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STEREOLOGY - A VIEW TOWARDS THE FUTURE

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ABSTRACT

Two directions in which stereology may evolve in the future are suggested. First, the use of ideas of mathematical morphology enables the estimation of functions describing the local properties of a structure. Secondly, the integration of stereology into model building within its user disciplines enables prediction of properties of a diverse range of materials on the basis of their geometrical structure.

Keywords: Stereology, mathematical morphology, modelling.

1. INTRODUCTION

Twenty-five years ago a group of diverse scientists, brought together through the enterprise of Hans Elias, distilled a collection of techniques for interpreting sections and projections which were used in common by their various disciplines. Those pioneers of stereology had a vision of the scope of the new subject in extracting structural information which has not yet been completely fulfilled. Indeed some potential users of stereology seem disappointed when they discover its limitations.

One cause of disappointment is the focus upon global rather than local properties. While useful as a summary, a few global parameters such as V_V and S_V cannot be expected to encapsulate the intricacies of a particular structure. Examples of local properties which have received attention from stereologists are spherical particle size and membrane thickness. Each of these can be recovered in a distributional sense from cross-section data. The techniques of mathematical morphology offer the possibility of stereological estimation of many other facets of local structure. Rather than estimating only parameters, entire functions can be estimated, providing much richer information about a structure.

Although stereology may not have yet reached its full potential, it has been suggested (Ripley, 1981, Chap. 9) that it may be heading towards obsolescence. Certainly, such technological developments as X-ray tomography (Shepp and Kruskal, 1978) and the tandem scanning reflected light microscope (Boyde et al., 1982) enable direct 3-dimensional measurements in some situations.

I would argue that stereology will survive in the era of 3-dimensional image analysis, although its role may tend to be more interpretative and less predictive. Traditionally stereology has been concerned with providing higher dimensional predictions on the basis of lower dimensional information. However it can also play the reverse role of providing a lower dimensional interpretation of a complicated higher dimensional property. In fact integral geometry, one of the parent subjects of stereology, fits better into the latter framework than the former. Software for 3-dimensional image analysis will probably exploit some stereological principles. Finally, stereology has a role to play in model building which is likely to remain important during the next twenty-five years.

2. STEREOLOGY AND MATHEMATICAL MORPHOLOGY

Mathematical morphology, which has developed in parallel with stereology (see Serra, 1982), does focus upon the local properties of images, although not necessarily with the aim of 3-dimensional or statistical interpretation. One of its basic ideas is the use of structuring elements, which can be regarded as local sampling frames which are moved continuously across an image, each position resulting in a value of 0 or 1. This idea falls readily into the framework of stereology.

Stereological formulae are essentially applications of Fubini's theorem in which part of the integration can be expressed as expectation with respect to a lower dimensional sampling scheme. Let F denote a flat (usually a plane) or a grid of parallel flats, and let T denote a structuring element which can be contained in Some F. Let μ and ν be measures of flats and structuring elements respectively; often these measures are motion invariant but not necessarily. Finally, μ_T and ν_F denote measures of flats containing T, and of structuring elements contained in T.

It will be assumed (after appropriate normalisation if necessary) that the above measures satisfy the Fubini-type relationship

$$\nu_F(dT)\,\mu(dF) = \mu_T(dF)\,\nu(dT) \tag{1}$$

and also that the total integral

$$\int \mu_T(dF)$$

is constant for all T.

Let X be a bounded, deterministic specimen set containing a feature set Y, and let f be a measurable function assigning 0 or 1 to each intersection $Y \cap T$. By integrating f with respect to each side of (1), we obtain

$$\int \{\int f(Y \cap T)\ \nu_F\ (dT)\}\ \mu(dF) = \int \mu_T(dF) . \int f(Y \cap T)\,\nu(dT) \tag{2}$$

If $f(\phi)$ is defined as 0, there is no need to explicitly specify the ranges of integration in (2). If F is generated randomly with measure proportional to μ, restricted to flats intersecting X, then (2) can be expressed as

$$E(\theta_F) = \theta/c(X) \tag{3}$$

where
$$\theta_F = \int f(Y \cap T)\ \nu_F(dT)$$
$$\theta = \int f(Y \cap T)\ \nu\ (dT)$$

and
$$c(X) = \int_{F \cap X \neq \phi} \mu(dF) / \int \mu_T(dF)$$

In other words, the integral involving positions of the structuring element within a flat is proportional on average to the corresponding spatial integral. The constant of proportionality depends on the size and shape of the specimen.

A simple illustrative example is the case when T is a single point governed by uniform measure, F is a plane governed by motion invariant measure and f is the indicator function of Y. Equation (3) reduces to the familiar form

$$E(A(Y \cap F)) = V(Y)/M(X)$$

where A, V and M denote area, volume and mean caliper diameter respectively.

A more interesting example is constructed by letting T be a pair of points separated by a fixed distance r. The measure ν is specified by locating the first point according to surface measure over the boundary ∂Y, and then drawing a line through this point with density $\sin\theta$ relative to isotropic measure, where θ is the angle between the line and the tangent plane to ∂Y (assumed piecewise differentiable). The second point is located on the line at distance r from the first point, in the outwards direction relative to Y. Once again F is a plane and μ is isotropic uniform measure. In order to satisfy equation (1), ν_F is constructed by locating the first point according to perimeter measure on $\partial Y \cap F$, and the second point at distance r from the first in the outwards direction along a line through the first point and contained in F. The orientation of this line has density $\sin\theta$ with respect to isotropic measure in the plane. The function f indicates whether or not the second point belongs to Y.

In order to interpret the resulting version of equation (3), first note that

$$\int \nu(dT) = \pi S(\partial Y)$$

and
$$\int \nu_F(dT) = 2\ B(\partial Y \cap F)$$

Let us define the conditional volume fractions, which describe the structure at a distance of r from the feature boundary:

$$p(r \mid \partial Y) = \int f(Y \cap T)\nu(dT)/\pi S(\partial Y)$$

and
$$p_F(r \mid \partial Y \cap F) = \int f(\partial Y \cap T)\ \nu_F(dT)/2\ B(\partial Y \cap F).$$

Then (3) can be written as

$$2\ E\{B(\partial Y \cap F)\ p_{F'}(r \mid \partial Y \cap F)\} = \pi S(\partial Y)\ p\ (r \mid \partial Y)/2M(X). \qquad (4)$$

Equation (4) is similar in form to the classical stereological formula

$$E\{B(\partial Y \cap F)\} = \Pi S(\partial Y)/4M(X)$$

but deals with the second order properties of the structure. So far r has been kept fixed, but by allowing it to vary (4) can be seen as a method of estimating a function.

By taking F to be an isotropic uniform random line rather than a plane, the analogue of (4) is

$$E\{N(\partial Y \cap F)\ p_F\ (r \mid \partial Y \cap F)\} = S(\partial Y)p(r \mid \partial Y)/2\ M_2(X) \quad (5)$$

where N denotes number of boundary intercepts and M_2 denotes mean areal projection. In this case $p_F(r \mid \partial Y \cap F)$ is simply the proportion of intercepts for which the point at distance r in a direction away from Y also belongs to Y. Equations (4) or (5) can be used in the usual way to provide consistent ratio estimates of $p(r \mid \partial Y)$.

In the preceding example, μ was motion invariant but ν was not. Other examples can be constructed in which μ is not invariant: for example equations can be derived for vertical sections (Baddeley, 1984) or area-weighted sections. Also T could be a triple of points, or even more complicated. Equation (3) therefore encompasses a very broad class of stereological formulae. The matter of which of these are likely to be useful impinges upon the next section.

3. STEREOLOGICAL MODELLING

Although traditionally associated with images of real structures, stereological principles also have important implications for the construction and analysis of models, which exist only in the abstract. It could be argued that the technique of dimensional reduction in model construction predates stereology - mathematicians have for a long time fallen into a pattern of first solving a one-dimensional problem and then extending the conclusions to higher dimensions. Contemporary stereology offers a formalisation of this process and provides insight into some of the more subtle relationships between lower and higher dimensions.

Twenty-five years ago there was a need for stereology to detach itself from its user disciplines in order to be clarified and generalised. Having developed its own identity, there is now a need for stereology to become integrated into the quantitative modelling taking place within the various sciences which first gave rise to it. In the past applied stereology papers have exhibited a tendency to neglect the details of how the estimated parameters are to be used in subsequent analysis, concentrating instead on overcoming experimental difficulties associated with the application of stereological techniques. The divorce of stereology from subsequent modelling can lead to erroneous results. It is a well-known phenomenon in statistics that substitution of a mean value into a non-linear function does not in general yield the correct mean of the function. In the context of modelling the properties of a spatial structure, this problem can be circumvented by the two following methods. Either the stereological parameters can be redefined so as to vary linearly with the output of the model, or the variability inherent in the real structure can be built into the model.

An example of the former approach is Weibel's (1970) model for the estimation of the diffusion capacity of a lung. Although the arithmetic mean thickness of tissue barrier is an appropriate quantity to use in estimating tissue mass, the harmonic mean thickness is appropriate for estimating diffusion capacity. Both types of mean thickness can be estimated simply from cross-sections; the implicit model consists of planar barriers distributed in space.

The best known class of models following the second approach is probably that of random sphere models, with centres distributed according to a marked point process and radii specified by the marks. Mecke and Stoyan (1980) give a formal account of the underlying mathematical model, and numerous authors have addressed the numerical and statistical problems involved in inverting the integral equations which arise for the radius distribution. Authors are often vague as to their ultimate use of an estimated size distribution: there is a danger that the particular functional(s) of interest may be quite sensitive to the inversion procedure adopted.

An understanding of the relationship between the geometrical structure of a material and its physical or physiological properties indicates which stereological parameters or functions should be estimated. For example, in modelling the liberation of composite mineral ores during crushing and grinding (Davy, 1984), certain functions emerge as relevant summaries of particle size and shape, of the second order properties of the ore structure, and of the interaction between the fracturing process and the ore structure. In addition to determining appropriate structural summaries, the integration of stereology into model-building can in some situations result in the simplification of the model to a more tractable form. An example is given by Davy and Guild (1986, submitted for publication) in which stereological principles are used in constructing a finite element model to predict the stress and elastic properties of a composite material. The important structural property in this case turns out to be the distribution of interparticle distance, which is defined in terms of a Voronoi tessellation.

The rapid development of image analysis technology has created a need for models wich can usefully interpret the large volume of available data. Stereologists have a challenge to meet this need, but must be prepared to combine their theory with other branches of science and mathematics.

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