Intersections of non quasi-analytic classes of ultradifferentiable functions

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Abstract

As in [8] and [9], we define the intersections $E(\mathfrak{M})(\Omega)$, $D(\mathfrak{M})(K)$ and $D(\mathfrak{M})(\Omega)$ of non quasi-analytic classes by means of a matrix $\mathfrak{M}$. We prove that they differ from classical Beurling classes and that they coincide algebraically with the corresponding intersections of Roumieu classes. We next consider a few elementary properties and give a condition on $\mathfrak{M}$ under which these spaces are nuclear.

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1 Introduction

Intersections of non quasi-analytic classes have first been investigated by Chaumat and Chollet in [3] in the case $M_{j,p} = M_p^{a_j}$ where $(M_p)_{p \in \mathbb{N}_0}$ is a sequence with moderate growth and $(a_j)_{j \in \mathbb{N}}$ a sequence of positive numbers strictly decreasing to 0. They obtained a Whitney extension theorem, a Lojasiewicz theorem on regular situation, some theorem of division and preparation and a Whitney spectral theorem.

Later on Beaugendre studied extensively such intersections in [1] and [2] when the numbers $M_{j,p}$ are defined by means of a convex and increasing function $\Phi$ on $[0, +\infty[$ such that $\lim_{t \to \infty} \Phi(t)/t = \infty$. In particular he obtained extension results for Whitney jets and an explicit continuous linear extension map for Whitney jets.

We considered such intersections for general matrices $\mathfrak{M} = (M_{j,p})_{j \in \mathbb{N}, p \in \mathbb{N}_0}$ and obtained analytic and holomorphic extensions of Whitney jets in [8] and an explicit continuous linear extension map for Whitney jets in [9].

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