Weighted Banach spaces of holomorphic functions on the upper half-plane
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Abstract

In 2001 Bierstedt [1] asked if the weighted space of holomorphic functions $H^v(G)$ on the upper half-plane must have the approximation property under the conditions of Holtmanns [5]. Under these conditions he had shown that $H^v(G)^*$ and $H^v(G)$ are isometrically isomorphic. The problem remains open in general, but in the present paper we give a positive answer for weights with two additional conditions. Actually we can even show the existence of a basis.

1. Introduction

In 1993 Bierstedt, Bonet und Galbis [2] investigated weighted spaces $H^v(G)$ of holomorphic functions for radial weights on balanced domains $G \subset \mathbb{C}^N, N \geq 1$. They showed that $H^v(G)$ has the bounded approximation property and that the polynomials are dense whenever they are contained in $H^v(G)$. For starshaped domains and admissible weights Kahalo and Vogt [6] had already proved the approximation property by a different method. More recently Stanev [11] studied weighted spaces of holomorphic functions on the upper half-plane. He gave a characterization when the spaces are not trivial, and with one of his examples one can construct a weighted space of holomorphic functions with an unbounded weight which has the bounded approximation property, see Example 23 below. In her thesis Holtmanns [5] investigated biduals of weighted spaces of holomorphic functions on the upper half-plane. She presented conditions on the weight $v$ such that $H^v(G)^*$ and $H^v(G)$ are isometrically isomorphic.

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