A proper description of clumping in hot star winds: the key to obtaining reliable mass-loss rates?

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Abstract: Small-scale inhomogeneities, or ‘clumping’, in the winds of hot, massive stars are conventionally included in spectral analyses by assuming optically thin clumps. To reconcile investigations of different diagnostics using this microclumping technique, very low mass-loss rates must be invoked for O stars. Recently it has been suggested that by using the microclumping approximation one may actually drastically underestimate the mass-loss rates. Here we demonstrate this, present a new, improved description of clumpy winds, and show how corresponding models, in a combined UV and optical analysis, can alleviate discrepancies between previously derived rates and those predicted by the line-driven wind theory. Furthermore, we show that the structures obtained in time-dependent, radiation-hydrodynamic simulations of the intrinsic line-driven instability of such winds, which are the basis to our current understanding of clumping, in their present-day form seem unable to provide a fully self-consistent, simultaneous fit to both UV and optical lines. The reasons for this are discussed.

1 Introduction

The winds from hot, massive stars are described by the radiative line-driven wind theory (Castor, Abbott & Klein 1975), in which the standard model assumes the wind to be stationary, spherically symmetric, and homogeneous. Despite its apparent success (e.g., Vink, de Koter, & Lamers 2000), this model is probably oversimplified. In particular, much evidence for a time-dependent, small-scale inhomogeneous wind (that is, a ‘clumped’ wind) has over the past years accumulated, from the theoretical as well as the observational side (for an overview, see Hamann, Feldmeier & Oskinova 2008). In the following, we investigate and discuss the indirect evidence for wind clumping that has arisen from quantitative spectroscopy aiming to infer mass-loss rates from observations.

2 Deriving empirical mass-loss rates from hot star winds

The main mass-loss diagnostics of OB-star winds are UV resonance lines, H\textalpha line emission (and other recombination lines), and infra-red and radio continuum emission. Recently, X-ray emission lines have also been added to the set (e.g., Cohen et al. 2010). Here we will focus on resonance
lines and Hα, discussing the influence of optically thick clumping on these diagnostics and using the well-studied Galactic O supergiant λ Cep as a test bed.

2.1 Smooth wind models and microclumping

When smooth wind models are used, mass-loss rates inferred for a given star, but from different diagnostics, can vary substantially. For example, in an analysis by means of the unified NLTE model atmosphere code FASTWIND (Puls et al. 2005), we find that the phosphorus V (P V) UV resonance lines suggest a mass-loss rate approximately 20 times lower than the one required for a decent fit of the Hα emission. That is, depending on which diagnostic is used, the inferred mass-loss rate of λ Cep can vary by more than an order of magnitude. This inconsistency has been interpreted as a consequence of neglecting clumping when deriving these rates.

Wind clumping has traditionally been included in diagnostic tools by assuming statistically distributed optically thin clumps and a void inter-clump medium, while keeping a smooth velocity field. The main result of this microclumping approach is that mass-loss rates derived from smooth models and diagnostics that depend on the square of the density (such as Hα in OB-star winds) must be scaled down by the square root of the clumping factor, \( f_{\text{cl}}(r) \equiv \langle \rho^2 \rangle / \langle \rho \rangle^2 \) with the angle brackets denoting spatially averaged quantities. On the other hand, processes that depend linearly on the density (such as the UV resonance lines) are not directly affected by microclumping. Thus, with such an optically thin clump model, the only way to reconcile the above large deviations in rates would be to accept the very low mass-loss rate indicated by P V, and so assume very high clumping factors (in this case \( f_{\text{cl}} \approx 400! \)) to simultaneously reproduce Hα. Using this technique, we would derive a mass-loss rate for λ Cep that is roughly an order of magnitude lower than predicted by the line-driven wind theory (according to the mass-loss recipe in Vink et al. 2000). Similar results have been found by, e.g., Bouret, Lanz & Hillier (2005) and Fullerton, Massa & Prinja (2006).

Such low mass-loss rates would have enormous implications for massive star evolution and feedback, and of course also cast severe doubts on the validity of the line-driven wind theory. In addition, the extreme clumping factors we had to invoke to reproduce Hα are in stark contrast with theoretical predictions (e.g., Runacres & Owocki 2002, see also Fig. 2). A possible solution of this dilemma is that clumps are not optically thin for the diagnostic lines under consideration, which would lead to underestimated rates if microclumping were still assumed (Oskinova, Hamann & Feldmeier 2007; Owocki 2008; Sundqvist, Puls & Feldmeier 2010a).

2.2 Relaxing the microclumping approximation

Clump optical depths. The critical parameter determining the validity of the microclumping approximation is the clump optical depth (which of course is different for different diagnostics). Actually, for both resonance lines and Hα, we may quite easily obtain reasonable estimates for this quantity. In the following, all radii are given in units of the stellar radius and all velocities in units of the terminal speed. The radial clump optical depth for a resonance line may then, in the Sobolev approximation, be written as

\[
\tau_{\text{cl}}^{\text{res}} \approx \frac{\tau_{\text{sm}}^{\text{res}}}{f_{\text{vel}}} \approx \frac{\kappa_0 q f_{\text{cl}}}{r^2 \frac{d v}{d r}},
\]

where \( q \) is the ionization fraction of the considered ion and \( f_{\text{vel}} \) a velocity filling factor (Owocki 2008), defined as the ratio between the clump velocity span, \( \delta v \), to their velocity separation, in full analogy with the traditional volume filling factor. \( f_{\text{vel}} \) largely controls how a perturbed velocity field affects the line formation, and may be approximated by \( f_{\text{vel}} \approx |\delta v / \delta v_{\beta}| f_{\text{cl}}^{-1} \) (Sundqvist et al. 2010b), where \( \delta v_{\beta} \) is the clump velocity span of a corresponding model with a smooth velocity field. As seen from
the middle term in Eq. 1, \( f_{\text{vel}} \) also controls how the clump optical depth differs from the corresponding smooth one. \( \kappa_0 \) is a line-strength parameter proportional to the mass-loss rate and the abundance of the considered element, and taken to be constant within the wind (e.g., Puls, Vink & Najarro 2008). As an example, assuming a mass-loss rate \( 3.0 \times 10^{-6} \text{M}_\odot/\text{yr} \) and a solar phosphorus abundance, we get \( \kappa_0 \approx 3.0 \) for the blue component of P V in \( \lambda \) Cep. Note that Eq. 1 assumes that the clumps cover a complete resonance zone. This is reasonable for the wind’s inner parts, but may be questionable for its outer, slowly accelerating parts (see Sundqvist et al. 2010b). For our discussion here, however, a Sobolev treatment suffices. For H\( \alpha \), the analogy to \( \kappa_0 \) is the parameter \( A \) (Puls et al. 1996), which is proportional to the mass-loss rate squared and to the departure coefficient of the lower transition level. We may write the radial Sobolev clump optical depth for H\( \alpha \) as

\[
\tau_{\text{cl}} \approx \frac{f_{\text{cl}}}{f_{\text{vel}}} \approx A \frac{f_{\text{cl}}^2}{r^4 v^2 dr/dr}.
\]

Assuming unity departure coefficients, and a mass-loss rate as above, \( A \approx 1.5 \times 10^{-3} \) for H\( \alpha \) in \( \lambda \) Cep. Note the extra \( f_{\text{cl}} \) term in this expression, stemming from the \( \rho^2 \)-dependence of this diagnostic. In Fig 1, we plot the H\( \alpha \) and P V clump optical depths, using \( \kappa_0 \) and \( A \) as just given and \( f_{\text{cl}} = 9 \), which is a typical value. For simplicity, we also used \( q = 1 \) for the P V line formation and a smooth velocity field, \( v = 1 - 0.99/r \) (i.e., a ‘\( \beta =1 \)’ law), which gives \( f_{\text{cl}} = f_{\text{vel}}^{-1} \), when calculating the clump optical depths displayed in the figure. Clearly, within these approximations the clumps remain optically thick for P V throughout the entire wind. For H\( \alpha \), the \( \rho^2 \)-dependence makes the optical depth decrease faster with increasing radii, so this line is optically thick only in the lower wind. Based only on these simple estimates, we may therefore expect that resonance lines such as P V should be sensitive to deviations from microclumping in the complete line profile, whereas recombination lines such as H\( \alpha \) should be affected only in the line core. We note also that even if we were to reduce the ion fraction of P V to, say, \( q = 0.1 \), the clumps would still be optically thick. This illustrates the necessity of relaxing the microclumping approximation for typical mass-loss line diagnostics of hot star winds.

\[\begin{array}{c}
\tau_{\text{PV}} \quad \tau_{\text{H}} \\
0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0
\end{array}\]

Figure 1: Clump optical depths for H\( \alpha \) and P V line formation as functions of wind velocity. Stellar and wind parameters as for \( \lambda \) Cep, see text.

**Wind models.** To investigate effects on the line formation from optically thick clumps, a non-void inter-clump medium, and a non-monotonic velocity field, we create (pseudo-)2D and 3D wind models, following the ‘patch method’ of Dessart & Owocki (2002). In this technique, the full wind is

\[\text{\( A \) may be readily modified to handle other recombination lines than H\( \alpha \).}\]
‘patched’ together by assembling inhomogeneous, spherically symmetric, wind-snapshots in radially independent slices. We construct winds using both self-consistent, time-dependent, radiation-hydrodynamic (RH) simulations (computed following Feldmeier, Puls & Pauldrach 1997) and empirical, stochastic models. Line synthesis of resonance lines is carried out using the Monte-Carlo method described in Sundqvist et al. (2010a), whereas for Hα we have developed a new radiative transfer code, in which we solve the ‘formal integral’ within our 3D winds, assuming that the departure coefficients are unaffected by optically thick clumping, which should be reasonable for the O-star winds discussed here (Sundqvist et al. 2010b). Hydrogen departure coefficients, and ionization fractions of P V, are calculated with FASTWIND, accounting for radially dependent microclumping. Stellar rotation is treated by the standard convolution procedure of a constant $v \sin i$ (thus neglecting differential rotation), set to 220 km s$^{-1}$.

When creating our stochastic models, we take an heuristic approach and use a set of parameters to define the structured medium. If clumps are optically thick for the investigated diagnostic, the line formation will generally depend on more structure parameters than just $f_{cl}$. These parameters (for a two component medium) were defined and discussed in Sundqvist et al. (2010a). They are the inter-clump medium density, the physical distances between the clumps, $v_\beta \delta t$, set by the time interval $\delta t$ between the release of two clumps and in our geometry equal to the porosity length (Owocki, Gayley & Shaviv 2004), and finally the velocity filling factor, $f_{vel}$ (see previous paragraph). We stress that these parameters are essential for the radiative transfer in an inhomogeneous medium with optically thick clumps, and not merely ‘ad-hoc parameters’ used in a fitting procedure. We notice also that in our stochastic models they are used to define the structured wind, and so are independent of the origin to the inhomogeneities. This should be distinguished from the RH simulations, in which the structure arises naturally from following the time evolution of the wind and stems directly from the line-driven instability. For these simulations then, the averaged structure parameters are an outcome.

3 Results from an exemplary study of λ Cep

![Figure 2: Left and middle panels: Observed and modeled P V and Hα line profiles in λ Cep. Solid black lines are calculated from empirical, stochastic wind models, and blue dashed ones from RH simulations. The dashed-dotted line in the left panel illustrates corresponding results using the micro-clumping approximation. Observations (red dotted lines) are from Fullerton, et al. (2006) (P V) and Markova et al. (2005) (Hα). Right panel: Corresponding clumping factors.]

We have carried out a combined P V and Hα study, using both RH models and stochastic ones. We find that synthetic spectra computed directly from the RH models are unable to reproduce the diagnostic lines (Fig. 2). Two main problems are identified: i) the absorption toward the blue edge of P V is too deep, and ii) the core emission of Hα is much lower than observed. Note that changing the mass-loss rate for which the RH model is calculated ($\dot{M}=1.5 \times 10^{-6} M_\odot/yr$) does not resolve
these issues, or even alleviate them; if for example a higher rate is adopted, the wings of Hα become much too strong. Consequently, we apply our stochastic models, aiming to empirically capture the essence of the structured wind. By means of these models, we obtain consistent fits essentially by increasing the clumping in the lower wind, but also by adopting somewhat lower velocity spans of the clumps. The second point largely resolves the issue of reproducing the observed P V lines, and was extensively discussed in Sundqvist et al. (2010a). Regarding the first point, the observed Hα absorption trough followed by the steep incline to rather strong emission can only be reproduced by our models if clumping is assumed to start at a velocity only marginally lower than predicted by the RH models (see also Puls et al. 2006; Bouret et al. 2008), but with a much steeper increase with velocity (Fig. 2, right panel). Regarding the outermost wind, RH simulations by Runacres & Owocki (2002), which extend to much larger radii than those used here, indicate that the clumping factor there settles at approximately four. $f_{cl} \approx 4$ is consistent with our derived mass-loss rate (see below) and the constraints from radio emission derived by Puls et al. (2006), suggesting that the outermost wind is better simulated than the inner by current RH models. However, let us point out that the use of the so-called smooth source function formalism in our RH models probably leads to overly damped perturbations in the inner wind (e.g., Owocki & Puls 1999), which might at least partly explain the discrepancies between theoretical clumping factors and those inferred from observations.

The basic expectations from the clump optical depth estimates in Sect. 2.2 are confirmed by our detailed analysis. The P V line profiles are clearly weaker than those calculated using microclumping (Fig. 2, left panel), whereas Hα is affected only in the line core (not shown). Finally, our derived mass-loss rate for λ Cep is the same as the one used in our RH simulations. It is approximately two times lower than predicted by the line-driven wind theory, but a factor of five higher than the corresponding rate derived assuming microclumping. This suggests that only moderate reductions of current mass-loss predictions for OB-stars might be necessary, and illustrates how a correct description of clumping is pivotal for obtaining consistent, reliable estimates of mass-loss rates.

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References

**Discussion**

**S. Owocki:** Regarding your introduction of strong clumping in the inner wind, I should note that most instability simulations so far have been purposely conservative by allowing the instability to be self-excited, i.e. without external perturbations. Moreover, the damping from line-drag is estimated to completely cancel the strong instability near the base. But if this cancellation is not exact, and there is moderately strong net instability, then a model with base perturbations, e.g. from photospheric turbulence, could induce substantial wind clumping quite close to the surface.

**J. Sundqvist:** In our latest analysis we use the instability simulations by Feldmeier, which do include photospheric turbulence as base perturbations. But the clumping in the H\(\alpha\) forming regions is still too low to reproduce the observations. We do use the SSF formulation though. So in principle, I agree with you, and it should be made a high priority to try and develop self-consistent instability models that are able to reproduce the observed spectral features.