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KRASNOSELSKY-TYPE THEOREMS INVOLVING OUTWARD RAYS [1]

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1. INTRODUCTION.

We know that the kernel of a hunk $S \subseteq \mathbf{R}^n$ can be described as the intersection of the inner stems of its points of local nonconvexity (theorem 4.3 of [4]). Then it is natural to look for Krasnoselsky-type theorems to state the starshapedness of S by means of properties which involve subsets containing finite elements of the set. Helly's theorem needs to be applied to the sets that appear in the characterization, and these sets should be convex. Our problem now is that the inner stems of boundary points are not necessarily convex. To solve this, we prove what is called a "Krasnoselsky-type lemma" which consists in getting a new characterization of the kernel of the set as the intersection of the closures of the convex hull of the inner stems of its points of local nonconvexity. The planar case was solved by F. Toranzos (see [4]). Finally we obtain the Krasnoselsky-type theorems.

More formally, if $S \subseteq \mathbf{R}^n$ hunk then kerS = $\bigcap\limits_{t \in \mathrm{lnc}S} cl(conv(ins(t,S)))$. One of the inclusions is immediate, then we state the problem in the following way:

Let $S \subseteq \mathbf{R}^n$ be a hunk, $x \in S$. If $x \notin$ kerS, then there exists t a point of local nonconvexity of S such that $x \notin$ cl conv (ins(t,S)).

2.- BASIC DEFINITIONS AND NOTATIONS.

Unless otherwise stated all the points and sets considered here are included in $\mathbf{R}^n$ the real n-dimensional euclidean space. The interior, closure, boundary, complement and convex hull of a set S are denoted by: intS, clS, bdryS, CS and convS respectively. The open segment joining x and y is denoted (x,y). The substitution of one or both parentheses by square ones indicates the adjunction of the corresponding extremes.

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