A Real Analytic Schwartz’ Kernels Theorem

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Abstract

In this short paper, we study a few topological properties of the sheaf of real analytic functions on a real analytic manifold $M$. In particular, we show that its topological Poincaré-Verdier dual is the sheaf of hyperfunction densities on $M$. We also prove that if $N$ is a second real analytic manifold, then the continuous cohomological correspondences between the sheaf of real analytic functions on $M$ and the sheaf of hyperfunctions on $N$ are given by integral transforms whose kernels are hyperfunction forms on $M \times N$ of a suitable kind. This result may be viewed as a real analytic analogue of the well-known kernels theorem of Schwartz.

Introduction

Let $M$ be a real analytic manifold of dimension $m$ and let $X$ be a complexification of $M$. Denote $\omega_M$ the orientation sheaf of $M$ and $\mathcal{O}_X$ the sheaf of holomorphic functions on $X$. A classical pure codimensionality theorem due to M. Sato states that all the cohomology sheaves of the complex

$$\omega_M \otimes R\Gamma_M(\mathcal{O}_X)$$

vanish except for the $m$-th one. This non-vanishing cohomology sheaf is then defined to be the sheaf $\mathcal{B}_M$ of hyperfunctions on $M$.

This approach is at first glance completely different from the one followed by L. Schwartz to construct the sheaf of distributions on a smooth manifold. Recall that, if $M$ is a smooth manifold of dimension $m$, one defines the sheaf $\mathcal{D}b$ of distributions on $M$ by duality through the formula

$$\mathcal{D}b(U) = L(\Gamma_c(U; \omega_M \otimes C^\infty_M), \mathbb{C})$$

where $C^\infty_M$ is the sheaf of smooth $m$-forms on $M$ and $U$ is any open subset of $M$. 

395