]\,[3-2y(t+\Delta t, \tau)]\,y(t+\Delta t, \tau)^2$$

where $\Delta\tau = \Delta t/n$, n being the number of time steps to define the time duration Δt.
The time t is a stroboscopic external time of duration Δt, and τ is a computational internal time of duration $\Delta\tau$.
From outside the system, the successive values of y are: $y(t)$, $y(t+\Delta t)$, $y(t+2\Delta t)$, etc.
Inside the system, the successive values of y during the external duration Δt are: $y(t, \tau+\Delta\tau)$, $y(t, \tau+2\Delta\tau)$, $y(t, \tau+3\Delta\tau)$, ... , $y(t, \tau+n\Delta\tau) = y(t+\Delta t, \tau)$.

The figures 5abcd give the numerical simulation of eq. (9).

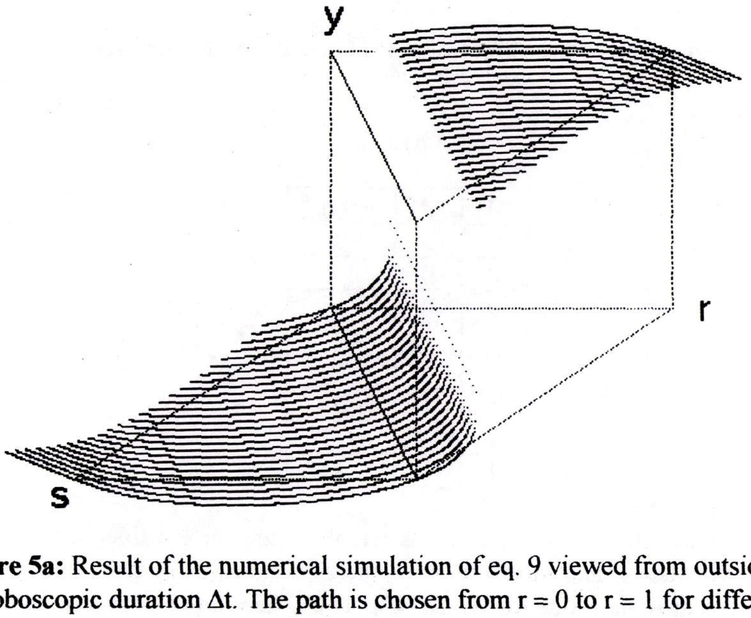

Figure 5a: Result of the numerical simulation of eq. 9 viewed from outside the system at stroboscopic duration Δt. The path is chosen from $r = 0$ to $r = 1$ for different values of s between $s = 0$ and $s = 1$.

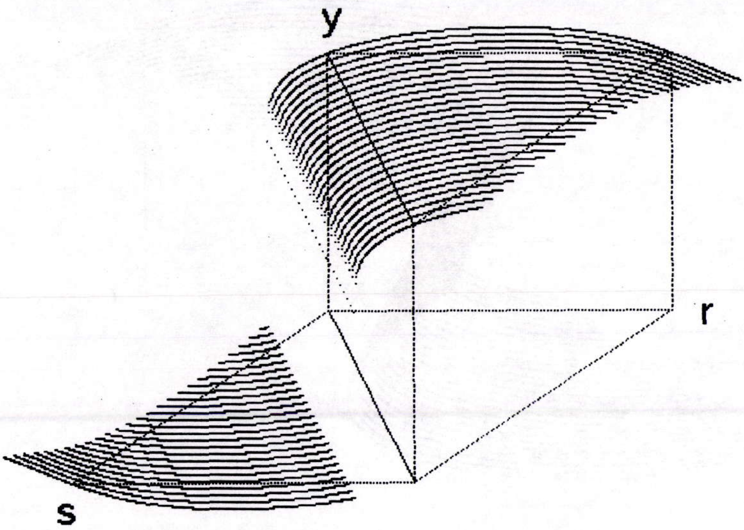

Figure 5b: The path is chosen from r = 1 to r = 0 for s between s = 0 and s = 1.

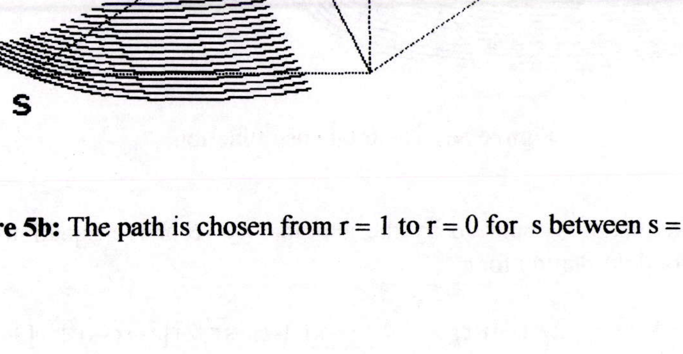

Figure 5c: The unstable part of the cusp.

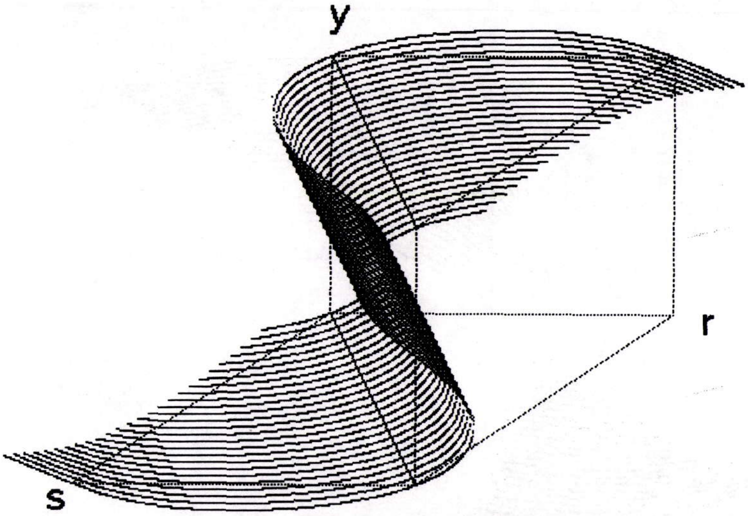

Figure 5d: The total cusp function.

In Figure 5c, the eq. 9 is replaced by the anticipatory recursive equation in using a backward temporal derivation for τ

(9a) $y(t+\Delta t,\tau+\Delta\tau) = 2y(t+\Delta t,\tau) - [\ (r-s)/2+(r-s)^2/2+[1-(r-s)^2]\ [3-2y(t+\Delta t,\tau)]\ y(t+\Delta t,\tau)^2\ ]$

The unstable part of the cusp becomes stable and the two stable parts become unstable. So in using anticipation, a system can reach unstable states.

There is an other way to compute the cusp from the catastrophe $C(y(t), r, s) = 0$, given by eq. 8, in embedding it in a recursive loop as follows

(8a) $y(t+\Delta t/n) = y(t) + (\Delta t/n)C(y(t), r, s)$

or

(8a') $y(t+\Delta t/n) = y(t) + (\Delta t/n)\ [-y\ (t)+ (r-s)/2 + (r-s)^2/2 + [1-(r-s)^2]\ [3-2y]\ y(t)^2]$

in choosing a number of recursive loops n such that at the nth step, $C(x(t), r, s) = 0$, so that $y(t+\Delta t) = y(t)$.

Figure 6 gives the numerical simulation of eq. 8a' : $y(t+\Delta t)$ for $(r - s) = -1$ to $+1$.

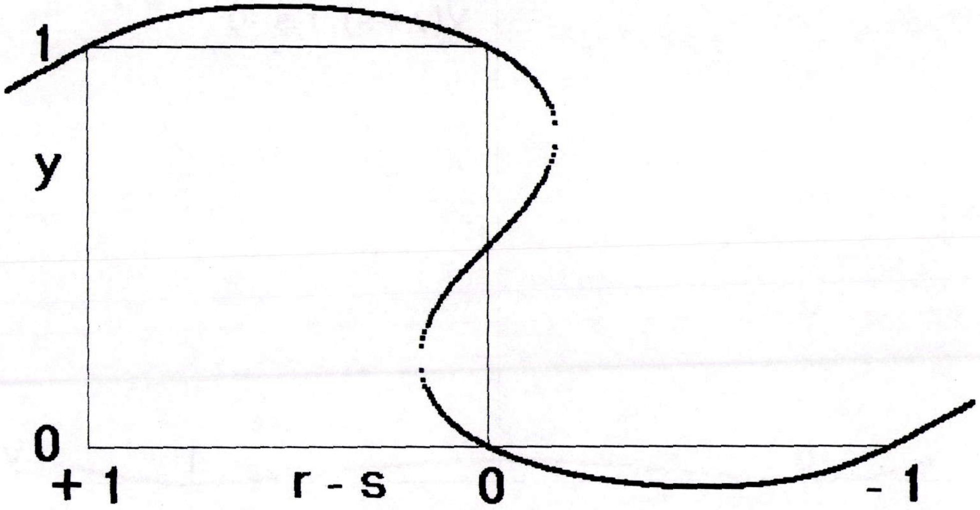

Figure 6: Numerical simulation of eq. 8a' and eq. 8a".

The unstable part of the cusp is computed from

$$(8a'') \quad y(t+\Delta t/n) = y(t) - (\Delta t/n)\,[-y(t) + (r-s)/2 + (r-s)^2/2 + [1-(r-s)^2]\,[3-2y]\,y(t)^2]$$

A potential V(y, r – s) can be obtained in integrating by y the equation 8 as follows

$$(8b) \quad V(y, r, s) = -\,[-y^2/2 + (r-s)y/2 + (r-s)^2/2 + [1-(r-s)^2]\,[1-y/2]\,y(t)^3\,] + V_0$$

where V_0 is an arbitrary constant potential and for which

$$(8c) \quad dV(y, r, s)/dy = -\,C(y, r, s)$$

The extrema of the potential correspond to C(y, r, s) = 0, the solutions are stable for minimum and unstable for maximum. Figure 7a is the potential V as a function of y for r – s = 0. There are two mimima corresponding to two stable solutions at y = 0 and y = 1. There is an unstable solution for y = 1/2. Figure 7b is the potential V as a function of y for r – s = – 1. There is one mimimum corresponding to one stable solution at y = 0. Figure 7c is the potential V as a function of y for r – s = + 1. There is one mimimum corresponding to one stable solution at y = 1.

Classically, the catastrophe is derived from a potential and so, only the external observer is taking into account. Our hyperincursive approach gives a dual description of catastrophes in taking into account both the internal and external observers.

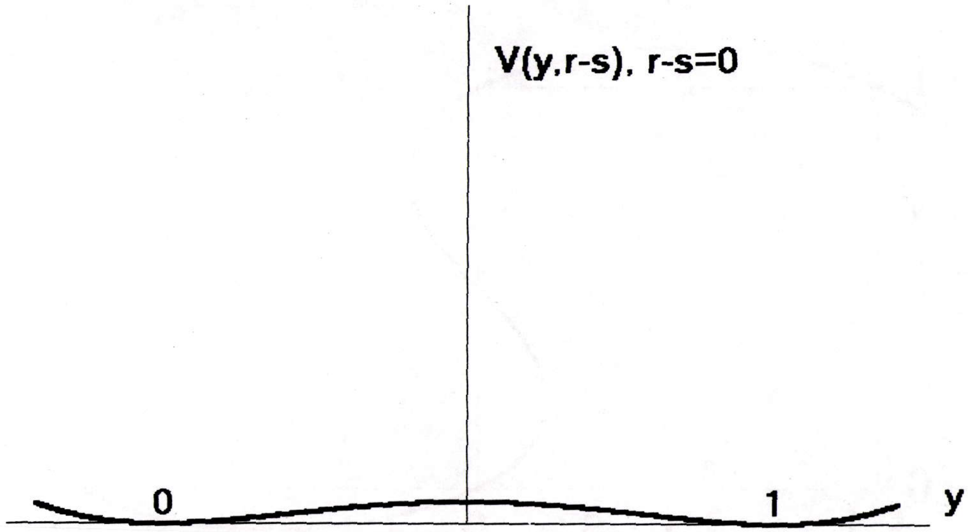

Figure 7a: Potential V as a function of y for r – s = 0.
There are two mimima corresponding to two stable solutions at y = 0 and y = 1.
There is an unstable solution for y = 1/2.

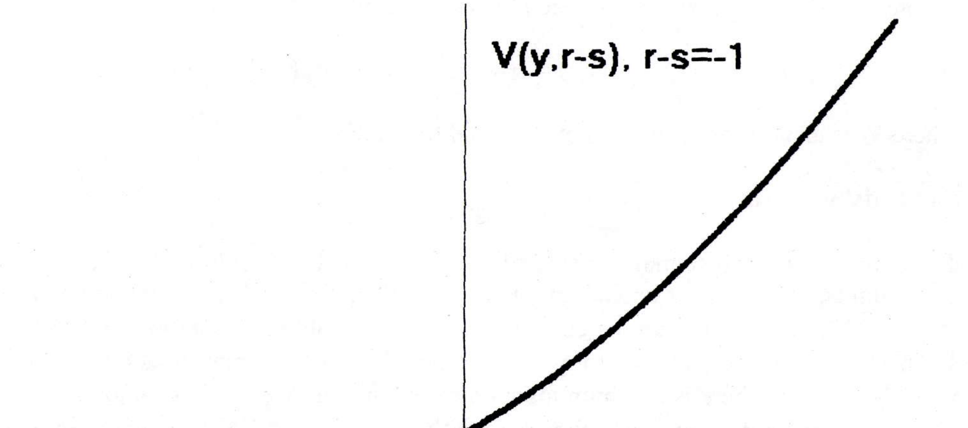

Figure 7b: Potential V as a function of y for r – s = – 1.
There is one mimimum corresponding to one stable solution at y = 0.

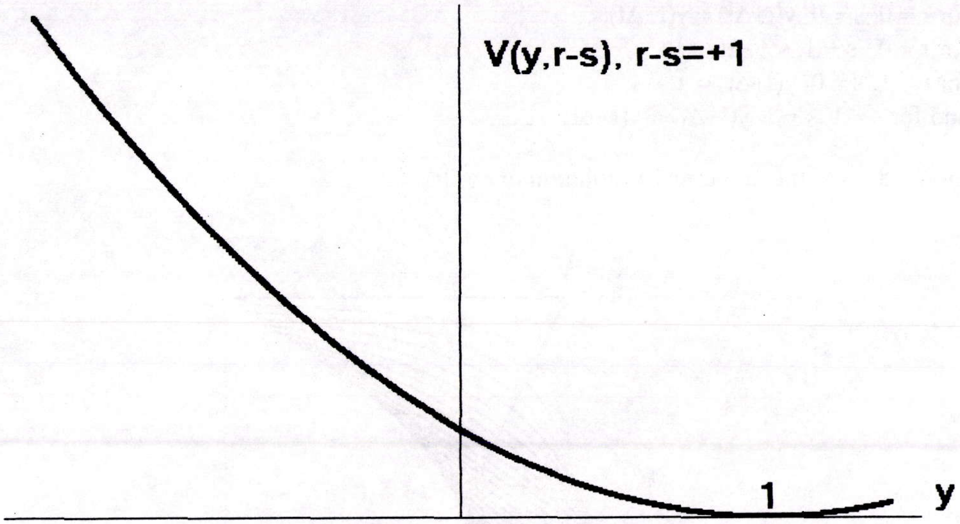

Figure 7c: Potential V as a function of y for r – s = + 1.
There is one mimimum corresponding to one stable solution at y = 1.

In the three solutions of the cusp, there are two stable solutions around y = 0 and y = 1 and one unstable solution around y = 1/2. So in the Table I, we can suppress the unstable solution to obtain the Table II:

TABLE II

r(t)	s(t)	y(t+Δt)
0	0	0
0	0	1
0	1	0
1	0	1
1	1	0
1	1	1

This Table II can be considered as a Truth Table or a Boolean Table because all the values of r, s and y are in the set of values {0,1}.

The relation 7 can be transformed into an algebraic linear relation in knowing that, for Boolean values, $x^2 = x$:

$$(10) \quad y(t+\Delta t) = r(1-s) + (1-r-s+2sr)]\, y(t+\Delta t)$$

So,
for $r = 0$, $s = 0$, $y(t+\Delta t) = y(t+\Delta t)$;
for $r = 0$, $s = 1$, $y(t+\Delta t) = 0$;
for $r = 1$, $s = 0$, $y(t+\Delta t) = 1$;
and for $r = 1$, $s = 1$, $y(t+\Delta t) = y(t+\Delta t)$.

Figure 8 gives the numerical simulation of eq. 10.

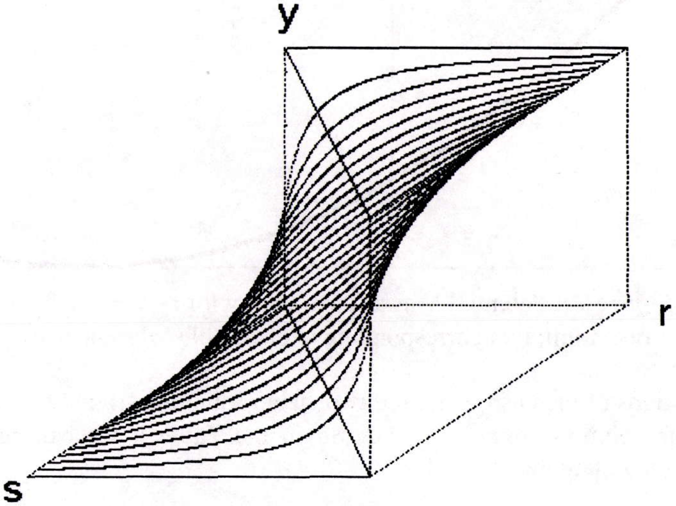

Figure 8: Simulation of the hyperincursive linear equation 10.
This is a digital cusp because for $r = s = 0$ and $r = s = 1$, $y = y$, two catastrophe paths.
For $r = 0$ and $s = 1$, $y = 0$, for $r = 1$ and $s = 0$, $y = 1$.
For the other values of r and s the paths from $y = 1$ to $y = 0$
and from $y = 0$ to $y = 1$ are smooth.

As shown by D. Dubois (1998, 1999), such an algebraic digital equation is equivalent to a Hyperincursive Heaviside Function

(11) $y(t+\Delta t) = \Gamma\, [\, r(t) - s(t) + y(t+\Delta t)\,]$

with the definition of a Heaviside Function:

(11a) $\Gamma(x) = 1$ for $x > 1$ and $\Gamma(x) = 0$ for $x \leq 0$.

Figure 9 gives the simulation of eq. 11.

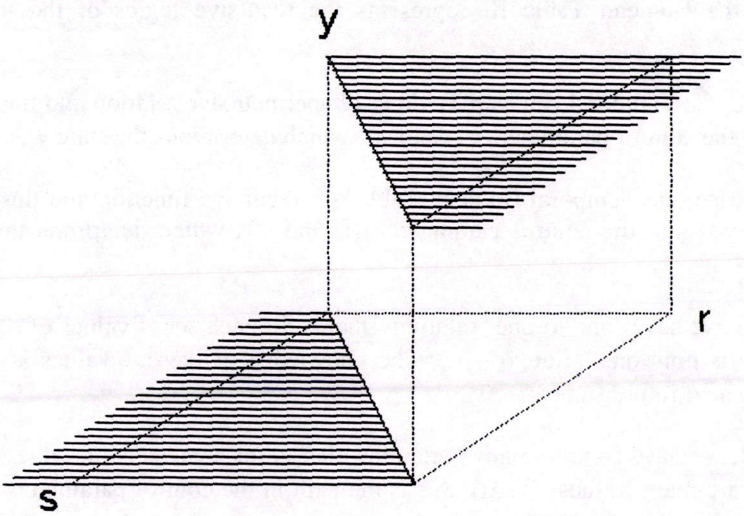

Figure 9: Simulation of eq. 11. This is a Hyperincursive Cusp.
For r – s = 0, there are two solutions y = 0 and y = 1.
For r – s > 0, there is one solution, y = 1 and for r – s < 0,
there is also one solution y = 0.

The hyperincursive equation 11 is not at all similar to the following recursive equation:

(12) $y(t+\Delta t) = \Gamma [r(t) - s(t) + y(t)]$

because, in this case, the Boolean table is no more identical to the Boolean Table II. Indeed, in this recursive equation, the different values of $y(t+\Delta t)$ are given in the following Table III:

TABLE III

r(t)	s(t)	y(t)	y(t+Δt)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

The Boolean Table II represents the stroboscopic hyperincursive logics of the external observer and the Boolean Table III represents the recursive logics of the internal observer.

In the first case, the Temporal Boolean Table is a hyperincursive relation and this is the path chosen in the control parameters $r(t)$ and $s(t)$ which determines the state $y(t+\Delta t)$.

In the second case, the Temporal Boolean Table is a recursive function and this is the knowledge of $y(t)$ and the control parameters $r(t)$ and $s(t)$ which determine the state $y(t+\Delta t)$.

In the Table III, we have "one-to-one" relations, that is for each set of values of $r(t)$, $s(t)$, and $y(t)$, there is only one value $y(t+\Delta t)$. The knowledge of these 3 values at time t determines the next future state $y(t+\Delta t)$.

In the Table II, we have "one-to-many" relations, that is for some sets of values of $r(t)$ and $s(t)$, there are many values $y(t+\Delta t)$: this is the path in the control parameters which determines which value of $y(t+\Delta t)$ is obtained.
The knowledge of these values at time t determines the value of $y(t+\Delta t)$ for $r(t) \neq s(t)$.
Uncertainties exist even if two successive sets of values of r and s are known: for example, $r(t) - s(t) = 0$ followed by $r(t+\Delta t) - s(t+\Delta t) = 0$ doesn't permit to anticipate $y(t+2\Delta t)$! In choosing these critical values of r and s, we can anticipate nothing about the state y without a knowledge of the initial value of y, because the external observer has no access to the internal dynamics of the system!

The conclusion which arises from this deep difference between the two descriptions is very important from methodological and epistemological points of view. The recursive description can be related to the Newtonian Mechanics, based on initial conditions, and the hyperincursive description can be related to the concept of path, after Richard Feynman, in Quantum Mechanics.

D. Dubois (1999) introduced what he called a Heaviside Cusp Function, noted $\Gamma^*(x)$, for which

$$(13) \quad \Gamma^*(x) = 1 \text{ for } x \geq 0 \text{ and } \Gamma^*(x) = 0 \text{ for } x \leq 0$$

which means that for $x = 0$ there are two values of the Heaviside function, 0 or 1, which is a hyperincursive property.

So eq. 11 can be described by such a Heaviside Cusp Function as

$$(14) \quad y(t+\Delta t) = \Gamma^* [\, r(t) - s(t) \,]$$

So the hyperincursive eq. 11 is transformed into a hyper recursive relation 14. The hyperincursive property of eq. 11 is then hidden in the relational property of the function Γ^*.

As described in D. Dubois (1998, 1999), such eqs. 11 and 14 represent an elementary flip-flop memory as used in computers, and also represent a neuronal memory in the framework of the McCulloch and Pitts formal neuron theory.

Conclusion

In a naturalist approach, anticipation of the prey capture can be viewed as the existence of an internal state p°, representing the real prey p, in the metabolism of the predator. p° have an advance on p in the direction of the positive v parameter (satiety). The capture dynamics, is mathematically the product of the deformation of a cycle resulting from a Hopf bifurcation into a hysteresis loop (predation loop). The variable v, initially a pure control variable, ends up acquiring slow dynamics dependant on a potential whose energy must proceed, through partition, from the initial potential V. This is the process of "internalisation" of an external variable which was suggested to Thom (1977) by the formation of mesoderm, from ectoderm, in vertebrate embryology. But in order to attain something like a global vision of physiological regulation, we can complicate this basic structure by a succession of new folds affecting the characteristic of the initial cusp. The duplication of the hysteresis loop generates an anticipative process with many loops subservient by other (recognition, capture, digestion, and so on).

In this framework, the appropriation of an instrument can be viewed as the duplication of the capture loop. The hungry monkey, who had experienced a feeling of frustration, when he had not succeeded to reach a banana with his hand, transforms his regulatory structure by complicating it. The segment JK develops an inflection that subsequently unfolds in two concatenated cycles: capture of a stick, capture of the prey. The hungry predator is now alienated by the image of the stick. The means becomes an end in themselves. The end, to take the banana, must be set aside in order to concentrate on the means of getting there. The tool is thus invested by a pregnance which was initially attached to the prey. Close in close the primitive pregnances distribute, and contribute to structure semantically an environment constituted of objects, tools, and linguistic symbols. The totality of these meanings intervene directly in the dynamics of anticipatory systems.

In this paper, a new interpretation of the catastrophe theory is given in the framework of hyperincursion: a hyperincursive function is an extension of the recursive function in which the state y(t+1) at time t+1 is computed from a function of itself

$$y(t+1) = f([\, y(t+1), r(t), s(t)]$$

where r(t) and s(t) are control parameters defined at the time t.

A Hyperincursive Cusp Function can be modelled by a Heaviside Cusp Function

$$y(t+1) = \Gamma\ [r(t) - s(t) + y(t+1)]$$

A Hyperincursive Boolean Table can be built and a hyperincursive algebraic linear function can model a cusp

$$y(t+1) = (1-r(t))(1-s(t))y(t+1) + r(t)(1-s) + r(t)s(t)y(t+1)$$

which represents an elementary flip-flop one bit memory.

A recursive process defines the successive states from its initial conditions and a hyperincursive process defines the successive states from the path chosen in the control parameters space. The recursive process is related to an internal observer and the hyperincursive process is related to an external observer.
Classically, the catastrophe is derived from a potential and so, only the external observer is taking into account.
Our hyperincursive approach gives a dual description of catastrophes in taking into account both the internal and external observers.

References

Bertalanffy (Von), L. (1961). Les problèmes de la vie. Paris: Gallimard.

Canguilhem, G. (1989). *La connaissance de la vie.* Paris : Vrin.

Collectif. (1978) *Aristotle on Mind and Senses.* VII Symposium Aristotelium. Lloyd G.E.R., Owen G.E.L. Cambridge: Cambridge University Press.

Dubois, D.M. (1998) Introduction to Computing Anticipatory Systems. *International Journal of Computing Anticipatory Systems.* Edited by Daniel M. Dubois, Published by CHAOS, Vol. 2, pp. 3-14.

Dubois D. M. (1998) *Computing Anticipatory Systems with Incursion and Hyperincursion*, Conference Proceedings CP 437, CASYS'97 - COMPUTING ANTICIPATORY SYSTEMS: CASYS -- First International Conference, Edited by Daniel M. Dubois, Published by The American Institute of Physics, pp. 3-29.

Dubois D. M. (1998) *Boolean Soft Computing by Non-linear Neural Networks with Hyperincursive Stack Memory*, Invited Paper, in Computational Intelligence: Soft Computing and Fuzzy-Neuro Integration with Applications, Edited by Okyay Kaynak, Lotfi A. Zadeh, Burhan Türksen, Imre J. Rudas, NATO ASI Series, Series F: Computer and Systems Sciences, vol. 162, Springer-Verlag, pp. 333-351.

Dubois D. M. (1999) *Hyperincursive McCulloch and Pitts Neurons for Designing a Computing Flip-Flop Memory*, Conference Proceedings CP 465, COMPUTING ANTICIPATORY SYSTEMS: CASYS'98 -- Second International Conference, Edited by Daniel M. Dubois, Published by The American Institute of Physics, pp. 3-21.

Goldstein, K. (1983) *La structure de l'organisme*. Paris: Gallimard, 285p.

Goodwin, B., Holder, N., Wyles, C. (1983) Development and Evolution. Cambridge: Cambridge University Press.

Husserl, E. (1950) *Idées directrices pour une phénoménologie*. Paris: Gallimard.

Petitot, J. (1992) *Physique du sens*. Paris: Edition CNRS, 450p.

Piaget, J. (1967) *Biologie et connaissance*. Paris: Gallimard, 509p.

Pichot, A. (1980) *Eléments pour une théorie de la biologie*. Paris : Maloine.

Pichot, A. (1993) *Histoire de la notion de vie*. Paris: Gallimard, 973p.

Polanyi, M. (1969) *Knowing and being*. Chicago: University Press.

Rosen, R. (1985) *Anticipatory Systems*. Pergamon Press: New-York.

Searle, J. (1983) *Intentionality. An essay of in the Philosophy of Mind*. Cambridge: Cambridge University Press.

Thom, R. (1977) *Stabilité Structurelle et Morphogénèse*. Paris : InterEdition.

Thom, R. (1988) *Esquìsse d'une Sémiophysique*. Paris: InterEdition.

Thom, R. (1990) *Apologie du logos*. Paris: Hachette, 664p.

Uexküll (Von), J. (1972) *Mondes animaux et mondes humains*. Paris: Denoël Gontier, 240p.

Varela, L. (1989). *Principles of biological autonomy*. Elsevier: New-York.